① Jordon-wigner string $e^{i\pi\int_{-\infty}^{x}\rho(x')dx'}$

$$\phi=\pi\int_{-\infty}^{x}\rho(x')dx' \Leftrightarrow \boxed{\frac{\partial\phi}{\partial x}=\pi\rho(x)}$$

$\boxed{\psi=\frac{F}{\sqrt{2\pi a}}e^{i\Phi}}$ 考虑 $\begin{cases}F^\dagger=F\\ F^2=1\end{cases}$ $\psi^\dagger=\frac{F}{\sqrt{2\pi a}}e^{-i\Phi}$

$\psi^\dagger\psi=\frac{F^2}{2\pi a}e^{-i\Phi(x)}e^{i\Phi(x)}=\frac{1}{2\pi a}$ ∵ $a\to0$ ∴ $\rho\to\infty$ 发散

测到的是大背景. 于是

定义 $\rho=\lim_{y\to0}\psi^\dagger(x+y)\psi(x)=\frac{F^2}{2\pi a}e^{-i\Phi(x+y)}e^{i\Phi(x)}$ $(y\gg a)$

$$=\frac{1}{2\pi a}e^{-i\Phi(x+y)+i\Phi(x)+\frac{1}{2}[\Phi(x+y),\Phi(x)]}$$

$$=\frac{1}{2\pi a}e^{\frac{1}{2}[\Phi(x+y),\Phi(x)]}e^{-i(\frac{\partial\Phi}{\partial x})y}$$

$$=\frac{1}{2\pi a}e^{\frac{1}{2}[\Phi(x+y),\Phi(x)]}(1-iy\frac{\partial\Phi}{\partial x})$$

$$=\underbrace{\text{const}}_{发散}-\frac{iy}{2\pi a}e^{\frac{1}{2}[\phi(x+y)-\Phi(x)]}(\frac{\partial\Phi}{\partial x})$$

$$=\infty+\frac{1}{2\pi}(\frac{\partial\Phi}{\partial x})$$

$:\psi^\dagger(x+y)\psi(x):=A-\langle0|A|0\rangle$ ∵ $\langle0|\Phi|0\rangle=0$

$$=\frac{1}{2\pi}(\frac{\partial\Phi}{\partial x}).$$

字典: ① $\rho\to\rho=:\psi^\dagger\psi:\to\frac{\partial\Phi}{\partial x}$
0次导 → 1次导

直观 ② Luttinger liquid: $(\psi^\dagger\partial\psi)\longrightarrow(\partial_x\Phi)^2$
1次导 ⟶ 2次形(声子).

证明②: $\lim_{y\to0}\psi^\dagger(x+y)\psi(x)=\text{const}+(\frac{\partial\Phi}{\partial x})^2$

③

配对 pairing

背散射 $k_F\to-k_F$

$V\cos(2k_Fx)\psi^\dagger(x)\psi(x)$

$\psi(x)=\underbrace{e^{ik_Fx}}_{fast}\underbrace{\psi_R}_{slow}+e^{-ik_Fx}\psi_L$

slow: 可作梯度展开

$V\cos(2k_Fx)\,(e^{-ik_Fx}\psi_R^\dagger + e^{ik_Fx}\psi_L^\dagger)(e^{ik_Fx}\psi_R + e^{-ik_Fx}\psi_L)$

无贡献: $\psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L$ trivial

有贡献: $V\cos(2k_Fx)e^{-2ik_Fx}\psi_R^\dagger\psi_L + h.c. = \frac{1}{2}(\psi_R^\dagger\psi_L + h.c.)$

$\frac{1}{2}(1+e^{-i4k_Fx})$

$= \frac{F_LF_R}{2\pi a}e^{-i\Phi_L}e^{-i\Phi_R} + h.c.$

$V\cos(2k_Fx) \overset{打开}{\Longleftrightarrow}$ gap 对应 S.G model 中 A很大.

