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$\psi \sim e^{i\Phi} \Leftrightarrow \begin{cases} \psi^2(x)=0 \\ \{\psi(x), \psi^+(y)\} = \delta(x-y) \end{cases} \Big|_{\text{at } \boxed{x=y}}$

$\begin{cases} \psi^+(x)\,\psi(x) \sim e^{-i\Phi} e^{i\Phi} = 1 \\ \psi^2(x) = e^{i\Phi} \cdot e^{i\Phi} = e^{2i\Phi} \neq 0 \\ \frac{F}{\sqrt{2\pi a}} \leftarrow \text{cutoff} \end{cases}$

① ψ (fermi 场) 反对易关系.

② Hilbert 空间 - - 对应

③ 关联函数一样 $\Rightarrow$ 两体关联一样 $\Leftrightarrow$ 多体关联一样 $\Leftrightarrow$ 实验观测也一样.

近似 ~ 严格 $\quad b_i \to a_i$

$\Downarrow$ 直观图像 $\quad \Phi = \underbrace{\alpha \int_{-\infty}^{x} \rho(x)\,dx}_{\text{string(改变统计性质)}} + \underbrace{\theta(x)}_{\text{local phase fluctuation}}$

$\begin{cases} \alpha = \pi \Rightarrow \text{fermion} \\ \alpha \neq \pi \Rightarrow \text{anyon} \end{cases}$

① Klein factor

② Hilbert space

③ Normal ordering $:\_\_:$ $\begin{cases} (1)\ :A: = A - \langle 0|A|0\rangle \\ (2)\ \psi^+(x)\,\psi(x) = \lim_{a\to 0} \psi^+(x+a)\psi(x) - \langle 0|\psi^+(x+a)\psi(x)|0\rangle \\ (3)\ :e^{i\Phi}: = e^{i\Phi_+} e^{i\Phi_-}, \quad (\Phi = \Phi_+ + \Phi_-) \end{cases}$

三个等价定义.

产生湮灭

证明:

$\nearrow$

* Mahan 书

$e^A e^B = e^{A+B+\frac{1}{2}[A,B]}, \quad ([A,B] = \text{const})$

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$[\rho(x), \psi(y)] = -\psi(y)\delta(x-y)$ 对易/反对易关系.

$\Leftrightarrow [\rho_q, \psi(x)] = -e^{-iqx}\psi(x)$

$\Leftrightarrow [b_q, \psi(x)] = -\sqrt{\frac{2\pi}{L|q|}} e^{-iqx}\psi(x) = -\alpha_q(x)\psi(x)$

$[b_q, \psi(x)]|N\rangle_0 = -\alpha_q(x)\psi(x)|N\rangle_0$

$b_q\psi(x)|N_0\rangle - \psi(x) b_q|N_0\rangle = -\alpha_q(x)\psi(x)|N\rangle_0$

$$\boxed{b_q\psi(x)|N_0\rangle = -\alpha_q(x)\psi(x)|N\rangle_0}$$

H_{N-1}, b_q 的本征态 联想

相干态 $\Leftrightarrow$ 经典近似 $\underline{|a|\alpha\rangle = \alpha|\alpha\rangle}$ $\begin{cases} a \to \alpha \\ a^+ \to \alpha^* \end{cases}$

$|\alpha\rangle \sim e^{\alpha a^+}|0\rangle$ 由 $e^{\alpha^* a}|0\rangle = |0\rangle$

$\sim e^{\alpha a^+} e^{\alpha^* a}|0\rangle$ $\because [\alpha^+, a] = 0$.

$\sim e^{\alpha a^+ + \alpha^* a}|0\rangle$

$b_q \leftrightarrow -\alpha_q$ $\therefore \underbrace{\psi(x)|N\rangle_0}_{H_{N-1}} \sim e^{-\sum_q \alpha_q(x) b_q^+}|N-1\rangle_0$

$\sim e^{-\sum_q \alpha_q^* b_q}|N-1\rangle_0$

$\psi(x) = Fe^{i\Phi}$ 代入

$Fe^{i\Phi(x)}|N\rangle_0 \sim e^{-\sum_q \alpha_q(x) b_q^+}|N-1\rangle_0$

其中 $(F|N\rangle_0 = |N-1\rangle_0)$ $e^{i\Phi} \Leftrightarrow \Phi = \Phi^+$ 厄密场

x)|0) 得 $e^{i\Phi(x)}\cancel{|N-1\rangle_0} \sim e^{-\sum_q \alpha_q b_q^+}\cancel{|N-1\rangle_0}$

$$\boxed{\psi(x) = \frac{F}{\sqrt{2\pi a}} e^{i\Phi}}$$ 无可调自由度

证明:

令 $\underbrace{\psi(x)|N\rangle_0}_{N-1} = \Lambda \cdot e^{\sum_q \alpha_q(x) b_q^+}|N-1\rangle_0$

$= \Lambda \cdot e^{\sum_q \alpha_q(x) b_q^+} \cdot F|N\rangle_0$ $F^+F = FF^+ = 1$

$\langle N|F^+ = \langle N-1|$ $\langle N|F^+\overset{N-1}{\psi(x)|N\rangle_0} = \Lambda\langle N|F^+ e^{\sum_q \alpha_q(x) b_q^+} \cdot F|N\rangle_0$

$$\boxed{\psi(x) = \frac{1}{\sqrt{L}}\sum_k e^{ikx} c_k}$$ 只有 $q = \frac{2\pi N}{L}$ 有贡献.

$$\langle N|F^\dagger\psi(x)|N\rangle_0 = \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}x}$$

$$= \Lambda \langle N| e^{\sum_q \alpha_q(x) b_q^\dagger} |N\rangle_0 = \Lambda$$

~~等价于求 $\langle 0|e^{\alpha a^\dagger}|0\rangle$~~ $\therefore \Lambda = \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}}$

$$\psi(x)|N\rangle_0 = \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{b_+} |N-1\rangle_0$$

$$= \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{b_+(x)} |N\rangle_0$$

湮灭作用到基态

$$= \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{b_+(x)} e^{b_-(x)} |N\rangle_0$$

$$= \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{b_+ + b_- + \boxed{\frac{1}{2}[b_+(x), b_-(x)]}} |N\rangle_0$$

该常数发散，可以选取适当截断

$$\frac{1}{2}[b_+(x), b_-(x)] = \frac{1}{2}\sum_q \alpha_q^*(x)\alpha_q(x)[b_q^\dagger, b_q]$$

$$= -\frac{1}{2}\sum_q \alpha_q^*(x)\alpha_q(x)$$

$\alpha_q = \sqrt{\frac{2\pi}{L|q|}} e^{iqx}$

$$= -\frac{1}{2}\sum_q \left(\sqrt{\frac{2\pi}{L|q|}}\right)^2 e^{-iqx+iqx}$$

$$= -\frac{1}{2}\sum_q \frac{2\pi}{L|q|}$$

$q = \frac{2\pi n}{L}$

$$= -\frac{1}{2}\sum_{n\geq 1}\frac{1}{n} \to \infty$$

选取规范场 $\alpha_q = \sqrt{\frac{2\pi}{L|q|}} e^{iqx - \frac{a}{2}q}$ $a>0$

$$= -\frac{1}{2}\sum_q \frac{2\pi}{L|q|} e^{-aq}$$

$\ln(1-x) = \sum_{n=1}^{+\infty}\frac{x^n}{n}$

$$= -\frac{1}{2}\sum_q \frac{1}{n} e^{-\frac{2\pi a n}{L}}$$

$$= \frac{1}{2}\ln\left(1 - e^{-\frac{2\pi a}{L}}\right)$$

$$= \ln\left|\sqrt{\frac{2\pi a}{L}}\right|$$

代回 $\psi(x)|N_0\rangle = \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{b_+ + b_-} \sqrt{\frac{L}{2\pi a}}$

$$= \frac{F}{\sqrt{2\pi a}} e^{i\text{通}}$$

等价表示：1) $\psi \sim e^{i\Phi}$，$\frac{F}{\sqrt{2\pi a}}$省略

2) $\psi = \frac{F}{\sqrt{2\pi a}} e^{i\Phi}$ 最完整表示，或 $= \frac{1}{\sqrt{2\pi a}} e^{i\Phi}$

3) Normal ordering $\psi = \frac{F}{\sqrt{L}} :e^{i\Phi}: = \frac{F}{\sqrt{L}} e^{i\Phi_+} e^{i\Phi_-}$

