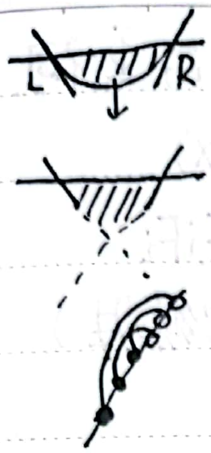
图象:  $H=\psi^\dagger(-\frac{d^2}{dx^2}+\mu)\psi$ + interaction

① ↓线性化

$\mathcal{L}_0=\bar{\psi}i\partial\psi$ or $H_F=v_F[\psi_R^\dagger(-i\partial_x)\psi_R-\psi_L^\dagger(-i\partial_x)\psi_L]+V_{int}$

↓② Tomonga-Luttinger

$\rho_q=\sum_n C_n^\dagger C_{n+q}$, $[\rho_q,\rho_{q'}^\dagger]=(-\frac{qL}{2\pi})$

⇓ Bose ③ $\rho_q\propto b_q$

$H_B=\sum_q vq\, b_q^\dagger b_q+V_{int}$

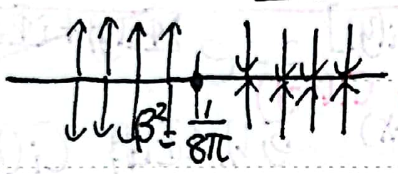
④

$\mathcal{L}=\frac{1}{2}(\partial_t\phi)^2-\frac{v^2}{2}(\partial_x\phi)^2-V_{int}$

⑤

SG: $\mathcal{L}=\frac{1}{2}(\partial\phi)^2+A\sin(\beta\phi)$ ← $\psi_F=f(\phi)=\frac{Fe^{i\Phi}}{\sqrt{2\pi a}}$

⑥ ↓ $\boxed{RG}$



$\beta^2=\frac{1}{8\pi}$

BKT

SG model

F: majorana operator

Hein factor

Boson field ⇒ Fermi field ⇒ 可行性: eg: 1d Fermi/Boson $Z_F=Z_B$

$\sigma_i=e^{i\sum_{j=-\infty}^{i-1}c_j^\dagger c_j}c_i$, $c_i=e^{i\sum_{j=-\infty}^{i-1}\sigma_j^\dagger\sigma_j}\sigma_i$ 自旋与玻色类比

$\sigma_i\Rightarrow e^{i\sum_{j=-\infty}^{i-1}\pi c_j^\dagger c_j}b_i\Rightarrow e^{i\pi\sum_{j=-\infty}^{i-1}b_j^\dagger b_j}\cdot e^{i\theta_i}\sqrt{\rho_i}\Rightarrow e^{i\pi\int_{-\infty}^{i}\rho(x)dx+i\theta}\sqrt{\rho}$

$\psi_i\sim e^{i\Phi_i}$, $\Phi_i\sim\underbrace{\pi\int_{-\infty}^{i}\rho(x)dx}_{\text{string(改变统计性质)}}+\theta_i$ phase fluctuation

快速了解基本物理:

① 基本条件 $\psi_{i,F}=\frac{F_i e^{i\Phi_i(x)}}{\sqrt{2\pi a}}$, 有多个场(i)

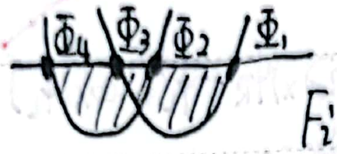
1) 满足Fermi统计性质: ① $\{\psi_i(x),\psi_j(y)\}=0$

② $\{\psi_i(x),\psi_i^\dagger(y)\}=\delta_{ij}\delta(x-y)$

2) 关联函数一样: $\langle\psi_1(x_1)\psi_2(x_2)\psi_3(x_3)\psi_4(x_4)\rangle$ 只要两体关联相同才能保证多体关联相同. ⇒ 实验观测不变.

3) 了解 Hilbert space 关系.

已知 $\Phi_i(x)$ 为 Bose field ⇒ 不同 fermi point 附近的 Φ_i 间.



$$\psi_i(x)\psi_j(y) = -\psi_j(y)\psi_i(x)$$

$$F_iF_j e^{i\Phi_i + i\Phi_j} = e^{i\Phi_i + i\Phi_j} F_jF_i$$

$F_iF_j = -F_jF_i$ 提供 （F_i 必须性）

$i=j$, $\psi_i(x)\psi_i(y) = -\psi_i(y)\psi_i(x)$

$$\Rightarrow \cancel{F_i^2} e^{i\Phi_i(x)} e^{i\Phi_i(y)} = -\cancel{F_i^2} e^{i\Phi_i(y)} e^{i\Phi_i(x)}$$

等式：

$e^A e^B = e^{A+B+\frac{1}{2}[A,B]}$ ，其中 $[A,B]$ 是常数）Bake-Hausdoff 公式.

