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$= \frac{1}{2\pi a} e^{\frac{1}{2}[\ ]} \quad \langle 0| e^{-i(\Phi_+(x+\varepsilon)-\Phi_+(x))+i(\Phi_-(x+\varepsilon)-\Phi_-(x))}|0\rangle$

$= \langle 0| e^{i(\Phi_+(x+\varepsilon)-\Phi_+(x))} e^{i\Phi_- - i\Phi_+}|0\rangle$

$\rho \sim \frac{1}{2\pi a} e^{-i\Phi(x+\varepsilon)+i\Phi(x)} \sim \frac{1}{2\pi a} e^{-i(\frac{\partial\Phi}{\partial x})\varepsilon} = \frac{1}{2\pi a}\left(1 - i\frac{\partial\Phi}{\partial x}\varepsilon + \frac{1}{2}\left(\frac{\partial\Phi}{\partial x}\right)^2 \varepsilon^2 + \cdots\right) = \frac{1}{2\pi a} - \frac{1}{2\pi}(\partial_x\Phi)$

为什么 $\psi \sim e^{i\Phi}$ 是正确的？

1) 反对易

2) Hilbert space

3) $Z_F = Z_B$ (Bloch, 1933)

☆ $\{\psi(x), \psi^+(y)\} = \delta(x-y)$

☆ 关联函数一样：$\langle \psi^+(x)\psi(y)\rangle = \frac{1}{2\pi a}\langle e^{-i\phi(x)} e^{i\phi(y)}\rangle$

⇒ 应用 $\xrightarrow{\text{字典}}$ $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\cos(\beta\phi)$

$[\Phi(x), \Phi(y)] = \ln\left(\frac{x-y-ia}{y-x-ia}\right) \quad \Phi_+(x) + \Phi_-(x)$

$\Leftrightarrow \quad \cancel{\Phi(x)} \ [\Phi_+(x), \Phi_-(y)] + [\Phi_-(x), \Phi_+(y)]$

$\Leftrightarrow \quad [\Phi_+(x), \Phi_-(y)] = \ln(x-y-ia)$

1) 关联函数一样

2) 给出 $[\Phi(x), \Phi(y)]$ 表达式

3) 翻译 $\psi \to \Phi$ 场 明确物理意义

$\langle \psi^+(x)\psi(y)\rangle = \frac{1}{L}\sum e^{-iq(x-y)}\langle 0|C_q^+ C_q|0\rangle$

$= \frac{1}{L}\sum_{q\le 0} e^{-iq(x-y)+aq} \quad \int_{-\infty}^{0} e^{Aq}\, dq = \frac{1}{A}$

$= \frac{1}{2\pi}\int_{-\infty}^{0} e^{[a-i(x-y)]q}\, dq = \frac{1}{2\pi}\frac{1}{a-i(x-y)} = \frac{i}{2\pi}\frac{1}{x-y+ia}$

$= \frac{1}{2\pi a}\langle e^{-i\phi(x)} e^{i\phi(y)}\rangle = \frac{1}{2\pi a} e^{K}$

$e^{K} = \frac{ia}{x-y+ia}$ or $K = \ln(ia) - \ln(x-y+ia)$

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) g = \delta(x-y) \quad g \sim \frac{1}{2\pi}\ln r$

$\Phi = i\sum_{q>0}\sqrt{\frac{2\pi}{Lq}}\left(e^{-iqx} b_q^+ - e^{iqx} b_q\right)$

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Remands:

1) $x \sim a + a^\dagger$, $p \sim i(a - a^\dagger)$

2) $p \sim \left(\frac{\partial \Phi}{\partial x}\right) \sim \sum_{q>0} P_q e^{-iqx} + P_q^\dagger e^{iqx}$

3) 为什么 $\frac{1}{\sqrt{L}}$

第一步: $[\Phi(x), \Phi(y)] = -\sum_{q>0} \frac{2\pi}{Lq}\left[e^{-iqx} b_q^\dagger - e^{iqx} b_q,\ e^{-iqy} b_q^\dagger - e^{+iqy} b_q\right]$

