DATE

BKT相变

XY model（∞阶相变） $\mathcal{L} = \frac{1}{2}(\partial_x\phi)^2 + \frac{1}{2}(\partial_y\phi)^2 + A\cos(\beta\phi)$

补充：以前用Landau相变　　经典问题.

Beyond Landau 理论：1) Anderson localation.

2) BKT相变 phase trasation

3) QHE / FQHE

$$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 + \frac{1}{2}(\partial_x\phi)^2 + A\cos(\beta\phi)$$

运动方程 $\partial_t^2\phi = \partial_x^2\phi + A\beta\sin(\beta\phi)$　sine-Gordon Eq（可解）

时空 vortex $\Longleftarrow$ 对应很多孤粒子的解.

$\mathcal{L}(\psi^+,\psi)$ $\xrightarrow{\psi \sim e^{i\phi}}$ $\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}(\partial_x\phi)^2 - A\cos(\beta\phi)$

研究这个model相变（BKT）　wick转动后变+

key point：A不是常数，与能标有关.

$+V\cos(\phi)$ → 相互作用.

$Z = \int D\phi\, e^{-S}$，$S = \int dt\,dx\left[\frac{1}{2}(\partial_t\phi)^2 + \frac{1}{2}(\partial_x\phi)^2 + A\cos(\beta\phi)\right]$.

标度分析　$\int dt\,dx(\partial_t\phi)^2 = \int dt'\,dx'\,(\partial_{t'}\phi'(x',t'))^2$

取 $t' = \lambda t$，$x' = \lambda x$　$= \int dt\,dx\,\partial_t(\phi'(\lambda x, \lambda t))^2$

即 $\phi'(\lambda x', \lambda t') = \phi(x,t) \Longrightarrow \boxed{[\phi] = 0}$　ϕ无量纲，则β也无量纲

$\int dt\,dx\, A\cos(\beta\phi)$

$\Downarrow$ 动量^2

$\int \underline{dx\,dy}\, y\, \underline{\Lambda^2} \cos(\beta\phi)$

$\int dt\,dx\, y\, \frac{1}{a^2}\cos(\beta\phi)$

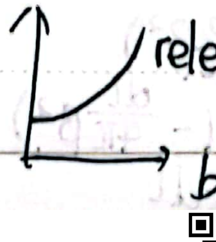
$x \to bx$，$t \to bt$

$\int A b^2 \cos(\beta\phi)\,dt\,dx$

$= A(b)\int \cos(\beta\phi)\,dt\,dx$

$A(b) = Ab^2$.　-o—k—o—o

relevant.　-o—k'=k(b)—o



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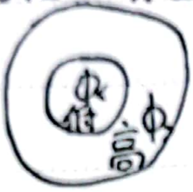
$$Z=\int D\phi\, e^{-\int \frac{1}{2}(\partial\phi)^2+A\cos(\beta\phi)} = \int D\phi_< D\phi_> e^{-(S_< + S_> + \delta S)}$$

方法：实空间处理



$$S_< = \int dx\, \frac{1}{2}(\partial\phi_<)^2 + A\cos(\beta\phi_<)$$
$$S_> = \int \frac{1}{2}(\partial\phi_>)^2\, dx$$
$$\delta S = \int A\cos[\beta(\phi_< + \phi_>)] - A\cos(\phi_<)$$

$$\cos[\beta(\phi_< + \phi_>)] - \cos\beta\phi_<$$
$$= \cos(\beta\phi_<)\cos(\beta\phi_>) - \sin(\beta\phi_<)\sin(\beta\phi_>) - \cancel{\cos(\phi_<)}$$ $\phi_>$ 小量泰勒展开

$$= \cos\beta\phi_< (1 - \tfrac{1}{2}\beta^2\phi_>^2) + \cdots$$
$$= -\frac{\beta^2}{2}\phi_>^2\cos(\beta\phi_<) - \sin(\beta\phi_<)(\beta\phi_>)$$

$$Z = \int D\phi_< e^{-S_<}\langle e^{-\delta S}\rangle_> \times \text{const}$$

$$\langle e^{-\delta S}\rangle = e^{-\langle \delta S\rangle_>^c + \frac{1}{2}\langle \delta S^2\rangle_>^c}$$ 粗糙处理