证明：$(1+A+\frac{1}{2}A^2)(1+B+\frac{1}{2}B^2) = 1+A+B+\frac{1}{2}[AB]$

$$e^{\cancel{i\Phi_i(x)+i\Phi_i(y)} - \frac{1}{2}[\Phi_i(x),\Phi_i(y)]} = -e^{\cancel{i\Phi_i(y)+i\Phi_i(x)} - \frac{1}{2}[\Phi_i(y),\Phi_i(x)]}$$

记为 $C_i(x-y)$

则 $e^{-\frac{1}{2}C_i(x-y)} = e^{-\frac{1}{2}C_i(y-x)}$ 可以证明 $C_i(x-y) = -C_i(y-x)$

则 $C_i(x-y) = i\pi H(x-y) = \begin{cases} i\pi, & x-y>0 \\ -i\pi, & x-y<0 \end{cases}$

$\Leftrightarrow [\Phi_i(x),\Phi_i(y)] = i\pi H(x-y)$. 注：$x=y$ 不成立.

出现一个问题：$\psi = e^{i\Phi(x)}$ ，$\psi^2(x)=0$ $\cancel{\Rightarrow}$ $(e^{i\Phi})^2 = 0$

用别的方式消除.

考虑条件 1) ② $\{\psi_i(x), \psi_j^+(y)\} = \delta_{ij}\delta(x-y)$

如果 $i\neq j$ ，$F_i e^{i\Phi_i}$ ，$F_j^+ e^{-i\Phi_j}$ ，$\{F_i, F_j^+\} = \delta_{ij}$

问题在 $i=j$ $\{\psi_i(x),\psi_i^+(y)\} = F_iF_i^+ e^{i\Phi_i(x)} e^{-i\Phi_i(y)} + F_i^+F_i e^{-i\Phi_i(y)} e^{i\Phi_i(x)}$

$$= F_iF_i^+ e^{i\Phi_i(x) - i\Phi_i(y) + \frac{1}{2}[\Phi_i(x),\Phi_i(y)]} + F_i^+F_i e^{-i\Phi_i(y) + i\Phi_i(x) - \frac{1}{2}[\Phi_i(x),\Phi_i(y)]}$$

$$= e^{i\Phi_i(x) - i\Phi_i(y) + \frac{1}{2}[\Phi_i(x),\Phi_i(y)]} \times \{F_iF_i^+ + F_i^+F_i e^{-i\pi H(x-y)}\}$$

$= 0$ if $x-y \neq 0$ 要求 $F_iF_i^+ = F_i^+F_i$

⇒ $\boxed{a}$ 两个粒子可以待在一起的最小距离 ⇒ $\Lambda = \frac{\pi}{a}$

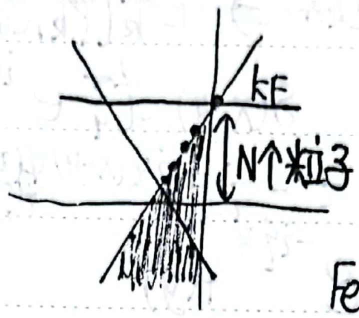
$\boxed{x=y}$ $\{\psi_i(x), \psi_i^\dagger(x)\}$ 有问题 (同一支, $x=y$)

ref: miranda. Introduction to Bosonization, 2003.
shankar 书 chap 17-18

☆重点:

① Hilbert space

② Klein factor



Fermion $|N\rangle_0$ ↙ Ground State

$|N\rangle$ 数目相同有激发

Fermion N个粒子的激发态.

$|N\rangle \propto e^{\sum_q \alpha_q b_q^\dagger} |N\rangle_0$ (α_q 为系数)

b_q or ρ_q

$\rho_q^\dagger \propto b_q^\dagger$ 对所有 q 或 α_q 成立.

$\psi|N\rangle \sim |N-1\rangle \in H_{N-1}$ $H = H_0 \oplus H_1 \oplus H_2 \oplus \cdots = \bigoplus_i H_i$ (空间独立)

$\psi = Fe^{i\Phi}$, $\boxed{Fe^{i\Phi}|N\rangle}$ $b_q \in H_N$ $F|H_N\rangle \to |H_{N-1}\rangle$

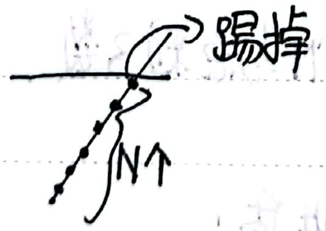
$\psi_i = \frac{F_i}{\sqrt{2\pi a}} e^{i\Phi_i(x)}$

F_i 作用:
① 反对易(不同场之间) ⇒ $F_iF_j = -F_jF_i$ 同时 $F_iF_i^\dagger = F_i^\dagger F_i$
② $F|N\rangle \sim |N-1\rangle$
③ 唯一确定成 $F|N\rangle_0 = F|N-1\rangle_0$
$[F, b_q] = 0$ N个GS到(N-1)个GS

*: $\Phi(x) \sim b_q, b_q^\dagger \sim \rho_q, \rho_q^\dagger$

$\rho_q = \sum_n c_n^\dagger c_{n+q}$ 整体表现为Boson行为

F



⇒ $[\Phi_i(x), \Phi_j(y)] = i\pi\Theta(x-y)\delta_{ij}$.