$= -\frac{2\pi}{L}\sum_{q>0}\left(\frac{1}{q} e^{-iq(x-y)-aq} - \frac{1}{q} e^{iq(x-y)-aq}\right)$ ① $q = \frac{2\pi n}{L}$ ② 求和化积分

$= -\int_0^{+\infty} dq \frac{e^{-iq(x-y)-aq} - e^{iq(x-y)-aq}}{q} = \int_0^{+\infty} \frac{e^{Aq} - e^{Bq}}{q} dq$

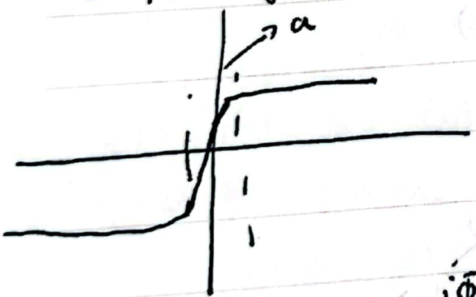
$A = i(x-y) - a$ $B = -i(x-y) - a$

① $= -\sum_{n\geq 1}\left(\frac{1}{n} e^{(-i(x-y)-a)\frac{2\pi n}{L}} - \frac{1}{n} e^{(i(x-y)-a)\frac{2\pi n}{L}}\right)$

$\ln(1-x) = -\sum_{n\geq 1} \frac{1}{n} x^n = \ln\left(1 - e^{\frac{2\pi}{L}(-i(x-y)-a)}\right) - \ln\left(1 - e^{\frac{2\pi}{L}(i(x-y)-a)}\right)$

$= \ln\left(\frac{2\pi}{L}(i(x-y)+a)\right) - \ln\left(\frac{2\pi}{L}(i(y-x)+a)\right)$

$= \ln(x-y-ia) - \ln(y-x-ia) = \ln\left(\frac{x-y-ia}{y-x-ia}\right)$



关联函数: $\langle e^{-i\Phi(x)} e^{i\Phi(y)} \rangle$ 合→分 $:e^{i\Phi}: = e^{\Phi_+} e^{\Phi_-}$

Normal ordering $:A: = A - \langle 0|A|0\rangle$

正规化: $\langle 0| e^{-i\Phi(x)+i\Phi(y)} \cancel{\frac{1}{2}[} + \frac{1}{2}[\Phi(x),\Phi(y)] |0\rangle = e^{\frac{1}{2}[\Phi(x),\Phi(y)]} \langle 0| e^{-i\Phi(x)+i\Phi(y)} |0\rangle$

$\boxed{e^A = e^{A_+ + A_-} = e^{A_+} e^{A_-} e^{-\frac{1}{2}[A_+, A_-]}}$

$= e^{\frac{1}{2}[\Phi(x)\Phi(y)]} e^{-\frac{1}{2}[A_+, A_-]} \langle 0| e^{A_+} e^{A_-} |0\rangle = \frac{i}{2\pi(x-y-ia)}$

$K = \frac{1}{2}[\Phi(x), \Phi(y)] - \frac{1}{2}[A_+, A_-] = \frac{1}{2}[\Phi(x), \Phi(y)] + \frac{1}{2}[\Phi_+(x) - \Phi_+(y), \Phi_-(x) - \Phi_-(y)] = \frac{1}{2}\ln\left(\frac{x-y-ia}{y-x-ia}\right) + \frac{1}{2}\left[2\ln(-ia) - \ln(x-y-ia) - \ln(y-x-ia)\right] = \ln(-ia) - \ln(y-x-ia)$

$$\langle \psi^\dagger(x)\psi(y)\rangle = \frac{i}{2\pi}\frac{1}{x-y+ia} \qquad \langle \psi(y)\psi^\dagger(x)\rangle = \frac{i}{2\pi}\frac{1}{y-x+ia}$$

$$\langle\{\psi(y),\psi^\dagger(x)\}\rangle = \delta(x-y) = \frac{i}{2\pi}\left(\frac{1}{x-y+ia}+\frac{1}{y-x+ia}\right)$$

