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$\psi \sim e^{i\Phi} \Longleftrightarrow \begin{cases} \psi^2(x) = 0 \\ \{\psi(x), \psi^\dagger(y)\} = \delta(x-y) \end{cases}$ at $x=y$

$\begin{cases} \psi^\dagger(x)\psi(x) \sim e^{-i\Phi} e^{i\Phi} = 1 \\ \psi^2(x) \neq 0 \end{cases}$

cut off : a

① ψ场 反对易关系

② ~~Hubbard~~ Hilbert 空间有 1-1 对应

③ 关联函数一样 ⇒ 二体关联一样

近似 ~ 严格 多体也一样

⇓

实验观测也一样

$\Phi = \frac{\alpha}{\pi}\int_{-\infty}^{x} \rho(x)\,dx + \theta(x)$ | $\alpha = \pi$ fermion; $\alpha \neq \pi$ anyon

string local phase fluctuation

① klein factor

② Hilbert space

③ Normal-ordering : :

三个等价定义 ① $:A: = A - \langle 0|A|0\rangle$

② $\psi^\dagger(x)\psi(x) = \lim_{a\to 0} \psi^\dagger(x+a)\psi(x)$

③ $:e^{i\Phi}: = e^{i\Phi_+} e^{i\Phi_-}$

$[\rho(x), \psi(y)] = -\psi(y)\,\delta(x-y)$ 对易/反对易

$\Longleftrightarrow$ ~~$[\rho(x)$~~ $[\rho_q, \psi(x)] = e^{-iqx}\psi(x)$

$\Longleftrightarrow$ $[b_q, \psi(x)] = -\sqrt{\frac{2\pi}{L|q|}}\, e^{-iqx}\psi(x) = -\alpha_q(x)\psi(x)$

$[b_q, \psi(x)]\,|N\rangle_0 = -\alpha_q(x)\psi(x)\,|N\rangle_0$

$b_q\psi(x)|N\rangle_0 - \psi(x) b_q|N\rangle_0 = -\alpha_q(x)\,\psi(x)\,|N\rangle_0$

$b_q\,\underbrace{\psi(x)\,|N\rangle_0}_{H_{N-1}} = -\alpha_q\psi(x)\,|N\rangle_0$

相干态 ⟺ 经典近似

$a|\alpha\rangle = \alpha|\alpha\rangle$ | $a \to \alpha$, $a^\dagger \to \alpha^*$

$$ {}_0\sum_\alpha \alpha_q^* b_q $$

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$$ \frac{\psi(x)|N\rangle_0}{H_{N-1}} \sim e^{-\sum_q \alpha_q b_q^\dagger} \Longrightarrow |N-1\rangle_0 \qquad \psi(x) = F e^{i\Phi} $$

$$ F e^{i\Phi(x)} |N\rangle_0 \sim e^{-\sum_\alpha \alpha_q(x) b_q^\dagger} |N-1\rangle_0 $$

$$ F|N\rangle_0 = |N-1\rangle_0 \qquad e^{i\Phi(x)} \sim e^{-\sum_q \alpha_q b_q^\dagger} $$

$$ \psi(x) = \frac{F}{\sqrt{2\pi a}} e^{i\Phi} $$ 无可调自由度

$$ e^A e^B = e^{A+B+\frac{1}{2}[A,B]} \quad (\text{if } [A,B] = \text{const}) $$

令 $\psi(x)|N\rangle_0 = \Lambda \exp[\sum_q \alpha_q(x) b_q^\dagger] |N-1\rangle_0$

$$ = \Lambda \exp[\sum_q \alpha_q(x) b_q^\dagger] F |N\rangle_0 $$

$$ {}_0\langle N| F^\dagger = {}_0\langle N-1| \qquad {}_0\langle N| F^\dagger \psi(x) |N\rangle_0 = \Lambda\, {}_0\langle N| \exp[\sum_q \alpha_q b_q^\dagger] |N\rangle_0 $$

$$ \psi(x) = \frac{1}{\sqrt{L}} \sum_k e^{ikx} c_k $$

$$ \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}x} = \Lambda\, {}_0\langle N| \exp(\sum_q \alpha_q(x) b_q^\dagger) |N\rangle_0 $$ （只有0阶）

$$ \Lambda = \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}} \qquad \psi(x)|N\rangle_0 = \frac{1}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{\varphi_+} |N-1\rangle_0 $$

$$ = \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{\varphi_+(x)} e^{\varphi_-(x)} |N\rangle_0 = \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{\varphi_+ + \varphi_- + \frac{1}{2}[\varphi_+(x), \varphi_-(x)]} |N\rangle_0 $$

$$ \frac{1}{2}[\varphi_+(x), \varphi_-(x)] = \frac{1}{2} \sum_q \alpha_q^*(x) \alpha_q(x) [b_q^\dagger, b_q] = -\frac{1}{2} \sum_q \alpha_q^*(x) \alpha_q(x) $$

