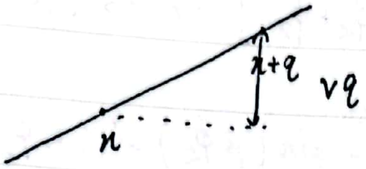
$$\rho_q = \sum_n C_n^{+} C_{n+q} \qquad \begin{cases} F \\ B \end{cases}$$



$$[H_F, \rho_q] = -vq\rho_q$$

$$[H_B, \rho_q] = -vq\rho_q$$

$$\int_0^{+\infty} \ln(1+e^{-x})\,dx = \frac{\pi^2}{12}$$

$$\int_0^{+\infty} -\ln(1-e^{-x}) = \frac{\pi^2}{6}$$

$$\psi = e^{i\Phi} \qquad \mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\cos(\phi)$$

RG

BKT相变:

XY model

相变(∞阶相变)

补充: Landau 相变理论 Beyond Landau 理论

1) Anderson localization

2) BKT

3) QHE/FQHE

$$\mathcal{L} = \frac{1}{2}(\partial_x\phi)^2 + \frac{1}{2}(\partial_y\phi)^2 + A\cos(\beta\phi)$$

$$\partial_t^2\phi = \partial_x^2\phi + A\beta\sin(\beta\phi)$$ (Sine-Gordan Eq) 可解

$$\mathcal{L}(\psi^{+},\psi) \xrightarrow{\psi\sim e^{i\Phi}} \quad \mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}(\partial_x\phi)^2 - A\cos(\beta\phi)$$

研究这个 model 相变 (BKT)

A 不是常数, 和能标有关.

$$Z = \int D\phi\, e^{-S} \qquad S = \int dt\,dx\,\mathcal{L}$$

标度分析 $d=2$

$$\int dt\,dx\,(\partial_t\phi)^2 = \int dt'\,dx'\,(\partial_{t'}\phi'(x',t'))^2$$

$$t' = \lambda t \qquad x' = \lambda x \qquad = \int dt\,dx\,\partial_t(\phi'(\lambda x, \lambda t))^2$$

$$\int dt\,dx\, y\frac{1}{a^2}\cos(\beta\phi) \qquad \left.\begin{matrix} x\to bx \\ t\to bt \end{matrix}\right\} \int Ab^2\cos(\beta\phi)\,dt\,dx$$

$$Z = \int D\phi\, e^{-\int \frac{1}{2}(\partial\phi)^2 + A\cos(\beta\phi)}$$

方法: 实空间处理

$$= \int D\phi_< D\phi_> \, e^{-(S_< + S_> + \delta S)}$$

$$S_< = \int dx\ \frac{1}{2}(\partial\phi_<)^2 + A\cos(\beta\phi_<)$$

$$S_> = \int dx\ \frac{1}{2}(\partial\phi_>)^2 \qquad \delta S = \int A\cos[\beta(\phi_< + \phi_>)] - A\cos(\phi_<)$$

第一种处理:

$$\cos[\beta(\phi_< + \phi_>)] - \cos\beta\phi_< = \cos\beta\phi_< \cos\beta\phi_> - \sin(\beta\phi_<)\sin(\beta\phi_>)$$

$$-\cos\beta\phi_< = -\frac{\beta^2}{2}\phi_>^2\cos(\beta\phi_<) - \sin(\beta\phi_<)(\beta\phi_>)$$

$$\langle e^{-\delta S}\rangle = e^{-\langle\delta S\rangle_>^c + \frac{1}{2}\langle\delta S^2\rangle_>^c}$$

$$\delta S = -\frac{A}{2}\beta^2\int dx\ \cos(\beta\phi_<)\phi_>^2$$

$$\langle\psi^+(x)\psi(y)\rangle \sim g(x-y)$$

$$\langle\phi_>^2(x)\rangle = \langle\phi_>^2(0)\rangle = \sum_{b\Lambda<|k|<\Lambda}\frac{1}{k^2} = \left(\frac{1}{2\pi}\right)^2 \times \frac{2\pi\Lambda^2(1-b)}{\Lambda^2}$$

$$= \frac{1-b}{2\pi}$$

$$Z = \int D\phi_<\ e^{-\int dx \frac{1}{2}(\partial\phi_<)^2 + A\cos(\beta\phi_<) - \frac{A\beta^2}{2}\frac{1}{2\pi}(1-b)\cos(\beta\phi_<)}$$

$$= \int D\phi_<\ e^{-\int dx\ \frac{1}{2}(\partial\phi_<)^2 + A\left(1-\frac{1-b}{4\pi}\beta^2\right)\cos(\beta\phi_<)}$$

$$= \int D\phi\ e^{-\int dx\ \frac{1}{2}(\partial\phi)^2 + Ab^{-2}\left(1-\frac{1-b}{4\pi}\beta^2\right)\cos(\beta\phi)}$$