Remarks: ϕ^4, ϕ^3, SG model
1+2阶　2+3阶　1阶

高阶处理 X.G.Wen 书

$$\delta S = -\frac{A}{2}\beta^2\int dx\, \cos(\beta\phi_<)\phi_>^2$$
$$\langle \delta S\rangle_>^c = -\frac{A\beta^2}{2}\int dx\, \cos(\beta\phi_<) \times \langle \phi_>^2(x)\rangle.$$

由 $\langle \psi^+(x)\psi(y)\rangle \sim g(x-y)$ 知 $\langle\phi_>^2(x)\rangle = \langle \phi_>^2(0)\rangle.$

$$= \sum_{b\Lambda < |k| \le \Lambda} \frac{1}{k^2}$$
$$= \left(\frac{1}{2\pi}\right)^2 \cdot \frac{2\pi\Lambda(\Lambda - b\Lambda)}{\Lambda^2}$$ 面积

接上：$\langle \delta S\rangle_>^c = -\frac{A\beta^2}{2}\frac{1-b}{2\pi}\int dx\, \cos(\beta\phi_<)$

$$Z = \int D\phi_< e^{-\int dx \frac{1}{2}(\partial\phi_<)^2 + A\cos(\beta\phi_<) - \frac{A\beta^2}{2}\cdot\frac{1}{2\pi}\cdot(1-b)\cos(\beta\phi_<)}$$
$$= \int D\phi_< e^{-\int dx \frac{1}{2}(\partial\phi_<)^2 + A(1-\frac{1-b}{4\pi}\beta^2)\cos\phi_<}$$ $x \to \frac{1}{b}x$, $t \to \frac{1}{b}t$
$$= \int D\phi_< e^{-\int dx \frac{1}{2}(\partial\phi_<)^2 + Ab^{-2}(1-\frac{1-b}{4\pi}\beta^2)\cos(\beta\phi)}$$
$$= \int D\phi\, e^{-\int dx \frac{1}{2}(\partial\phi)^2 + \underbrace{Ab^{-2}(1-\frac{1-b}{4\pi}\beta^2)}_{\text{ʋ}}\cos(\beta\phi)}$$

此处 $b = 1 - dl = 1 - \frac{d\Lambda}{\Lambda} = 1 - d\ln\Lambda.$

$$A(b) = Ab^{-2}(1 - \frac{1-b}{4\pi}\beta^2)$$
$$= A(1+2dl)(1 - \frac{dl}{4\pi}\beta^2) = A[1 + (2 - \frac{\beta^2}{4\pi})dl] = A - \beta dl$$

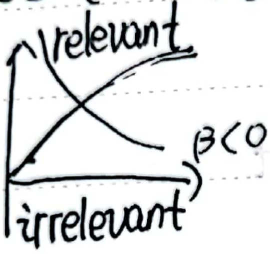
$$A(1-dl) = A - \frac{dA}{d\ln\Lambda}dl \qquad \boxed{\frac{dA}{d\ln\Lambda}dl = (\frac{\beta^2}{4\pi} - 2)A} = A(\frac{k_BT}{4\pi J} - 2)$$

二.

