ϕ^4理论：计算 m, λ,

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2}_{\text{Gaussian}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{perturbation}}$$

1) propagator of $\mathcal{L}_0$

2) operator $O \Leftrightarrow \langle O\rangle_S = \dfrac{\int D\phi\, e^{iS} O}{\int D\phi\, e^{iS}}$

3) $\int D\phi\, e^{iS}\phi(x_1)\cdots\phi(x_e)$

① Cumulent expansion $Z = \langle e^{tX}\rangle = e^{\Omega}$

Z：特征函数/配分函数　　Ω：自由能

$X = \frac{\lambda}{4!}\int\phi^4 dx$　，X可以为变量也可以是场

$\Omega = \sum_{r=1}^{+\infty}\frac{t^r}{r!}k_r$ (明确意义) $\langle X^4\rangle = k_1^4 + 6k_2^2 + 4k_1k_3 + k_4$

$$\langle O\rangle_H = \frac{\int D\phi\, e^{-H_0-u} O}{\int D\phi\, e^{-H_0-u}} = \sum_{n=0}^{+\infty}\frac{(-1)^n}{n!}\langle O u^n\rangle_0^c$$

$$= \frac{\int D\phi\, e^{iS_0+iS_I} O}{\int D\phi\, e^{iS_0+iS_I}} = \sum_{n=0}^{+\infty}\frac{i^n}{n!}\langle O S_I^n\rangle_0^c$$

解决问题：① 分母问题 ⇒ 扣除无贡献的相互作用

② 平均 $\langle O\rangle_S \longrightarrow \langle O\rangle_{S_0} = \langle O\rangle_0$　转化为Gaussian积分

③ $\langle O u^n\rangle_0^c \Leftarrow$ connected diag 连接图　剩下的就是

= Cumulent展开

$$\langle\phi(x)\phi(y)\rangle = \frac{1}{V^2}\sum_{k_1k_2} e^{ik_1x}e^{ik_2y}\langle\phi(k_1)\phi(k_2)\rangle$$

有贡献的唯一的一项 $k_2 = -k_1$

$$= \frac{1}{V^2}\sum_k\langle\phi^*(k)\phi(k)\rangle = \frac{1}{V^2}\frac{1}{(i\lambda_k)} = \frac{1}{V^2}\frac{1}{-i(k^2-m^2)/V} = \frac{i/V}{k^2-m^2}$$

$$S_0 = \frac{1}{V}\sum_{k_0>0}(k^2-m^2)\phi^*(k)\phi(k)$$

$$= \sum_k \lambda_k\phi^*(k)\phi(k)$$

T 某些书中会引入 T，时序算子不讨论.

$g_0(x-y)=\langle\phi(x)\phi(y)\rangle=\frac{1}{V}\sum_k\frac{ie^{iq(x-y)}}{q^2-m^2}$

$\langle\phi(x)\phi(y)\rangle=\frac{\int D\phi\,\phi(x)\phi(y)e^{iS_0+iS_I}}{\int D\phi\, e^{iS_0+iS_I}}$

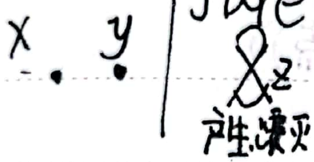
表示 $\langle O\rangle=\phi(x)\phi(y)=\sum_{n=0}^{\infty}\frac{i^n}{n!}\langle OS_I^n\rangle_0^c$

$S_I=-\frac{\lambda}{4!}\int\phi^4(z)\,dz$

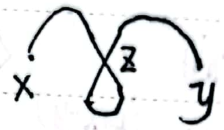
$\langle OS_I\rangle_0$ 与 $\langle OS_I\rangle_0^c$ 区别：

$\frac{\int D\phi\,\phi(x)\phi(y)S_I e^{iS_0}}{\int D\phi\, e^{iS_0}}$

$=\frac{-\lambda}{4!}\int dz\frac{\int D\phi(x)\phi(y)\phi^4(z)e^{iS_0}}{\int D\phi\, e^{iS_0}}$

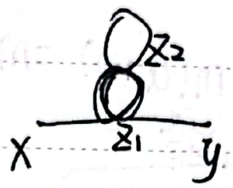


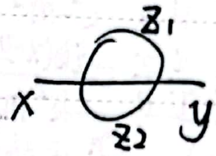
非连接图



连接图

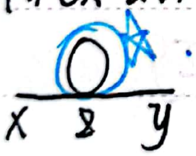
例：$\langle OS_I^2\rangle_0^c$





$\langle O\rangle=\langle\phi(x)\phi(y)\rangle_0+i\left(\frac{-\lambda}{4!}\right)\int\langle\phi(x)\phi(y)\phi^4(z)\rangle_0^c\,dz.$

一阶项只有一个图，与 2d $\delta(\vec{x})$ potential 联系起来.