之$[\Phi_i(x), \Phi_i(y)]$

能量截断 $\sqrt{2\pi a}$ 的作用: $x=y$ 时给出正确对易关系.

DATE 连续

$\rho(x)=\psi^\dagger(x)\psi(x) \Leftrightarrow [\rho(x),\psi(y)] = -\psi(y)\delta(x-y)$

离散 $[C_i^\dagger C_i, C_i] = C_i^\dagger C_i C_i - C_i C_i^\dagger C_i = \begin{cases} -C_i & \text{Fermi} \\ -C_i & \text{Bosen} \end{cases}$

~~令~~ $\cancel{\psi(y)=\frac{1}{\sqrt{L}}\sum_q e^{iqy}C_q}$

~~同时~~ $\cancel{\rho(x)=\frac{1}{L}\sum_k \rho_k e^{ikx} \Rightarrow \frac{1}{L}\sum_k [\rho_k, C_q] e^{ikx+iqy} = \sum_l e^{ily}\delta(x-y)}$

$\rho(x)=\frac{1}{L}\sum_q e^{iqx}\rho_q$, $\delta(x-y)=\frac{1}{L}\sum_q e^{iq(x-y)}$

$\Leftrightarrow \sum_q [\rho_q,\psi(y)]e^{iqx} = -\sum_q e^{iq(x-y)}\psi(y)$

$[\rho_q,\psi(y)] = -e^{-iqy}\psi(y)$

$\cancel{[\rho_q,\psi(y)] = e^{-iqy}\psi(y)}$

$[\rho_q,\psi(y)]|N\rangle_0 = -e^{-iqy}\psi(y)|N\rangle_0 = \alpha_q(y)\psi(y)|N\rangle_0$

即：$\rho_q \underbrace{\psi(y)|N\rangle_0}_{\in H_{N-1}} - \psi(y)\underbrace{\rho_q|N\rangle_0}_{\in H_N} = \alpha_q(y)\underbrace{\psi(y)|N\rangle_0}_{\in H_{N-1}}$

挪动粒子位置.

$\psi(y)|N\rangle_0 \propto \underbrace{e^{\sum_q \alpha_q b_q^\dagger}|N-1\rangle_0}$ 用于构造 N-1 个粒子.

$\psi(y)|N\rangle_0 = \Lambda(y)e^{\sum_{q>0}\alpha_q b_q^\dagger}|N-1\rangle_0$, $\begin{cases} \Lambda(y) \text{待定} \\ \text{求和时} q>0 \end{cases}$

$F^\dagger\psi(y)|N\rangle_0 = \Lambda(y)e^{\sum_{q>0}\alpha_q b_q^\dagger}\underbrace{F^\dagger|N-1\rangle_0}_{=|N\rangle_0}$

${}_0\langle N|F^\dagger\psi(y)|N\rangle_0 = N(y)\,{}_0\langle N|e^{\sum_q \alpha_q b_q^\dagger}|N\rangle_0$

$\psi(y)=\frac{1}{\sqrt{L}}\sum_q e^{iqy}\boxed{C_q}$ 只有一个有贡献.

只有 $\boxed{\frac{1}{\sqrt{L}}e^{\frac{i2\pi N}{L}}}$ 态有贡献. N为总粒子数.

相干态. $|\alpha\rangle \propto e^{\alpha a^\dagger}|0\rangle$ 性质:

$\begin{cases} e^{\alpha a}|0\rangle = (1+\alpha a+\frac{\alpha^2}{2}a^2+\cdots)|0\rangle = |0\rangle \\ \cancel{e^{\alpha^* a}e^{\alpha a^\dagger}} \end{cases}$

$\langle\alpha|\alpha\rangle = \mathcal{N}^2\langle 0|e^{\alpha^* a}e^{\alpha a^\dagger}|0\rangle$

$=\mathcal{N}^2\langle 0|e^{\alpha^* a+\alpha a^\dagger+\frac{|\alpha|^2}{2}[a,a^\dagger]}|0\rangle$

$=\mathcal{N}^2 e^{\frac{1}{2}|\alpha|^2}\langle 0|e^{\alpha^* a+\alpha a^\dagger}|0\rangle$

$=\mathcal{N}^2 e^{\frac{1}{2}|\alpha|^2}\langle 0|e^{\alpha a^\dagger}e^{\alpha^* a}e^{\frac{1}{2}|\alpha|^2}|0\rangle$

$= \delta^2 e^{-|\alpha|^2}$ $\therefore \cancel{A} r = e^{-\frac{1}{2}|\alpha|^2}$

$\psi = F e^{[a_q, b_q^+]\Phi(y)}$ $\boxed{x=y}$ 关联函数.