关联函数证明：$\psi = \frac{F}{\sqrt{2\pi a}} e^{i\Phi(x)} \Rightarrow \psi^2(x)$ 如何定义/计算

$\rho = \psi^\dagger(x)\cdot\psi(x)$?

$$\Phi = \pi\int_{-\infty}^{x} \rho(x)\,dx + \theta(x) = \underbrace{\phi(x)}_{\text{密度场构成}} + \underbrace{\theta(x)}_{\text{相位涨落}}$$

$$e^{-i\Phi(x)} e^{i\Phi(x)} = 1$$

$$\begin{cases} \{\psi(x), \psi(y)\} = 0, & \text{if } x \to y \\ \{\psi(x), \psi^\dagger(y)\} = \delta(x-y) & \text{if } x \to y \end{cases}$$ 此时F抵消，不起作用.

$$\begin{cases} \varphi = \frac{i}{\sqrt{L}} \sum_{q>0} \frac{e^{iqx}}{\sqrt{q}} b_q \\ \varphi^\dagger = \frac{i}{\sqrt{L}} \sum_{q>0} \frac{e^{-iqx}}{\sqrt{q}} b_q^\dagger \end{cases} \qquad \boxed{\Phi = \varphi + \varphi^\dagger}$$

$$\{\psi(x), \psi^\dagger(y)\} = \psi(x)\psi^\dagger(y) + \psi^\dagger(y)\psi(x)$$
$$= \frac{1}{2\pi a}\left(e^{i\Phi(x)} e^{-i\Phi(y)} + e^{-i\Phi(y)} e^{i\Phi(x)}\right)$$
$$= \frac{1}{2\pi a} e^{i\Phi(x) - i\Phi(y)} \{ e^{\frac{1}{2}[\Phi(x), \Phi(y)]} + e^{\frac{1}{2}[\phi(y), \phi(x)]}$$

要求 $[\Phi(x), \Phi(y)] = i\pi\,\mathrm{sgn}(x-y)$

$$= \left(\frac{i}{\sqrt{L}}\right)^2 \left[\frac{e^{iqx - \frac{a}{2}q}}{\sqrt{q}} b_q - \frac{e^{-iqx - \frac{a}{2}q}}{\sqrt{q}} b_q^\dagger, \frac{e^{iq'y - \frac{a}{2}q'}}{\sqrt{q'}} b_{q'} - \frac{e^{-iq'y - \frac{a}{2}q'}}{\sqrt{q'}} b_{q'}^\dagger \right]$$

$q' = q$ 才有意义 $$= \left(\frac{i}{\sqrt{L}}\right)^2 \sum_{q>0} \left(\frac{e^{iq(x-y) - aq}}{q} - \frac{e^{-iq(x-y) - aq}}{q} \right)$$

$$= \frac{1}{L} \sum_{q>0} \frac{1}{q} \left(e^{q(i(x-y) - a)} - e^{q(-i(x-y) - a)} \right)$$

$\sum_n \frac{1}{n} e^{na} = -\ln(1-e^{a})$

$[\Phi(x), \Phi(y)] = \frac{1}{L}\sum_{n\geq 1} \frac{L}{2\pi n}\left(e^{\frac{2\pi n}{L}(i(x-y)-a)} - e^{\frac{2\pi n}{L}(-i(x-y)-a)}\right)$

$= \frac{1}{2\pi}\left[\ln(1-e^{\frac{2\pi}{L}(i(x-y)-a)}) - \ln(1-e^{\frac{2\pi}{L}(-i(x-y)-a)})\right]$

$\ln(1-e^{a}) = \ln a$

可证明：$\{\psi(x), \psi^{+}(y)\} = \frac{1}{2\pi a} e^{i\Phi(x) - i\Phi(y)} \times \{c(x-y) + c(y-x)\}$

其中 $c(x-y) = e^{\frac{1}{2}[\Phi(x), \Phi(y)]}$

$\sim e^{\{\ln(\frac{2\pi}{L}(i(x-y)-a)) - \ln(\frac{2\pi}{L}(-i(x-y)-a))\}}$

$\sim e^{\ln(i(x-y)-a) - \ln(-i(x-y)-a)}$

$\sim e^{\ln\left(\frac{i(x-y)-a}{-i(x-y)-a}\right)} \longleftrightarrow \boxed{\mathrm{sign}(x-y)}$