$= \frac{F_LF_R}{2\pi a}e^{-i\Phi_L+i\Phi_R} - \frac{F_LF_R}{2\pi a}e^{i\Phi_L-i\Phi_R}$ $\quad e^{i\theta}+e^{-i\theta}=2\cos\theta$

$(iF_LF_R)^2$

$= i^2F_LF_RF_LF_R$

$= (-1)(-1)F_L^2F_R^2$

$= 1$

$= \frac{iF_LF_R}{\pi a}\sin(\Phi_L-\Phi_R)$

$= \frac{1}{\pi a}\sin(\Phi_L-\Phi_R)$

$\psi_R^\dagger\psi_L^\dagger + h.c. = \frac{F_RF_L}{2\pi a}e^{-i\Phi_R-i\Phi_L} - \frac{F_RF_L}{2\pi a}e^{i\Phi_R+i\Phi_L}$

$= \frac{iF_RF_L}{\pi a}\sin(\Phi_R+\Phi_L)$

$= \frac{1}{\pi a}\sin(\Phi_R+\Phi_L)$

④ 考虑自旋 $\psi_\uparrow^\dagger\psi_\downarrow + h.c. \to (e^{-ik_Fx}\psi_{R\uparrow}^\dagger + e^{ik_Fx}\psi_{L\uparrow}^\dagger)(e^{ik_Fx}\psi_{R\downarrow} + e^{-ik_Fx}\psi_{L\downarrow})$

IF $k_F^\uparrow = k_F^\downarrow$

扔 $\to \psi_{R\uparrow}^\dagger\psi_{R\downarrow} + \psi_{L\uparrow}^\dagger\psi_{L\downarrow} + h.c.$

$= \frac{F_{R\uparrow}F_{R\downarrow}}{2\pi a}e^{-i\Phi_{R\uparrow}+i\Phi_{R\downarrow}} + h.c. + \frac{F_{L\uparrow}F_{L\downarrow}}{2\pi a}e^{-i\Phi_{L\uparrow}+i\Phi_{L\downarrow}} +$

$\propto \frac{iFF}{\pi a}\sin(\quad) + (\quad)$.

⑤ 多体相互作用:

1) ρ·ρ 相互作用

2) normal ordering.

slow fast

$\rho = \psi^\dagger\psi = \psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L + e^{ik_Fx}\psi_L^\dagger\psi_R$
$+ e^{-ik_Fx}\psi_R^\dagger\psi_L$

fast.

相互作用：① $n_i n_{i+1}$（无spin粒子）$\xleftrightarrow{JW}$ XXZ model

② $n_{i\uparrow} n_{i\downarrow}$（二分量）

共同特点 $\rho\cdot\rho$ 　　　$m \times m$

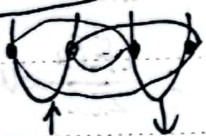
$\int \sin(Qx)\cdot dx = 0$ 但 $\int \sin^2(Qx)\cdot dx \neq 0$.

$(s+f)^2 = s\cdot s + f\cdot f + s\cdot f + f\cdot s$

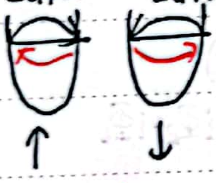
$\rho = \partial_x\phi_R + \partial_x\phi_L + e^{-2ik_F}\psi_L^\dagger\psi_R + e^{2ik_F}\psi_R^\dagger\psi_L$

两个粒子在同一地方

$\rho_\uparrow\rho_\downarrow = (\partial_x\phi_R^\uparrow + \partial_x\phi_L^\uparrow)(\partial_x\phi_R^\downarrow + \partial_x\phi_L^\downarrow) + (e^{-2ik_Fx}\psi_{L\uparrow}^\dagger\psi_{R\uparrow} + e^{2ik_Fx}\psi_{R\uparrow}^\dagger\psi_{L\uparrow})(e^{-2ik_Fx}\psi_{L\downarrow}^\dagger\psi_{R\downarrow} + e^{2ik_Fx}\psi_{R\downarrow}^\dagger\psi_{L\downarrow})$

明确物理意义：①  密度·密度相互作用 trivial

② $2ik_Fx$ 　$-2ik_Fx$



↑ 　↓ 　　$\pm 4ik_Fx$

$\psi_{R\uparrow}^\dagger\psi_{L\uparrow}\psi_{L\downarrow}^\dagger\psi_{R\downarrow}$

$$= \frac{F_{R\uparrow}\cdot F_{L\uparrow}\cdot F_{L\downarrow}\cdot F_{R\downarrow}}{(2\pi a)^2} e^{-i\Phi_{R\uparrow}+i\Phi_{L\uparrow}+i\Phi_{L\downarrow}-i\Phi_{R\downarrow}} + h.c.$$

