$$= \sum_{q,q'} e^{iq\cdot x + iq'y} \frac{1}{a} \delta(q,q').$$

$$\left(a = \frac{Q_k}{V} \right).$$

$$= \frac{1}{V} \sum_{q} e^{iq(x-y)} \frac{1}{Q_k}$$

2021.3.10 第三周第二节课.

parseval formula.

$$f(x) = \frac{a_0}{\sqrt{2}} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$\langle f, f \rangle = \int f^2(x)\, dx$$

or $\int f^2(x)\, dx = ? \int f^2(k)\, dk.$

今天的内容: 1. Green's function.

Propogator. $D(x-y) = \frac{1}{V} \sum_{k} (\quad)$ 成对出现.

图表示.

2. Interaction.

Cumulant Expansion (Moment Expansion).

物理: linked cluster expansion.

意义.

$$\phi(x) = \frac{1}{V} \sum_{k} e^{ik\cdot x} \phi(k)$$, $k = (k_0, \vec{k})$. 对应之前的 ω_k. $x = (t, -\vec{x})$.

$$S_0 = \int \mathcal{L}_0\, dx$$

$$\int \frac{1}{2} \left(\frac{\partial\phi}{\partial t}\right)^2 dx = \frac{1}{2V^2} \sum_{k,k'} \phi(k)\phi(k') (ik_0)(ik_0') \int e^{i(k+k')\cdot x}\, dx$$

$$= \frac{1}{2V^2} \sum_{k,k'} \phi(k)\phi(k')(ik_0)(ik_0')\, \delta(k+k') \cdot (2\pi)^d$$

$$= \frac{1}{2V^2} \sum_{k} \phi(k) \frac{V}{(2\pi)^d} \int \phi(k')(ik_0)(ik_0')\, \delta(k+k')\, dk'$$

要求 $k' = -k$. $= + \frac{1}{2V} \frac{1}{(2\pi)^d} \sum_{k} \phi(k)\, \phi(-k)\, k_0^2$

$\Rightarrow k_0' = -k_0;$

$$= + \frac{1}{V} \frac{1}{(2\pi)^d} \sum_{k\geq 0} \phi(k)\, \phi^*(k)\, k_0^2$$

对应关系(类比).

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2$$

$$= \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial t}\right)^2 - (\nabla \cdot \phi)^2 \right] - \frac{m^2}{2}\phi^2$$

$$S_0 = \int \mathcal{L}_0\, dx \xrightarrow{\text{Parseval}} \frac{1}{V} \cdot \frac{1}{(2\pi)^d} \int_{k_0 \geq 0} \phi^*(k)\, \phi(k)\, (k_0^2 - \vec{k}^2 - m^2)$$ (时序).

类比 $\rightarrow$ $$\int D\phi\, e^{iS[\phi]} = \int D\phi_k\, D\phi_k^*\, e^{\frac{i}{V}\sum_k (k_0^2 - \vec{k}^2 - m^2)\phi_k^* \phi_k}$$

$\Leftrightarrow \int dx\, P(x).$

求 $\langle \phi(x)\phi(y) \rangle$

$$= \frac{1}{V^2} \sum_{q,q'} \langle \phi(q)\phi(q') \rangle e^{i(q\cdot x + q'\cdot y)}$$

$$\langle x \rangle = \frac{\int x p(x)\, dx}{\int p(x)\, dx}$$

$$\langle \phi(x)\phi(y) \rangle = \frac{1}{V^2} \sum_{q,q'} e^{i(q\cdot x + q'\cdot y)} \frac{\int D\phi_k^* D\phi_k\, e^{\frac{i}{V}\sum_k (k_0^2 - \vec{k}^2 - m^2)\phi_k^* \phi_k} \cdot \phi_q \phi_{q'}}{\int D\phi_k^* D\phi_k\, e^{\frac{i}{V}(\cdots)\phi_k^* \phi_k}}$$

$$\int_{k\neq q} D\phi_k^* D\phi_k \int D\phi_q^* D\phi_q\, e^{iS_0}\, \phi_q^* \phi_q$$

$$\int_{k\neq q} D\phi_k^* D\phi_k \int D\phi_q^* D\phi_q\, e^{iS_0}$$

$$\Downarrow$$

特点: 1) 表达式复杂;

2) 对角的. Gauss型的积分: (复形式).

$$= \sum_q \frac{-i}{q_0^2 - \vec{q}^2 - m^2} e^{iq\cdot(x+y)} \cdot \frac{1}{(2\pi)^d};$$

$$\frac{1}{A} = \frac{V}{i(q_0^2 - \vec{q}^2 - m^2)}$$

$= D(x-y)$. or $G(x-y)$. Propogator.