$\Longrightarrow [\Phi(x), \Phi(y)] \sim \boxed{i\pi\, \mathrm{sign}(x-y)}$ ~

Density 如何计算？

$\rho = \psi^{+}(x)\psi(x) \longrightarrow \infty$ 发散.

split operator

$= \lim_{\varepsilon\to 0} \psi^{+}(x+\varepsilon)\psi(x) - \langle 0|\psi^{+}(x+\varepsilon)\psi(x)|0\rangle$

$= \frac{1}{2\pi a} e^{-i\Phi(x+\varepsilon)} e^{i\Phi(x)} - \frac{1}{2\pi a}\langle 0| e^{-i\Phi(x+\varepsilon)} e^{i\Phi(x)}|0\rangle$

$\langle 0| e^{\alpha a^{+} + \alpha^{*} a} e^{\beta a^{+} + \beta^{*} a}|0\rangle$

mathematica

$\frac{I}{q}$

$\propto \langle 0| e^{(\alpha+\beta)a^{+} + (\alpha^{*}+\beta^{*})a}|0\rangle$

$\propto \langle 0| e^{(\alpha+\beta)a^{+}} \cdot e^{(\alpha^{*}+\beta^{*})a}|0\rangle$

$\propto$ 常数 $\times 1$ 场 ... slow filed

求常数 $\frac{1}{2\pi a}\langle 0| e^{-i\Phi(x+\varepsilon)} \cdot e^{i\Phi(x)}|0\rangle$?

$= \frac{1}{2\pi a} \underbrace{e^{\frac{1}{2}[\Phi(x+\varepsilon)\ \Phi(x)]}}_{\text{发散}} \langle 0| e^{-i\Phi(x+\varepsilon) + i\Phi(x)}|0\rangle$

$\Phi(x) = \Phi_{+}(x) + \Phi_{-}(x)$

$= \frac{1}{2\pi a} e^{\frac{1}{2}[\quad]} \langle 0| e^{-i(\Phi_{+}(x+\varepsilon) - \Phi_{+}(x)) + i(\Phi_{-}(x+\varepsilon) - \Phi_{-}(x))}|0\rangle$

分开 $e^{xa^{+} + ya} = e^{xa^{+}} e^{ya} e^{+\frac{1}{2}xy}$

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$$e^{\frac{1}{2}[\Phi_+(x+\varepsilon)-\Phi_+(x),\ \Phi_-(x+\varepsilon)-\Phi_-(x)]}$$

$$\therefore \rho \sim \frac{1}{2\pi a} e^{-i\Phi(x+\varepsilon)+i\Phi(x)}$$

$$\sim \frac{1}{2\pi a} e^{-i(\frac{\partial \Phi}{\partial x})\varepsilon}$$

$$\doteq \frac{1}{2\pi a}\left(1 - i\frac{\partial \Phi}{\partial x}\varepsilon + \frac{1}{2}\left(\frac{\partial \Phi}{\partial x}\right)^2\varepsilon^2 + \cdots\right)$$

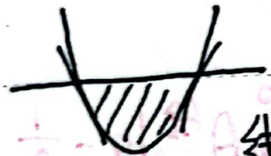
$$= \frac{1}{2\pi a} - \frac{1}{2\pi}(\partial_x \Phi)$$

意味着 $:\psi^\dagger\psi: \sim \partial\Phi$

$:\psi^\dagger\partial\psi: \sim (\partial\Phi)^2$

Dirac eq Bose-filed.

下次课: XXZ model | $\boxed{\text{KPZ eq}}$



$$H = \psi^\dagger\left(-\frac{\hbar^2}{2m} - \mu\right)\psi + V_{int}$$

线性化: Luttinger model

$\rho_q = \sum_n c_n^\dagger c_{n+q}$ ~ Boson

$$\psi = \frac{F}{\sqrt{2\pi a}} e^{i\Phi}$$ Boson field; cutoff

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\cos(\beta\phi)}$$ S-G model

字典/翻译 应用

↓ RG

$$[\Phi(x), \Phi(y)] = i\pi H(x-y)$$

为什么 $\psi \sim e^{i\Phi}$ 是正确的.