另一种方式

$$S_< = \int dx \frac{1}{2}(\partial_x \phi_<)^2$$
$$S_> = \int dx \frac{1}{2}(\partial \phi_>)^2$$
$$\delta S = \int A\cos[\beta(\phi_< + \phi_>)]$$

$$A(\Lambda) \propto (\quad) e^{(\frac{\beta^2}{4\pi}-2)\ln\Lambda} \propto e^{\beta \ln\Lambda}$$



$$\boxed{\beta^2 = 8\pi}$$

如何拉入温度？

Spin model $H = J\vec{S}_i\cdot\vec{S}_j \to J(\partial\phi)^2$

正确表述：$e^{-\beta H} = e^{-\beta J[\frac{1}{2}(\partial_x\phi)^2+\frac{1}{2}(\partial_y\phi)^2]+A\cos(\phi)]}$

引入 $\boxed{\phi=\frac{\phi}{\sqrt{\beta J}}}$ $= e^{-[\frac{1}{2}(\partial\phi)^2+A\cos(\frac{1}{\sqrt{\beta J}}\phi)]}$ $\qquad (\beta=\frac{1}{k_BT})$

相变点发生在 $\frac{k_BT}{J}=8\pi$ or $J=\frac{k_BT}{8\pi}$

Critical point $\boxed{k_BT=8\pi J}$

$$= \langle e^{\int A\cos(\beta\phi_<+\beta\phi_>)}\rangle_> = e^{\langle\int dx\, A\cos\beta(\phi_<+\phi_>)\rangle_>}$$ 再展开

$$= e^{\frac{A}{2}\int dx\,(e^{i\beta\phi_<}\langle e^{i\beta\phi_>}\rangle_> + h.c.)}$$

$$= e^{\frac{A}{2}\int dx\,(e^{i\beta\phi_<}\cdot e^{-\frac{\beta^2}{2}\langle\phi_>^2\rangle}+h.c.)}$$

$$= e^{\boxed{A\cdot e^{-\frac{\beta^2}{2}\langle\phi_>^2\rangle}}\int\cos(\beta\phi_<)dx}.$$

重整化后的 P352 – P854 .

Ref: ① Shanker 书 chap 18.4.2.

② X.G. Wen 书.

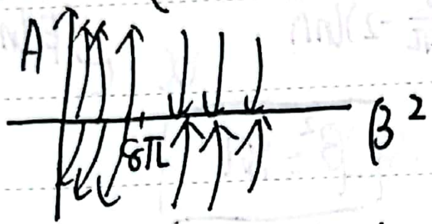
考虑2阶：$\langle \int dx\,dy\, \cos[\beta(\phi_< + \phi_>)]_x \cos(\beta(\phi_< + \phi_>))_y \rangle_>$

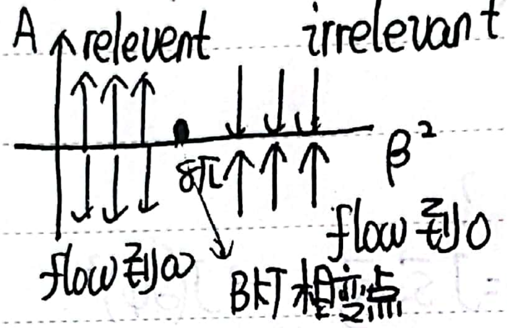
$\langle \phi_>(x)\phi_>(y)\rangle \propto \frac{1}{x-y}$, $y = x+z$

$= \int dx\,dz \langle \cos(\beta\phi(x)) \cos(\beta\phi(x+z)) \rangle_>$

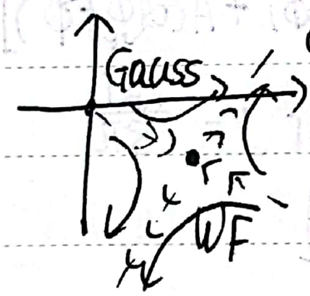
$$\frac{dA}{d\ln n} = \left(\frac{\beta^2}{4\pi} - 2\right)A + (\quad)A^2$$