$(F_1F_2F_3F_4)^\dagger = F_4F_3F_2F_1$
$= -F_3F_2F_1F_4$
$= F_1F_2F_3F_4$

$$= \frac{F_{R\uparrow}\cdot F_{L\uparrow}\cdot F_{L\downarrow}\cdot F_{R\downarrow}}{4\pi^2a^2}(e^{i\delta\Phi}+e^{-i\delta\Phi})$$

$$= \frac{F\cdot F\cdot F\cdot F}{2\pi^2a^2}\cos(\delta\Phi)$$

$$\approx \frac{1}{2\pi^2a^2}\cos(\delta\Phi)$$

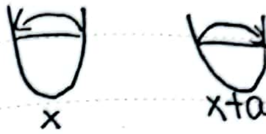
Ref: shanker chap 17.18

⑥. $\sum_i n_i n_{i+1}$

$= \sum_i c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}$, $c_i = \sqrt{a}\,\psi(x-ia)$

$= a\int \psi^\dagger(x)\psi(x)\psi^\dagger(x+a)\psi(x+a)$

由
$$\begin{cases}\partial\phi_R(x) + \partial\phi_L(x) + e^{-2ik_Fx}\psi_R^\dagger(x)\psi_L(x) + h.c. = \rho(x)\\ \partial\phi_R(x+a) + \partial\phi_L(x+a) + e^{-2ik_F(x+a)}\psi_R^\dagger(x+a)\psi_L(x+a) + h.c. = \rho(x+a)\end{cases}$$



x 　　x+a

剩下：$\psi_R^\dagger(x)\psi_L(x)\psi_L^\dagger$

x

$(e^{-2ik_Fx}\psi_{L\downarrow}^\dagger\psi_{R\downarrow} + e^{2ik_Fx}\psi_{R\downarrow}^\dagger\psi_{L\downarrow})$

$e^{-i(\partial\Phi)a}$
$= e^{-i\Phi(x+a)+i\Phi(x)}$
$= 1 - ia\partial\Phi + \ldots$

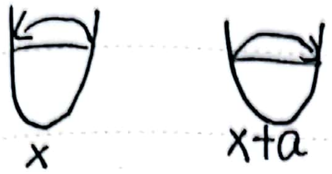
$\mathcal{L} = \frac{1}{2}(\partial$

① A=0

② A→

$\mathcal{L} = \frac{1}{2}($

Edwards



$$e^{-2ik_Fx+2ik_F(x+a)}=e^{2ik_Fa}$$

剩下：$\psi_R^+(x)\psi_L(x)\psi_L^+(x+a)\psi_R(x+a)e^{2ik_Fa}+h.c.$

$$\left(e^{2ik_Fx}\psi_{L\downarrow}^+\psi_{R\downarrow}+e^{2ik_Fx}\psi_{R\downarrow}^+\psi_{L\downarrow}\right)$$

$\psi_R^+(x)\psi_R(x+a)\,\psi_L(x)\psi_L^+(x+a)=\frac{1}{(2\pi a)^2}e^{+i\Phi_R(x+a)-i\Phi_R(x)}\cdot e^{i\Phi_L(x)-i\Phi_L(x+a)}$

Density　Density

$\left(\lim_{a\to 0}\psi^+(x+a)\psi(x)\right)$ 类比

$$=\frac{1}{(2\pi a)^2}\left[1-i(\partial\Phi_R)a+\frac{a^2}{2}(\partial\Phi_R)^2\right]\left[1-i(\partial\Phi_L)a+\frac{a^2}{2}(\partial\Phi_L)^2\right]$$

$e^{-i(\partial\Phi)a}$

$=e^{-i\Phi(x+a)+i\Phi(x)}$

$=1-ia\partial\phi+\frac{a^2}{2}(\partial\phi)^2$

$i\Phi_{R\downarrow}$　$+h.c.$

$e^{4ik_F(2(-ia)+2a)}$

$\psi_L^+(x)\psi_R(x)\psi_L^+(x+a)\psi_R(x+a)$

$=e^{-2i\Phi_L+2i\Phi_R}+h.c$

$\propto\boxed{\cos(2\Phi_L-2\Phi_R)}$

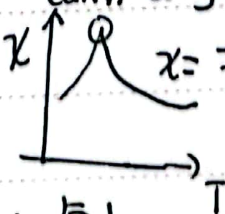
$\mathcal{L}=\frac{1}{2}(\partial\phi)^2+A\cos(\beta\phi)$

① $A=0$，声子

② $A\to$大，取$\beta\phi=2n\pi+\beta\delta\phi$ 小激发

$\mathcal{L}=\frac{1}{2}(\partial\delta\phi)^2-\frac{A}{2}\beta^2\delta\phi^2$

CuMn alloy



$\chi=\frac{C}{T-\Theta}$ Curie law

KPZ eq & spin glasses

parasi

$\Longrightarrow$ 无序 $\Longrightarrow$ Replica trick 复本．　n份copy 令 $n\to 0$．

Edwards　Brown运动．　Ref: kardar 书．