1) 反对易关系. $\psi\psi' = -\psi'\psi$

2) Hilbert Space $e^{i\Phi}|N\rangle_0 \in H_N$

3) $Z_F = Z_B$ (Bloch 1933)

4) ☆ $\{\psi(x), \psi^\dagger(y)\} = \delta(x-y)$

5) ☆ 关联函数一样. $\langle\psi^\dagger(x)\psi(y)\rangle = \frac{1}{2\pi a}\langle e^{-i\phi(x)} e^{i\phi(y)}\rangle$

$$[\Phi(x),\Phi(y)] = \ln\left(\frac{x-y-ia}{y-x-ia}\right)$$

$$\Phi(x) = \Phi_+(x) + \Phi_-(x)$$

$$[\Phi(x),\Phi(y)] = [\Phi_+(x),\Phi_-(y)] + [\Phi_-(x),\Phi_+(y)] = [\Phi_+(x),\Phi_-(y)] - [\Phi_+(y),\Phi_-(x)]$$

$$\Leftarrow [\Phi_+(x),\Phi_-(y)] = \ln(x-y-ia)$$

本节课目的：

1) 关联函数一样.

2) 给出 $[\Phi(x),\Phi(y)]$ 表达式.

$\Rightarrow$ 3) 翻译 $\psi \to \Phi$ 场 （要有明确物理意义）.

$$\langle\psi^+(x)\,\psi(y)\rangle = \frac{1}{L}\sum_{q\le 0} e^{-iq(x-y)}\langle 0|C_q^+C_q|0\rangle$$

$$= \frac{1}{L}\sum_{q\le 0} e^{-iq(x-y)+aq}$$

衰减因子.

$$= \frac{1}{2\pi}\int_{-\infty}^{0} e^{[a-i(x-y)]q}\,dq$$

$\int_{-\infty}^{0} e^{Aq}dq = \frac{1}{A}$.

$$= \frac{1}{2\pi}\,\frac{1}{a-i(x-y)}$$

$$= \frac{i}{2\pi}\,\frac{1}{(x-y)+ia}$$

$$= \frac{1}{2\pi a}\langle e^{-i\phi(x)}e^{i\phi(y)}\rangle$$

$$= \frac{1}{2\pi a}e^{k}$$

则 $e^k = \frac{ia}{x-y+ia}$ or $= \ln\frac{ia}{x-y+ia} = \ln(ia) - \ln(x-y+ia)$

*: 二维系统 $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g = \delta(x-y)$ 　2d 与 ln 有关.

$$g \sim \frac{1}{2\pi}\ln r$$

$$\Phi = i\sum_q\sqrt{\frac{2\pi}{Lq}}\left(e^{-iqx}b_q^+ - e^{iqx}b_q\right)$$

Remarks: 1) $x \sim a + a^+$

$p \sim i(a - a^+)$

扫描全能王 创建

2) $\rho \sim \left(\frac{\partial\phi}{\partial x}\right) \sim \rho_q e^{-iqx} + \rho_q^\dagger e^{iqx}$

3) 为什么 $\frac{1}{\sqrt{n}}$, $[\Phi(x),\Phi(y)] \propto \sum_q \frac{1}{L}(\quad)$

$\frac{1}{L}\sum_q \sim \frac{1}{2\pi}\int dq$

$\Phi(x)]$

第一步:求 $[\Phi(x),\Phi(y)] = -\sum_{q>0}\frac{2\pi}{Lq}\left[e^{-iqx}b_q^\dagger - e^{iqx}b_q,\ e^{-iqy}b_q^\dagger - e^{iqy}b_q\right]$

① $q=\frac{2\pi n}{L}$ $\quad = -\frac{2\pi}{L}\sum_{q>0}\left(\frac{1}{q}e^{-iq(x-y)-aq} - \frac{1}{q}e^{iq(x-y)-aq}\right)$

② 求和化积分. $\quad \overset{②}{=} -\int_0^{+\infty}dq\,\frac{e^{-iq(x-y)-aq} - e^{iq(x-y)-aq}}{q}$

$= \int_0^{+\infty}\frac{e^{Aq}-e^{Bq}}{q}dq \qquad (A = i(x-y)-a,\ B = -i(x-y)-a)$

① $= -\sum_{n\geq 1}\left(\frac{1}{n}e^{(-i(x-y)-a)\frac{2\pi n}{L}} - \frac{1}{m}e^{(i(x-y)-a)\frac{2\pi n'}{L}}\right)$