$$= \frac{i}{2\pi}\left(\frac{1}{x-y+ia}-\frac{1}{x-y-ia}\right) = \delta(x-y)$$

XXZ model

$$-J\sum_i \sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta\sum_i \sigma_i^z\sigma_{i+1}^z \qquad \text{Jordan - Wigner}$$

$$H = -t\sum_i (c_i^\dagger c_{i+1} + h.c.) + \mu c_i^\dagger c_i + U c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}$$

$$H_0 = [-2t\cos(ka)+\mu]\, c_k^\dagger c_k \qquad \mu=0$$

$$(-2t\cos(ka))' = 2ta\sin(ka) \qquad ka=\frac{\pi}{2}$$

$$= 2ta = v_F$$

$$H_0 = v_F\psi_R^\dagger\left(-i\frac{\partial}{\partial x}\right)\psi_R - v_F\psi_L^\dagger\left(-i\frac{\partial}{\partial x}\right)\psi_L = v_F k\, c_k^\dagger c_k - v_F k\, d_k^\dagger d_k$$

$$\mathcal{L} = \frac{1}{2}[(\partial_t\phi_R)^2 - v_F^2(\partial_x\phi_R)^2] + [\frac{1}{2}[(\partial_t\phi_L)^2 - v_F^2(\partial_x\phi_L)^2]$$

$$\sum_i c_i^\dagger c_{i+1} \longleftrightarrow \int \psi^\dagger(x)\left(\frac{\partial}{\partial x}\right)^2\psi \qquad c_i \sim \psi(x=ia)$$

有 $\sum_i c_i^\dagger c_i = \int \psi^\dagger(x)\psi(x)\,dx \qquad c_i = \Lambda\psi(x=ia)$

$$= \Lambda^2\sum_i(\psi^\dagger(x=ia)\psi(x=ia))$$

$$\sum_i -t\, c_i^\dagger c_{i+1} = -ta\sum_i \psi^\dagger(x=ia)\psi(x=(i+1)a) + h.c.$$

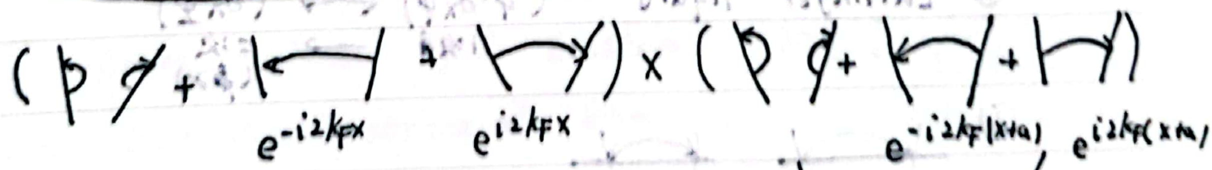
$$= -t\int\psi^\dagger(x=ia)\psi(x=ia+a)\,dx + h.c.$$

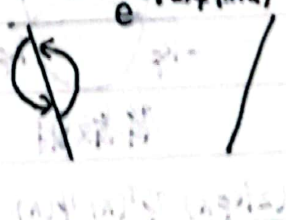
$$\simeq -t\int\psi^\dagger\frac{\partial^2}{\partial x^2}\psi$$

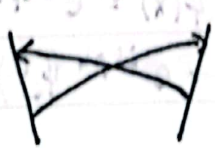
$$\sum_i c_i^\dagger c_i c_{i+1}^\dagger c_{i+1} = (\sqrt{a})^4\sum_i \psi^\dagger(x=ia)\psi(x=ia)\psi^\dagger(x=ia+a)$$

$$\psi(x-ia+a) = a\int dx\ \psi^\dagger(x)\psi(x)\psi^\dagger(x+a)\psi(x+a)$$

$$\underline{\psi = e^{ik_F x}\psi_R + e^{-ik_F x}\psi_L} \qquad \psi \sim \frac{F_i}{\sqrt{2\pi a}}e^{i\Phi_i}$$