符号 $\left\{ \begin{array}{l} x \longmapsto y \\ x \text{———} y. \end{array} \right.$

$$\delta(\vec{x}) = \frac{1}{V}\int e^{ik\cdot x}dx$$

$L(x)e^{ik\cdot x} \Rightarrow L(k)e^{ik\cdot x}$

$$G(x-y) = \frac{1}{V}\sum_k e^{ik\cdot x} G(k)$$

$$\frac{1}{V}\sum_k G(k) L e^{ik\cdot x} = \frac{1}{V}\sum_k e^{ik\cdot x}$$

$$G(k) = 1/L(k)$$

$$G(x-y) = \sum_q \underbrace{\frac{1}{L(q)}}_{\text{Lagrange.}} \cdot \underbrace{i}_{iS_0} \cdot \underbrace{e^{iq(x-y)}}_{\text{FT}}$$

$$L_I = L_0 + \frac{\lambda}{4!}\phi^4.$$

① 简化形式.
② 图表示.

补充：形式理论.

$$\int D\phi\, e^{iS_0 + \int J\phi} \Leftrightarrow \int dx\, e^{-\frac{A}{2}x^2 + Jx}$$

源

矩和累积量：

$$\langle e^{tx}\rangle = \frac{\int p(x)e^{tx}dx}{\int p(x)dx} = \sum_{n=0}^{+\infty}\frac{t^n}{n!}\langle x^n\rangle$$

moment. 矩.

缺点，$\langle x^n\rangle$ 会包含一阶、二阶 …所有的信息.

$$= e^{\Omega}, \quad \Omega = \sum_{r=1}^{+\infty}\mu_r\frac{t^r}{r!}$$

$$\langle x^n\rangle = \int x^n p(x)\,dx = m_n$$

$$Z = \langle e^{tx}\rangle = \sum_{n=0}^{+\infty}\frac{t^n}{n!}m_n = e^{K} = e^{\sum_{r=1}^{+\infty}k_r\frac{t^r}{r!}}$$

除去低阶关联，只保留3阶.

C: Connected.

$$k_1 = m_1$$

$$k_2 = m_2 - m_1^2$$

$$\langle x^3\rangle_c = k_3 = m_3 - 3m_2m_1 + 2m_1^3 = \langle x^3\rangle - 3\langle x^2\rangle\langle x\rangle + 2\langle x^3\rangle$$

一个抽象的证明:

Ref: Methods of QFT in Condensed Matter. Abrikosov.

累积量展开的物理意义: 在传播路径中, 周围的物理与传播无关.

$$\langle O\rangle = \frac{\int D\phi\; O\, e^{-H_0-U}}{\int D\phi\; e^{\overset{iS_0}{-H_0}\overset{iS_I}{-U}}}$$

operator

$$= \frac{\langle O e^{-U}\rangle_0}{\langle e^{-U}\rangle_0}$$

用分母来抵消掉周围无相互作用项的影响;

$$= \sum_{n=1}^{+\infty}\frac{(-1)^n}{n!}\langle O e^{-U}\rangle^{c}_{H_0}$$

连接图;

2022.3.14. 第4周. 第1节课.

目标: ϕ^4 理论. 计算 m. λ.

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{Gaussian}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{Perturbation}}.$$

1) Propogator. of $\mathcal{L}_0$

2) Operator. $\hat{O}$. $\quad \langle\hat{O}\rangle = \dfrac{\int D\phi\, e^{iS}\, O}{\int D\phi\, e^{iS}}$

3) $\int D\phi\, e^{iS} d\phi(x_1)\cdots d\phi(x_\ell)$.

上节课回顾:

① Cumulant expansion:

$$Z = \langle e^{tX}\rangle = e^{\Omega}$$

Z: 特征函数 / 配分函数.