等式 $-\ln(1-x) = \sum_{n\geq 1}\frac{1}{n}x^n$

$= \ln\left(1-e^{\frac{2\pi}{L}(-i(x-y)-a)}\right) - \ln\left(1-e^{\frac{2\pi}{L}(i(x-y)-a)}\right)$

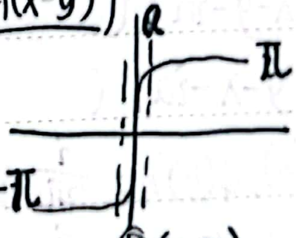
$\ln(1-e^x) \approx \ln(1-(1+x)) = \ln(-x)$

$= \ln\left(\frac{2\pi}{L}(i(x-y)+a)\right) - \ln\left(\frac{2\pi}{L}(i(y-x)+a)\right)$

$= \ln(x-y-ia) - \ln(y-x-ia)$

$= \ln\left(\frac{x-y-ia}{y-x-ia}\right) \Rightarrow \boxed{i\pi H(x-y)}$

考虑 $y=0$, $a=0.001$ 且图象



$[\Phi_+(x),\Phi_-(y)] = \ln(x-y-ia) \Rightarrow i\pi H(x-y)$

在 $x-y \gg a$ 时成立.

$\langle e^{-i\Phi(x)}, e^{i\Phi(y)}\rangle$

Normal ordering: $:A: = A - \langle 0|A|0\rangle$

正规化: $:e^{i\Phi}: = e^{\Phi_+}e^{\Phi_-}$

(去除发散) $\langle :e^{i\Phi}: \rangle = \langle 0|e^{\Phi_+}e^{\Phi_-}|0\rangle = 1$

合→分

$$\langle e^{-i\Phi(x)}, e^{i\Phi(y)}\rangle = \frac{1}{2\pi a}\langle 0| e^{-i(\Phi_+ + \Phi_-)} \cdot e^{i(\Phi_+ + \Phi_-)} |0\rangle$$

$$= \frac{1}{2\pi a}\langle 0| e^{-i\Phi(x) + i\Phi(y) + \frac{1}{2}[\Phi(x), \Phi(y)]} |0\rangle$$

$$= \frac{1}{2\pi a} e^{\frac{1}{2}[\Phi(x), \Phi(y)]} \langle 0| e^{-i\Phi(x) + i\Phi(y)} |0\rangle$$

$$e^{A} = e^{A_+ + A_-} = e^{A_+} e^{A_-} e^{-\frac{1}{2}[A_+, A_-]}$$

$$= \frac{1}{2\pi a} e^{\frac{1}{2}[\Phi(x), \Phi(y)]} e^{-\frac{1}{2}[A_+, A_-]} \qquad \langle 0| e^{A_+ A_-} |0\rangle = 1$$

$$= \frac{i}{2\pi (x-y-ia)}$$

$$= \frac{1}{2\pi a} e^{k} \qquad \left(k = \frac{1}{2}[\Phi(x), \Phi(y)] - \frac{1}{2}[A_+, A_-]\right)$$

$$= \frac{1}{2\pi a} e^{\ln\left(\frac{ia}{x-y+ia}\right)} \qquad \left(A_+ = -i\Phi_+(x) + i\Phi_+(y)\right)$$

$$= \frac{1}{2\pi a} \cdot \frac{ia}{x-y+ia} \qquad \left(A_- = -i\Phi_-(x) + i\Phi_-(y)\right)$$

$$= \frac{i}{2\pi} \cdot \frac{1}{x-y+ia}$$

则 $k = \frac{1}{2}[\Phi(x), \Phi(y)] + \frac{1}{2}[\Phi_+(x) - \Phi_+(y), \Phi_-(x) - \Phi_-(y)]$

$$= \langle \psi^\dagger(x) \psi(y)\rangle$$

$$k = \frac{1}{2}\ln\left(\frac{x-y-ia}{y-x-ia}\right) + \frac{1}{2}\left(2\ln(-ia) - \ln(x-y-ia) - \ln(y-x-ia)\right)$$