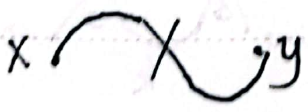
$\langle\phi(x)\phi(y)\phi^4(z)\rangle$

$=\langle\phi(x)\phi(z)\rangle_0\langle\phi(y)\phi(z)\rangle_0\langle\phi(z)\phi(z)\rangle_0$

$\begin{cases}\langle\phi(x)\phi(y)\rangle=\frac{1}{V}\sum_k\frac{ie^{ik(x-y)}}{k^2-m^2}\\ \langle\phi(z)\phi(z)\rangle=\frac{1}{V}\sum_k\left(\frac{i}{k^2-m^2}\right)\end{cases}$

$\begin{cases}g(x-y)=\langle\phi(x)\phi(y)\rangle_S\\ g_0(x-y)=\langle\phi(x)\phi(y)\rangle_{S_0}\end{cases}$

$g(x-y)=g_0(x-y)-\frac{i\lambda}{2}\cdot g_0(x-z)g_0(z-z)g_0(y-z)$



∞ 发散

$\boxed{\frac{1}{V}\sum_k\frac{i}{k^2-m^2}}$ $d=4$

抬高奇点

$=\frac{1}{V}\cdot\frac{V}{(2\pi)^d}\int dk_0\,d\vec{k}\cdot\frac{i}{k_0^2-\vec{k}^2-m^2+i\epsilon}$

$g\ \boxed{\frac{1}{V}\sum_k\frac{1}{E-\frac{\hbar^2k^2}{2m}}=1}$ $d=2$

$\begin{cases}k=(k_0,\vec{k})\\ k^2=k^{\mu}k_{\mu}=k_0^2-\vec{k}^2\end{cases}$

$\boxed{x^2 = m^2 - i\varepsilon}$

考虑：$\int \frac{1}{x^2-m^2}dx$

法一：$\int \frac{1}{x^2-m^2+i\varepsilon}dx$

法二：令 $x = iy$，
$$= i\int \frac{dy}{-y^2-m^2}$$
$$= -i\int \frac{dy}{y^2+m^2}$$

$$\frac{1}{V}\sum_k \frac{i}{k^2-m^2}$$

$$= \frac{1}{V}\cdot\frac{V}{(2\pi)^d}\int dk_0 d\vec{k}\,\frac{i}{k_0^2-\vec{k}^2-m^2+i\varepsilon}$$

$k_0 \to ik_0$ wick rotation

由格林函数积分解析延拓到虚轴

$$= \frac{i^2}{(2\pi)^d}\int dk_0 d\vec{k}\,\frac{1}{-k_0^2-\vec{k}^2-m^2}$$

$$= \frac{1}{(2\pi)^d}\int dk_E \frac{1}{k_E^2+m^2} = \frac{1}{(2\pi)^d}\Omega_d\int_0^{+\infty}\frac{k_E^{d-1}dk_E}{k_E^2+m^2} \to \infty$$

卷积：动量空间 $\langle\phi(\text{首端点})\phi(\text{尾端点})\rangle$

$$\begin{cases} g(x-y) = \frac{i}{V}\sum_k \frac{e^{ik(x-y)}}{k^2-m^2} \\ g_0(x-y) = \frac{i}{V}\sum_k \frac{e^{ik(x-y)}}{k^2-m_0^2} \end{cases}$$

$m(\lambda=0) = m_0$

小量

$$\boxed{g(k) = g_0(k) - \frac{i\lambda}{2}g_0(k)g_0(0)g_0(k)}$$

Dyson eq：$g = g_0 + g_0\Sigma g = g_0 + g_0\Sigma g_0 + g_0\Sigma g_0\Sigma g_0 + \cdots$

$$\begin{cases} g = \frac{1}{k^2-m^2-\Sigma} \\ g_0 = \frac{1}{k^2-m^2} \end{cases}$$

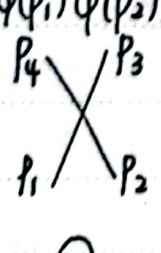
$g^{-1} - g_0^{-1} = k^2 - m^2 - \Sigma - (k^2-m^2) = -\Sigma$

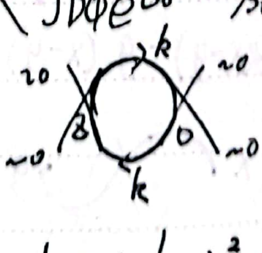
$1 - g_0^{-1}g = -\Sigma g$ $\quad g_0 - g = -g_0\Sigma g$

