快速了解基本物理:

① 基本条件 $\psi_{\alpha i}(x) = \frac{F_i}{\sqrt{2\pi a}} e^{i\Phi_i(x)}$. 有多个场

1) Fermion 统计性质

$\{\psi_i(x), \psi_j(y)\} = 0$

$\{\psi_i(x), \psi_i^+(y)\} = \delta_{ij}\,\delta(x-y)$

2) 关联函数: $\langle \psi_1(x) \psi_2^+(x_2) \psi_3^+(x_3) \psi_4(x_4) \rangle$ 一样

3) 了解 Hilbert space 关系

已知 $\Phi_i(x)$ Boson field ⇒ 不同 Fermi point 附近的 Φ_i 不同

$\psi_i(x)\psi_j(y) = -\psi_j(y)\psi_i(x)$

$F_i F_j e^{i\Phi_i + i\Phi_j} = -e^{i\Phi_i + i\Phi_j} F_j F_i$

| $F_i F_j = -F_j F_i$; F_i 必须性 | $i=j$: $\psi_i(x)\psi_i(y) = -\psi_i(y)\psi_i(x)$; $F_i^2 e^{i\Phi_i(x)} e^{i\Phi_i(y)} = -F_i^2 e^{i\Phi_i(y)} e^{i\Phi_i(x)}$ |

等式: $[A,B]$ 是常数 $e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$

$e^{i\Phi_i(x) + i\Phi_i(y) - \frac{1}{2}[\Phi_i(x), \Phi_i(y)]} = -e^{i\Phi_i(y) + i\Phi_i(x) - \frac{1}{2}[\Phi_i(y), \Phi_i(x)]}$

↑ $C_i(x-y)$

$e^{-i\pi} = e^{+\frac{1}{2}\pi}$

$C_i(x-y) = -C_i(y-x)$

$C_i(x-y) = i\pi H(x-y) = \begin{cases} i\pi & x-y>0 \\ -i\pi & x-y<0 \end{cases}$

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$\{\psi_i(x), \psi_j^+(y)\} = \delta_{ij}\delta(x-y)$　如果 $i\neq j$　$F_i e^{i\Phi_i}$　$F_j^+ e^{-i\Phi_j}$

$\{F_i, F_j^+\} = \delta_{ij}$

$i=j$　$F_i F_i^+ = F_i^+ F_i$

$x=y \Rightarrow a \Longleftrightarrow \Lambda = \frac{\pi}{a}$　$\{\psi_i(x), \psi_i^+(x)\}$ 有问题

☆ 重点: ① Hilbert space ② klein factor

$|N\rangle_0 \longrightarrow$ Ground state

$|N\rangle \Rightarrow$ N个粒子的(激发)态

$|N\rangle \propto \exp[\sum_q b\, \alpha_q b_q^+]\, |N\rangle_0$　b_q or ρ_q

$F|H_N\rangle \rightarrow |H_{N-1}\rangle$

| $\psi = F e^{i\Phi}$
| $\psi|N\rangle \sim |N-1\rangle \leftarrow H_{N-1}$

② 不同场之间反对易

定义 | $F|N\rangle_0 = |N-1\rangle_0$
| $[F, b_q] = 0$

$\rho(x) = \psi^+(x)\psi(x)$　$[\rho(x), \psi(y)] = -\psi(y)\delta(x-y)$

$\rho(x) = \frac{1}{L}\sum_q e^{iqx}\rho(q)$

$\delta(x-y) = \frac{1}{L}\sum_q e^{iq(x-y)}$

$[\rho(q), \psi(y)] = e^{-iqy}\psi(y)$

$\psi(y)|N\rangle_0 = \Lambda(y)\exp[\sum_{q=0}\alpha_q b_q^+]|N-1\rangle_0$

$F^+\psi(y)|N\rangle_0 = \Lambda(y)\exp[\sum_{q=0}\alpha_q b_q^+]F^+|N-1\rangle_0$

| · $\Lambda(y)$ 待定
| · 求和对 $q>0$

$|\alpha\rangle = A e^{\alpha a^+}|0\rangle$　$e^{\alpha a}|0\rangle = |0\rangle$

$\langle\alpha|\alpha\rangle = A^2 \langle 0|e^{\alpha^* a} e^{\alpha a^+}|0\rangle$

$= A^2 e^{\frac{1}{2}|\alpha|^2}\langle 0|e^{\alpha^* a + \alpha a^+}|0\rangle$

$= A^2 e^{\frac{1}{2}|\alpha|^2}\langle 0|e^{\alpha a^+}e^{\alpha^* a}e^{\frac{1}{2}|\alpha|^2}|0\rangle$

$= A^2 e^{\frac{1}{2}|\alpha|^2}\langle 0|e$

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$\psi \sim e^{i\Phi} \Longleftrightarrow \begin{cases} \psi^2(x) = 0 \\ \{\psi(x), \psi^\dagger(y)\} = \delta(x-y) \end{cases}$ at $x=y$

$\left(\begin{array}{l} \psi^\dagger(x)\psi(x) \sim e^{-i\Phi} e^{i\Phi} = 1 \\ \psi^2(x) \neq 0 \end{array} \right.$

cut off a

① ψ 场 反对易关系

② ~~Hubbard~~ Hilbert 空间有 1-1 对应

③ 关联函数一样 ⇒ 二体关联一样

近似 ~ 严格　　多体也一样

⇓

实验观测也一样

$\Phi = \frac{\alpha}{\pi} \int_{-\infty}^{x} \rho(x)\,dx + \theta(x)$

string　　local phase fluctuation

$\alpha = \pi$　fermion

$\alpha \neq \pi$　anyon

① Klein factor

② Hilbert space

③ Normal-ordering $:\ :$

三个等价定义　① $:A: = A - \langle 0|A|0\rangle$

② $\psi^\dagger(x)\psi(x) = \lim_{a\to 0} \psi^\dagger(x+a)\psi(x)$

③ $:e^{i\Phi}: = e^{i\Phi_+} e^{i\Phi_-}$

$[\rho(x), \psi(y)] = -\psi(y)\,\delta(x-y)$　对易/反对易

$\Longleftrightarrow$ ~~$[\rho(x)$~~ $[\rho_q, \psi(x)] = e^{-iqx}\psi(x)$

$\Longleftrightarrow$ $[b_q, \psi(x)] = -\sqrt{\frac{2\pi}{L|q|}}\, e^{-iqx}\psi(x) = -\alpha_q(x)\psi(x)$

$[b_q, \psi(x)]\,|N\rangle_0 = -\alpha_q(x)\psi(x)\,|N\rangle_0$

$b_q\psi(x)|N\rangle_0 - \psi(x)\,b_q|N\rangle_0 = -\alpha_q(x)\,\psi(x)\,|N\rangle_0$

$b_q\, \underbrace{\psi(x)\,|N\rangle_0}_{H_{N-1}} = -\alpha_q\psi(x)\,|N\rangle_0$

相干态 ⟺ 经典近似

$a|\alpha\rangle = \alpha|\alpha\rangle$　$\begin{array}{l} a \to \alpha \\ a^\dagger \to \alpha^* \end{array}$