$$b = 1 - dl = 1 - \frac{d\Lambda}{\Lambda} = 1 - d\ln\Lambda$$

$$A(b) = Ab^{-2}\left(1-\frac{1-b}{4\pi}\beta^2\right) = A(1+2dl)\left(1-\frac{dl}{4\pi}\beta^2\right)$$

$$= A\left[1+\left(2-\frac{\beta^2}{4\pi}\right)dl\right] = A - \frac{dA}{d\ln\Lambda}dl$$

$$\frac{dA}{d\ln\Lambda} = \left(\frac{\beta^2}{4\pi} - 2\right)A$$

spin model

$$H = J\vec{S_i}\cdot\vec{S_j} \Rightarrow J(\partial\phi)^2$$

$$e^{-\beta H} = e^{-\beta\left(J\left[\frac{1}{2}(\partial_x\phi)^2 + \frac{1}{2}(\partial_y\phi)^2\right] + A\cos(\phi)\right)}$$

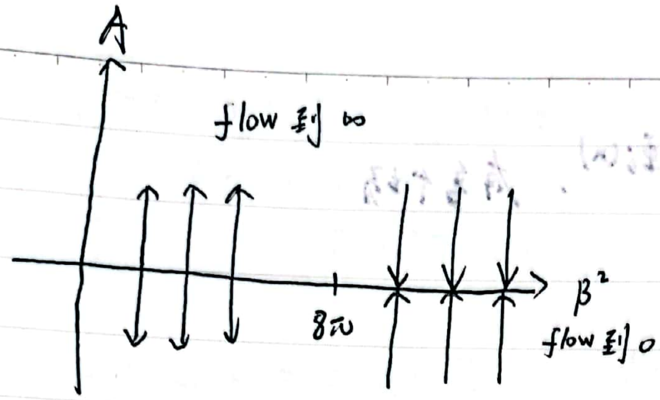
$$= e^{-\left[\frac{1}{2}(\partial\phi)^2 + A\cos\left(\frac{1}{\sqrt{\beta J}}\phi\right)\right]}$$

critical point $\qquad K_BT = 8\pi J$

Ref ① Shanker 书 18.4.2

② W X G 书

扫描全能王 创建



$H_F = \psi_R^\dagger(-i\partial_x)\psi_R - \psi_L^\dagger(-i\partial_x)\psi_L$

$\downarrow \rho_q$

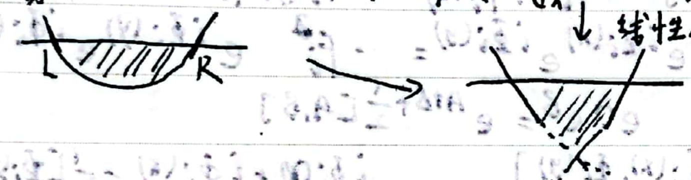
$H_B = \sum_q vq\, b_q^\dagger b_q$

$\mathcal{L} = \frac{1}{2}[(\partial_t\phi)^2 - v^2(\partial_x\phi)^2] + A\cos(\Phi)$

$H = \psi^\dagger\left(\frac{d^2}{dx^2}\right)\psi + V_{int}$

图象: $H = \psi^\dagger\left(-\frac{d^2}{dx^2} + \mu\right)\psi$ + interaction

↓ 线性化



$\mathcal{L}_0 = \bar\psi i\partial\psi$ or

$H = v_F[\psi_R^\dagger(-i\partial_x)\psi_R - \psi_L^\dagger(-i\partial_x)\psi_L]$
$+ V_{int}$

$\downarrow$

$\rho_q = \sum_n c_n^\dagger c_{n+q}$ & $[\rho_q, \rho_q^\dagger] = \frac{-qL}{2\pi}$

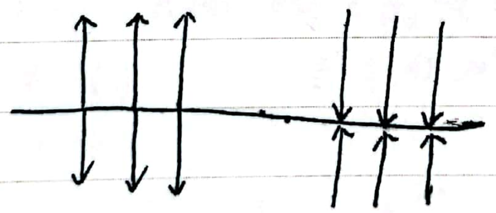
Boson $\downarrow$

$H_B = \sum_q vq\, b_q^\dagger b_q + V_{int}$

$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{v^2}{2}(\partial_x\phi)^2 - V_{int} \longrightarrow$ SG $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\sin(\beta\phi)$

$\beta^2 = \frac{1}{8\pi}$

→ $\boxed{\text{RG}}$



BKT
SG model

Boson field ⇒ Fermi field ⇒ 可行性. $Z_F = Z_B$

$\sigma_i = \exp\left[i\sum_{j=1}^{i-1} c_j^\dagger c_j\right] c_i \qquad c_i^\dagger c_i = b_i^\dagger b_i$

$\psi_i \sim e^{i\Phi_i} \qquad \Phi_i \sim \underbrace{\pi\int_{-\infty}^{i}\rho(x')\,dx'}_{\text{string}} + \underbrace{\theta_i}_{\text{phase fluctuation}}$