$$= \ln(-ia) - \ln(y-x-ia)$$

$$= \ln\left(\frac{ia}{x-y+ia}\right)$$

$$\psi(x)\psi(y) \propto (x-y-ia) \left(\qquad \right)$$

$$\psi(y)\psi(x) \propto (y-x-ia) \left(\qquad \right)$$

结论: $\langle \psi^\dagger(x), \psi(y)\rangle = \frac{i}{2\pi} \cdot \frac{1}{x-y+ia}$

$$\langle \psi(y), \psi^\dagger(x)\rangle = \frac{i}{2\pi} \cdot \left(\frac{1}{y-x+ia}\right)$$

$$\langle \{\psi(y), \psi^\dagger(x)\}\rangle = \delta(x-y)$$

$$= \frac{i}{2\pi}\left(\frac{1}{x-y+ia} + \frac{1}{y-x+ia}\right)$$

$\frac{1}{x-ia} = P \cdot \frac{1}{x} + i\pi\delta(x)$

$$= \frac{i}{2\pi}\left(\frac{1}{x-y+ia} - \frac{1}{x-y-ia}\right)$$

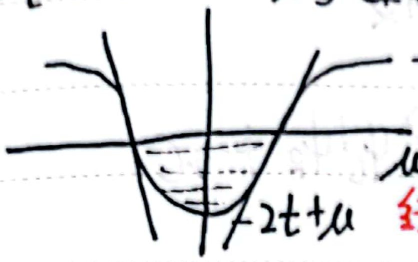
$$= \frac{i}{2\pi}\left(-i\pi\delta(x-y) - i\pi\delta(x-y)\right)$$

$$= \delta(x-y)$$

XXZ model: $-J\sum_i \sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y+\Delta\sum_i \sigma_i^z\sigma_{i+1}^z$

Jordan-Wigner 变换 $\boxed{H=-t\sum_i (c_i^\dagger c_{i+1}+h.c.)+\mu c_i^\dagger c_i+V c_i^\dagger c_i c_{i+1}^\dagger c_{i+1}}$

$H_0=[-2t\cos(\phi a)+\mu]\,c_k^\dagger c_k$



线性色散关系. $(-2t\cos(ka))'$
$=2ta\sin(ka)$
$(ka=\frac{\pi}{2})=2ta=v_F$

$H_0=v_F\,\psi_R^\dagger(-i\frac{\partial}{\partial x})\psi_R-v_F\,\psi_L^\dagger(-i\frac{\partial}{\partial x})\psi_L$

$=\underline{\underline{v_F\cdot k}}\,c_k^\dagger c_k-v_F\cdot k\,d_k^\dagger d_k$

$\Downarrow$

$-\ln(y-x-ia)]$ $\mathcal{L}=\frac{1}{2}[(\partial_t\phi_R)^2-v_F^2(\partial_x\phi_R)^2]+\frac{1}{2}[(\partial_t\phi_L)^2-v_F^2(\partial_x\phi_L)^2]$

$\sum_i c_i^\dagger c_{i+1}\longleftrightarrow\int\psi^\dagger(x)(\frac{\partial}{\partial x})^2\psi$

$c_i\sim\psi(x=ia)$ 有 $\sum_i c_i^\dagger c_i=\int\psi^\dagger(x)\psi(x)\,dx$

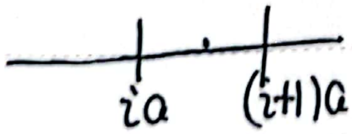
由 $c_i=\Lambda\psi(x=ia)$ ... $=\Lambda^2\sum_i\psi^\dagger(x=ia)\,\psi(x=ia)$

using $\int f(x)dx=\Delta x\sum_i f(x_i)$ $\therefore\Lambda^2=a$, $\Lambda=\sqrt{a}$.