$\to m^2 = m_0^2 + \Sigma$ ，$g = g_0 + g_0\Sigma g$

$O = \phi^4(x)$ or $= \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)$

$$\langle\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\rangle_S = \langle O\rangle + i\left(-\frac{\lambda}{4!}\right)\int dz\left\langle\frac{\int D\phi\, e^{iS_0}\phi^4(z)O}{\int D\phi\, e^{iS_0}}\right\rangle^C_{S_0}$$





$$\sim \frac{1}{V}\sum_k\left(\frac{1}{k^2-m^2}\right)^2$$

$$d=4 \to \left(\frac{1}{2\pi}\right)^d i\int dk_E \frac{1}{(k_E^2+m^2)^2} \propto \frac{k_E^3 dk}{(k_E^2+m^2)^2} \sim \int\frac{dk_E}{k_E} \sim \ln\Lambda$$

In

$\mathcal{L}$

重整

观

重整

$\int dk^d$

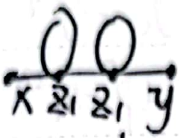
$\int \frac{d^d}{(2\pi}$

eg: 发散

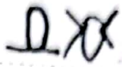
没有讲的东西

1) T算子

2) Peskin书数学技巧



$q \quad \propto \lambda^2 \underbrace{\left(\frac{1}{V}\sum_{k'}\frac{1}{k'^2-m^2}\right)}_{\propto \Lambda^2} \underbrace{\frac{1}{V}\sum_{k}\left(\frac{1}{k^2-m^2}\right)^2}_{\propto \ln\Lambda}$


$\begin{cases} \Lambda \text{ 能量截断} \\ \text{维度正规化} \\ \text{Feynman参数化} \end{cases}$

3) 流守恒

$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2+\lambda x^4$.

$|n\rangle \to$ perturbation

$E_n=\hbar\omega(n+\frac{1}{2})+\langle n|\lambda x^4|n\rangle+\cdots$

$H_0=\frac{p^2}{2m_r}+\frac{1}{2}m\omega_r^2x^2$ $\quad V=\frac{1}{2}m\delta\omega^2x^2+\lambda x^4+\frac{p^2}{2\mu}$ $\quad (\delta\omega^2=\omega^2-\omega_r^2)$

renormalized

$|n\rangle_r=$ $\quad E_n^r=\hbar\omega_c(n+1)+\langle n|V|n\rangle_r+\cdots$ $=0$.

$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{m_0^2}{2}\phi^2-\frac{\lambda_0}{4!}\phi^4$, $m_0\ \lambda_0$ at Λ

$=\mathcal{L}_R(m_R,\lambda_R)+\Delta\mathcal{L}(m_0,\lambda_0,m_R,\lambda_R)$ $\quad \infty-\infty=$ finite

重整化 $\quad$ 观测值 $\quad$ 用来抵消发散.

观测值 $\mu \Rightarrow \Big|^{m_R}_{\lambda_R}$, $\mu \ll \Lambda$

重整化 $\phi=\sqrt{Z}\phi_R$ $=\underbrace{\boxed{\frac{1}{2}(\partial_\mu\phi_R)^2-\frac{m_R^2}{2}\phi_R^2}}$ (观测值) $\frac{\lambda_R}{4!}\phi_R^4+\frac{1}{2}\delta_Z(\partial_\mu\phi_R)^2-\frac{\delta m^2}{2}\phi_R^2-\frac{\delta\lambda}{4!}\phi_R^4$ 微扰

 $\sim\int d\vec{k}\,\frac{1}{ck-k^2-m^2}\cdot\frac{1}{k^2-m^2}$

$\int d^dk\frac{1}{(k^2+m^2)^2}\sim\int_0^\infty dk\frac{k^{d-1}\epsilon}{(k^2+m^2)^2}$ $\swarrow$ $\int_0^\Lambda\frac{dk\,k^{d-1}}{(k^2+m^2)^2}\sim\int\frac{dk}{k}\propto\ln k$

$\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2}=\frac{\Gamma(1-\frac{d}{2})}{(4\pi)^{\frac{d}{2}}\cdot m^{2-d}}$

$\int^\Lambda\frac{dk_E}{k_E}\sim\ln\Lambda$ $\quad$ eg: $\int\frac{d^4k}{(2\pi)^d}\frac{i}{k^2-m^2}\sim\int_0^\Lambda\frac{dk\,k^3}{k^2+m^2}\sim\int dk\cdot k\sim\Lambda^2$