$$ = -\frac{1}{2} \sum_q \left(\sqrt{\frac{2\pi}{L|q|}}\right)^2 e^{-iqx+iqx} \quad \left(q = \frac{2\pi n}{L}\right) \quad \left(\alpha_q = \sqrt{\frac{2\pi}{L|q|}} e^{iqx - \frac{a}{2}q} \leftarrow q>0\right) $$

$$ = -\frac{1}{2} \sum_q \frac{2\pi}{Lq} e^{-aq} = -\frac{1}{2} \sum_q \frac{1}{n} e^{-\frac{2\pi a}{L} n} = \frac{1}{2} \ln(1 - e^{-\frac{2\pi a}{L}}) $$

$$ = \ln\left(\sqrt{\frac{2\pi a}{L}}\right) $$

$$ \frac{F}{\sqrt{L}} e^{i\frac{2\pi N}{L}} e^{\varphi_+ + \varphi_-} \sqrt{\frac{L}{2\pi a}} = \frac{F}{\sqrt{2\pi a}} e^{i\Phi} $$

等价表示：① $\psi \sim e^{i\Phi}$

② $\psi = \frac{F}{\sqrt{2\pi a}} e^{i\Phi}$

③ Normal-order $\frac{F}{\sqrt{L}} :e^{i\Phi}: = \frac{F}{\sqrt{L}} e^{i\Phi_+} e^{i\Phi_-}$ ( )

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关联函数证明: $\psi = \frac{F}{\sqrt{2\pi a}} e^{i\Phi(x)}$

$\Rightarrow$ $\psi^2(x) = 0$ 如何定义 / 计算

$\rho = \psi^+(x)\psi(x) = \frac{1}{\pi}(\partial_x \phi)$ $\quad \Phi = \pi\int_{-\infty}^{x} \rho(x)\,dx + \theta(x)$

$\{\psi(x), \psi(y)\} = 0$ of $x=y$

$\{\psi(x), \psi^+(y)\} = \delta(x-y)$ at $x \to y$

此时 F 抵消不起作用 $\quad \varphi = \frac{i}{\sqrt{L}} \sum_{q>0} \frac{e^{iqx}}{\sqrt{q}} b_q$

$\varphi^+ = -\frac{i}{\sqrt{L}} \sum_{q>0} \frac{e^{-iqx}}{\sqrt{q}} b_q^+$

$\{\psi(x), \psi^+(y)\} = \psi(x)\psi^+(y) + \psi^+(y)\psi(x) = \frac{1}{2\pi a}(e^{i\Phi(x)}e^{-i\Phi(y)} + e^{-i\Phi(y)} e^{i\Phi(x)}) = \frac{1}{2\pi a} e^{i\Phi(x) - i\Phi(y)} \{ e^{\frac{1}{2}[\Phi(x), \Phi(y)]} + e^{\frac{1}{2}[\Phi y, \Phi(x)]} \}$

$[\Phi(x), \Phi(y)] = -\frac{i}{\sqrt{2\pi}} \left(\frac{i}{\sqrt{L}}\right)^2 \left[\frac{e^{iqx - \frac{a}{2}q}}{\sqrt{q}} b_q - \frac{e^{-iqx - \frac{a}{2}q}}{\sqrt{q}} b_q^+, \frac{e^{iqy - \frac{a}{2}q}}{\sqrt{q}} b_q - \frac{e^{-iqy - \frac{a}{2}q}}{\sqrt{q}} b_q^+ \right]$

$= \frac{1}{L} \sum_{q>0} \frac{1}{q} [e^{q(i(x-y)-a)} + e^{q(-i(x-y)-a)}]$ $\quad \cancel{\sum_n \frac{1}{n} e^{-na}}$

$= \frac{1}{L} \sum_{n\geq 1} \frac{L}{2\pi n} (e^{\frac{2\pi n}{L}(i(x-y)-a)} + e^{\frac{2\pi n}{L}(-i(x-y)-a)})$

$= -\frac{1}{2\pi} [\ln(1 - e^{\frac{2\pi}{L}(i(x-y)-a)}) + \ln(1 - e^{\frac{2\pi}{L}(-i(x-y)-a)})]$

$\ln(1 - e^{-a}) = \ln a$ (近似)

$\{\psi(x), \psi^+(y)\} = \frac{1}{2\pi a} e^{i\Phi(x) - i\Phi(y)} \times \{ C(x-y) + C(y-x) \}$

$C(x-y) = e^{\frac{1}{2}[\Phi(x), \Phi(y)]} \sim e^{\left\{ \begin{array}{l} \ln(i(x-y)-a) \\ -\ln(i(x-y)-a) \end{array} \right\}}$

$\Rightarrow [\Phi(x), \Phi(y)] \simeq i\pi\, \mathrm{sgn}(x-y)$

Density 如何计算:

$\rho = \psi^+(x)\psi(x) = \lim_{\varepsilon \to 0} \psi^+(x+\varepsilon)\psi(x) - \langle 0 | \psi^+(x+\varepsilon) \cdot \psi(x) | 0 \rangle$

$= \frac{1}{2\pi a} e^{-i\Phi(x+\varepsilon)} e^{i\Phi(x)} - \frac{1}{2\pi a} \langle 0 | e^{-i\Phi(x+\varepsilon)} e^{i\Phi(x)} | 0 \rangle$

$\langle 0 | e^{\alpha a^+ + \alpha^* a} e^{\beta a^+ + \beta^* a} | 0 \rangle \propto \langle 0 | e^{(\alpha+\beta) a^+ + (\alpha^* + \beta^*) a} | 0 \rangle$

$\propto \langle 0 | e^{(\alpha+\beta) a^+} e^{(\alpha^* + \beta^*) a} | 0 \rangle \propto (\ \ ) \times$

求常数: $\frac{1}{2\pi a} \langle 0 | e^{-i\Phi(x+\varepsilon)} e^{i\Phi(x)} | 0 \rangle = \frac{1}{2\pi a} \underbrace{e^{\frac{1}{2}[\Phi(x+\varepsilon), \Phi(x)]}}_{\text{发散}}$

$\langle 0 | e^{-i\Phi(x+\varepsilon) + i\Phi(x)} | 0 \rangle$

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$= \frac{1}{2\pi a} e^{\frac{1}{2}[\ ]} \quad \langle 0| e^{-i(\Phi_+(x+\varepsilon)-\Phi_+(x))+i(\Phi_-(x+\varepsilon)-\Phi_-(x))}|0\rangle$

$= \langle 0| e^{i(\Phi_+(x+\varepsilon)-\Phi_+(x))} e^{i\Phi_- - i\Phi_+}|0\rangle$

$\rho \sim \frac{1}{2\pi a} e^{-i\Phi(x+\varepsilon)+i\Phi(x)} \sim \frac{1}{2\pi a} e^{-i(\frac{\partial\Phi}{\partial x})\varepsilon} = \frac{1}{2\pi a}\left(1-i\frac{\partial\Phi}{\partial x}\varepsilon+\frac{1}{2}\left(\frac{\partial\Phi}{\partial x}\right)^2\right.$
$\left.\varepsilon^2+\cdots\right) = \frac{1}{2\pi a} - \frac{1}{2\pi}(\partial_x\Phi)$

为什么 $\psi \sim e^{i\Phi}$ 是正确的？

1) 反对易
2) Hilbert space
3) $Z_F = Z_B$ (Bloch, 1933)

☆ $\{\psi(x), \psi^+(y)\} = \delta(x-y)$

☆ 关联函数一样：$\langle \psi^+(x)\psi(y)\rangle = \frac{1}{2\pi a}\langle e^{-i\phi(x)} e^{i\phi(y)}\rangle$

⇒ 应用 $\xrightarrow{\text{字典}}$ $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\cos(\beta\phi)$

$[\Phi(x), \Phi(y)] = \ln\left(\frac{x-y-ia}{y-x-ia}\right) \qquad \Phi_+(x)+\Phi_-(x)$

$\Longleftrightarrow \quad [\Phi_+(x), \Phi_-(y)] + [\Phi_-(x), \Phi_+(y)]$

$\Longleftrightarrow \quad [\Phi_+(x), \Phi_-(y)] = \ln(x-y-ia)$

1) 关联函数一样
2) 给出 $[\Phi(x), \Phi(y)]$ 表达式
3) 翻译 $\psi \to \Phi$ 场　明确物理意义

$\langle \psi^+(x)\psi(y)\rangle = \frac{1}{L}\sum e^{-iq(x-y)}\langle 0|C_q^+ C_q|0\rangle$

$= \frac{1}{L}\sum_{q\le 0} e^{-iq(x-y)+aq} \qquad \int_{-\infty}^{0} e^{Aq}\,dq = \frac{1}{A}$

$= \frac{1}{2\pi}\int_{-\infty}^{0} e^{[a-i(x-y)]q}\,dq = \frac{1}{2\pi}\frac{1}{a-i(x-y)} = \frac{i}{2\pi}\frac{1}{x-y+ia}$

$= \frac{1}{2\pi a}\langle e^{-i\phi(x)} e^{i\phi(y)}\rangle = \frac{1}{2\pi a} e^{K}$

$e^K = \frac{ia}{x-y+ia} \quad \text{or} \quad K = \ln(ia) - \ln(x-y+ia)$

$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) g = \delta(x-y) \qquad g \sim \frac{1}{2\pi}\ln r$

$\Phi = i\sum_{q>0}\sqrt{\frac{2\pi}{Lq}}\left(e^{-iqx} b_q^+ - e^{iqx} b_q\right)$