= 

forward scattering $\sim (\partial\phi^2) \sim \rho^2$ 

backward scattering 

$$(s\cdot s+\underbrace{s\cdot f+f\cdot s}_{0}+f\cdot f)$$

$$h.c.+\psi_L^\dagger\psi_R=\frac{F_LF_R}{2\pi a}e^{-i\Phi_L+i\Phi_R}+\frac{F_RF_L}{2\pi a}e^{-i\Phi_R+i\Phi_L}$$
$$=\frac{F_LF_R}{2\pi a}\left(e^{i\Phi_L-i\Phi_R}-e^{i\Phi_R-i\Phi_L}\right)=\frac{iF_LF_R}{\pi a}\sin(\Phi_L-\Phi_R)$$

今天目的：如何直接写出来：

$$\left.\begin{matrix}\mathcal{L}[\psi]\\ H(\psi)\end{matrix}\right\}\overset{?}{\Longrightarrow}\left|\begin{matrix}\mathcal{L}(\Phi)\\ \text{物理意义}\end{matrix}\right.$$

字典：ψ 相互作用 $\Longrightarrow$ Bosonized $\mathcal{L}$

关键：：normal ordering

① Jordan-Wigner string

$$e^{i\pi\int_{-\infty}^{x}\rho(x')dx'}\quad \phi=\pi\int_{-\infty}^{x}\rho(x)\,dx'\Longleftrightarrow\frac{\partial\phi}{\partial x}=\pi\rho$$

$$\psi^\dagger=\frac{F}{\sqrt{2\pi a}}e^{-i\Phi}\quad(F^\dagger=F\quad F^2=1)\qquad \psi^\dagger\psi=\frac{F^2}{2\pi a}e^{-i\Phi(x)}e^{i\Phi(x)}=\frac{1}{2\pi a}$$

$$\rho=\lim_{a\to0}\psi^\dagger(x+a)\psi(x)=\frac{F^2}{2\pi a}e^{-i\Phi(x+a)}e^{i\Phi(x)}$$
$$=\frac{1}{2\pi a}e^{-i\Phi(x+a)+i\Phi(x)+\frac{1}{2}[\Phi(x+a),\Phi(x)]}=\frac{1}{2\pi a}e^{\frac{1}{2}[\Phi(x+a),\Phi(x)]}e^{-i(\frac{\partial\Phi}{\partial x}}$$
$$=\frac{1}{2\pi a}e^{\frac{1}{2}[\psi(x+a),\psi(x)]}\left(1-ia\frac{\partial\Phi}{\partial x}\right)$$
$$=\text{const}-\frac{iy}{2\pi a}e^{\frac{1}{2}[\Phi(x-y),\Phi(x)]}\left(\frac{\partial\Phi}{\partial x}\right)$$
$$=\infty+\frac{1}{2\pi}\left(\frac{\partial\Phi}{\partial x}\right)$$

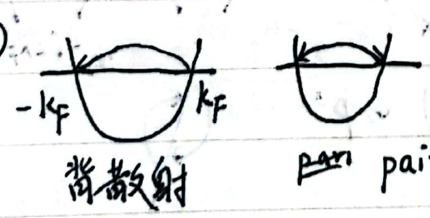
$$:\psi^\dagger(x+y)\psi(x):=A-\langle0|A|0\rangle=\frac{1}{2\pi}\left(\frac{\partial\Phi}{\partial x}\right)$$

字典. ① $\rho\to\rho=:\psi^\dagger\psi:\to\left(\frac{\partial\Phi}{\partial x}\right)$

② Luttinger liquid $(\psi^\dagger\partial_x\psi)\longleftrightarrow(\partial_x\Phi)^2$

1次幂 $\longrightarrow$ 二次(幂2)

③



背散射 ~~pan~~ pairing

$$V\cos(2k_Fx)\,\psi^\dagger(x)\psi(x)\qquad \psi(x)=e^{ik_Fx}\psi_R+e^{-ik_Fx}\psi_L$$

$$V\cos(2k_Fx)\left(e^{-ik_Fx}\psi_R^\dagger+e^{ik_Fx}\psi_L^\dagger\right)\left(e^{ik_Fx}\psi_R+e^{-ik_Fx}\psi_L\right)$$