BKT相变图：

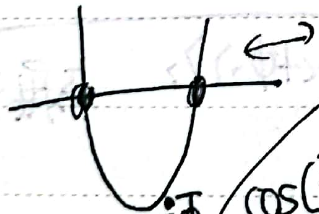




类比ϕ^4 theory的flow：



$$\boxed{\frac{k_BT}{J} = 8\pi}$$



$H_F = \psi_R^\dagger(-i\partial_x)\psi_R - \psi_L^\dagger(-i\partial_x)\psi_L$

↓定义ρ_q

$\psi \sim e^{i\Phi}$, $\cos(\Phi)$

$H_B = \sum_q v q\, b_q^\dagger b_q$

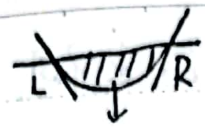
⇓

$\Phi \sim \Phi + \text{const}$

$\sim \Phi + 2\pi$.

$$\mathcal{L} = \frac{1}{2}[(\partial_t\phi)^2 - v^2(\partial_x\phi)^2] + A\cos(\Phi)$$ 晶格振动满足的形式.

ϕ与Φ存在差别.

图象：

$H = \psi^+(-\frac{d^2}{dx^2}+\mu)\psi$ + interaction

① ↓线性化

$\mathcal{L}_0 = \bar\psi i\partial\psi$ or $H_F = v_F[\psi_R^+(-i\partial_x)\psi_R - \psi_L^+(-i\partial_x)\psi_L] + V_{int}$

↓ ② Tomonga – Luttinger

$\rho_q = \sum_n C_n^+ C_{n+q}$, $[\rho_q, \rho_{q'}^+] = (\frac{-qL}{2\pi})$

⇓ Bose ③ $\rho_q \propto b_q$

$H_B = \sum_q v_q b_q^+ b_q + V_{int}$

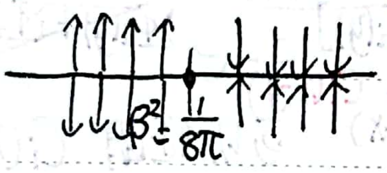
④

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{v_F^2}{2}(\partial_x\phi)^2 - V_{int}$

⑤

SG: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + A\sin(\beta\phi)$ ← $\psi_F = f(\phi_B) = \frac{Fe^{i\Phi}}{\sqrt{2\pi a}}$

⑥ ↓ $\boxed{RG}$



$\beta^2 = \frac{1}{8\pi}$

BKT

SG model

F: majorana operator

Hein factor

Boson field ⇒ Fermi field ⇒ 可行性：eg: 1d Fermi / Boson $Z_F = Z_B$

$\sigma_i = e^{i\sum_{j=-\infty}^{i-1} c_j^+ c_j} C_i$, $C_i = e^{i\sum_{j=-\infty}^{i-1} c_j^+ c_j}\sigma_i$ 自旋与玻色类比

$\sigma_i \rightleftharpoons e^{i\sum_{j=-\infty}^{i-1}\pi c_j^+ c_j} b_i \rightleftharpoons e^{i\pi\sum_{j=-\infty}^{i-1} b_j^+ b_j} e^{i\theta_i}\sqrt{\rho_i} \rightleftharpoons e^{i\pi\int_{-\infty}^{i}\rho(x')dx' + i\theta}\sqrt{\rho}$

$\psi_i \sim e^{i\Phi_i}$, $\Phi_i \sim \underbrace{\pi\int_{-\infty}^{i}\rho(x')dx'}_{\text{string（改变统计性质）}} + \theta_i$ phase fluctuation

快速了解基本物理：

① 基本条件 $\psi_{i,F} = \frac{F_i e^{i\Phi_i(x)}}{\sqrt{2\pi a}}$, 有多个场 (i)

1) 满足 Fermi 统计性质：① $\{\psi_i(x), \psi_j(y)\} = 0$

② $\{\psi_i(x), \psi_j^+(y)\} = \delta_{ij}\delta(x-y)$

2) 关联函数一样：$\langle\psi_1(x_1)\psi_2(x_2)\psi_3(x_3)\psi_4(x_4)\rangle$ 只要两体关联相同才能保证多体关联相同 ⇒ 实验观测椟。

3) 了解 Hilbert space 关系.