$\sum_i -t c_i^\dagger c_{i+1}$

$=-ta\sum_i\psi^\dagger(x=ia)\,\psi(x=(i+1)a)+h.c.$

$=-t\int\psi^\dagger(x=ia)\,\psi(x=ia+a)\,dx+h.c.$



此处 泰勒展开

$\approx -t\int\psi^\dagger\frac{\partial^2}{\partial x^2}\psi$

$\sum_i c_i^\dagger c_i\,c_{i+1}^\dagger c_{i+1}=(\sqrt{a})^4\sum_i\psi^\dagger(x=ia)\,\psi(x=ia)\,\psi^\dagger(x=ia+a)\,\psi(x=ia+a)$

$=a\int dx\,\psi^\dagger(x)\,\psi(x)\,\psi^\dagger(x+a)\,\psi(x+a)$

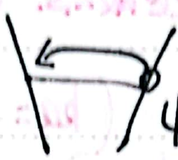
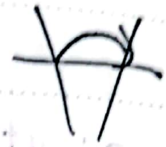
第四节：离→连

④： $\psi = e^{ik_Fx}\psi_R + e^{-ik_Fx}\psi_L$

快 慢

$\psi_i = \frac{F_i}{\sqrt{2\pi a}} e^{i\Phi_i}$

$\psi^\dagger(x)\psi(x) = \psi_R^\dagger\psi_R + \psi_L^\dagger\psi_L + e^{-2ik_Fx}\psi_L^\dagger\psi_R + h.c.$

$\psi_L^\dagger\psi_L$  $+$  $\psi_R^\dagger\psi_R$ $+$  $\psi_L^\dagger\psi_R e^{-2ik_Fx}$ $+$ 

*：不重要

但 $\psi^\dagger(x)\psi(x)\,\psi^\dagger(x+a)\psi(x+a)$

$= (\underbrace{\ \ +\ \ }_{slow} + \underbrace{\ \ + \ \ }_{fast}) \times (\underbrace{\ \ +\ \ }_{slow} + \underbrace{\ \ +\ \ }_{fast})$

e^{-i2k_Fx} e^{i2k_Fx} $e^{-i2k_F(x+a)}$ $e^{i2k_F(x+a)}$

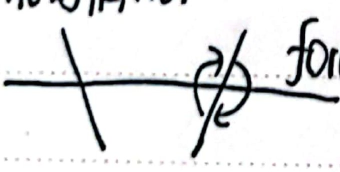
当 $4k_Fa = 2\pi$ 时 $e^{-4ik_Fx - 2ik_Fa} = e^{-2\pi i - 2ik_Fa}$ $x = ia$

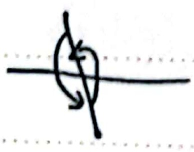
$= (s+f) \times (s+f)$

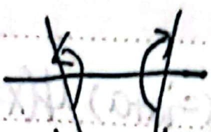
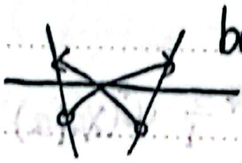
$= s\cdot s + \cancel{s\cdot f} + \cancel{f\cdot s} + f\cdot f$ 而 $f\cdot f$ e.f. $\int \sin^2(k_Fx)dx = \frac{1}{2}\left(\int(1-\cos 2k_Fx)dx\right)$

$\int(\quad)e^{2ik_Fx}dx \to 0$

可能的情况：

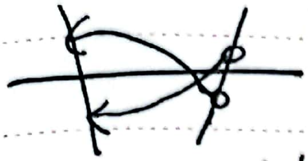
 forward scattering $\sim (\partial\phi)^2 \sim \rho^2$

 slow · slow

  backward scattering (背散射)

$\partial\phi \sim \rho$ $\partial\phi \sim \rho$

DATE



相差 $4k_F$ 的动量，存在其它补偿机制.

$$h.c + \psi_L^+ \psi_R = \frac{F_L F_R}{2\pi a} e^{-i\Phi_L + i\Phi_R} + \frac{F_R F_L}{2\pi a} e^{i\Phi_R - i\Phi_L}$$

$$= \frac{F_L F_R}{2\pi a}\left(e^{i\Phi_L - i\Phi_R} - e^{i\Phi_R - i\Phi_L}\right)$$

$e^{i\theta} - e^{-i\theta}$

$= 2i\sin(\theta)$

$$= \frac{iF_L F_R}{\pi a} \sin(\Phi_L - \Phi_R)$$

$\phi \to \phi + \text{const}$

不可能 $\frac{1}{2}(\partial\phi)^2 + A\phi^2$

$\because e^{i\phi} = e^{i\phi + i2\pi}$

slow $\Rightarrow$ forward scattering $\leftrightarrow \partial\phi$

fast $\Rightarrow$ backward scattering $\leftrightarrow \cos\phi$, $\sin\phi$

接下来: spin glasses

KPZ eq