

Gapless spin-fluid ground state in a random,
quantum Heisenberg magnet

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We examine the spin- S quantum Heisenberg magnet with Gaussian-random, infinite-range exchange interactions. The quantum-disordered phase is accessed by generalizing to $SU(M)$ symmetry and studying the large M limit. For large S the ground state is a spin-glass, while quantum fluctuations produce a spin-fluid state for small S . The spin-fluid phase is found to be generically gapless - the average, zero temperature, local dynamic spin-susceptibility obeys $\bar{\chi}(\omega) \sim \log(1/|\omega|) + i(\pi/2)\text{sgn}(\omega)$ at low frequencies.

Random quantum spin systems offer a useful laboratory for studying the fascinating interplay between strong interactions and disorder. Though not as complex or intractable as metal-insulator transition systems, they are still rich enough to display a host of unusual physical phenomena. Moreover, they can be realized in a number of experimental systems, many of which have been studied intensively in recent years [1–5].

It is useful to distinguish two different types of possible ground states of a random quantum magnet: (a) a state with magnetic long-range-order ($\langle \hat{\mathcal{S}}_i \rangle \neq 0$ where $\hat{\mathcal{S}}_i$ is the spin operator on site i) which can be a spin-glass, ferromagnet or an antiferromagnet; (b) a quantum disordered (or ‘spin-fluid’) state in which $\langle \hat{\mathcal{S}}_i \rangle = 0$ due to the presence of strong quantum fluctuations. Many properties of the magnetically ordered phase can be described by a semiclassical analysis. In contrast, the spin-fluid phase and its zero-temperature phase transition to the magnetically ordered phase are intrinsically quantum mechanical, and their properties are only very poorly understood. This paper shall mainly focus on the properties of the spin-fluid phase.

In early studies of random-exchange spin-1/2 Heisenberg spin chains by a numerical renormalization group method, Ma *et.al.* and others [6] noted that the low temperature spin susceptibility $\chi(T)$ behaved approximately like $T^{-\alpha}$ with $\alpha < 1$. This behavior, and their analysis, suggested that the quantum disordered phase generically possessed gapless excitations: the low energy excitations arose from a significant probability of finding a pair of spins which were essentially decoupled from the rest of the system, and with only a weak, mutual, effective exchange interaction. Subsequently, the results of a numerical analysis by Bhatt and Lee [7] of a three-dimensional random-exchange spin-1/2 Heisenberg antiferromagnet could be well fit with the same functional form with $\alpha \approx 0.66$. Experiments [8] on many lightly doped semiconductors have also found similar behavior in the low temperature spin susceptibility. More recently, Doty and Fisher [9,10] have obtained numerous exact results on random quantum spin chains; in particular, Fisher [10] proved that the random-exchange, spin-1/2 Heisenberg chain has $\chi \sim 1/(T \log^2(1/T))$ and is gapless.

In this paper we introduce a new solvable, random-exchange, quantum Heisenberg mag-

net - its solution reduces to the determination of the properties of an integro-differential equation, which is a difficult, though not impossible task. Our model possesses infinite-range exchange interactions, and is thus a solvable limit which is complementary to the spin chains. The spin-fluid phase of our model is generically gapless; however the physical mechanism of the gaplessness appears to be quite different from that of the random spin chains. Which of these two limits is closer to realistic three dimensional models remains an open question. Finally, our model is expected to display a transition to a spin glass phase. We have not yet succeeded in unraveling the nature of this transition and that of the replica symmetry breaking in the spin-glass phase - these are issues we hope to address in a future publication.

The main result discussed in this paper is that the $T = 0$, average, local dynamic spin susceptibility of our model has the following form over the entire quantum disordered phase:

$$\bar{\chi}(\omega) = X \left[\log \left(\frac{1}{|\omega|} \right) + i \frac{\pi}{2} \text{sgn}(\omega) \right] + \dots \quad (1)$$

where X is a constant to be determined below, and the omitted terms are subdominant in the limit $|\omega| \rightarrow 0$. A notable feature of this form is that it is identical to the ‘marginal’ Fermi liquid susceptibility proposed on phenomenological grounds by Varma *et. al.* [11] as a description of the electronic properties of the cuprates. It is not completely unreasonable to begin a study of the low-lying spin fluctuations in the cuprates by using the infinite-range quantum spin model described below; however, at present we have no arguments which can determine whether, or how, the ‘marginal’ spectrum will survive in more realistic models with charge carriers and finite-range interactions. Nevertheless, to our knowledge, ours is so far the only bulk model to display the ‘marginal’ spectrum over an entire phase, and one might hope that mathematical structure of the mean-field theory is of broader significance.

We consider the ensemble of Hamiltonians

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j \quad (2)$$

where the sum over i, j extends over N sites, the exchange constants J_{ij} are mutually

uncorrelated and selected with probability $P(J_{ij}) \sim \exp(-J_{ij}^2/(2J^2))$, the $\hat{\mathcal{S}}$ are the spin-operators of the group $SU(M)$, and the states on each site belong to a representation labeled by the integer n_b ($n_b = 2S$ for $SU(2)$; more generally n_b is the number of columns in the Young tableau of the representation [13]). This model has been considered previously by Bray and Moore [12] for the group $SU(2)$: they found strong evidence in favor of the presence of spin-glass order at $T = 0$ for all values of S . Accessing the spin-fluid phase therefore requires considerations of groups other than $SU(2)$: following a technique which has been successful in clean antiferromagnets [13,14], we generalize to the group $SU(M)$ and study the phase diagram in the $n_b - M$ plane. (We have also studied the properties of random $Sp(M)$ [15] magnets with results that are very similar to the simpler $SU(M)$ case considered here). There are three interesting limits in the $n_b - M$ plane:

(A) $n_b \rightarrow \infty$, M fixed: this is the semiclassical limit and yields ground states well within the magnetically ordered spin-glass phase.

(B) $M \rightarrow \infty$, n_b fixed: this takes us deep into the spin-fluid phase.

(C) $M \rightarrow \infty$, $n_b/M = \kappa$ fixed: this is in many ways the most interesting limit, because by varying κ one can interpolate between the spin-glass and spin-fluid phases. Moreover, one expects a phase-transition between these two ground states at a critical value of $\kappa = \kappa_c$.

The structure of the $N \rightarrow \infty$ limit was discussed in Ref. [12]. We express the partition function as a coherent-state path-integral [13], introduce n replicas, average the partition function, and the saddle-point reduces to the quantum mechanics of n replicas of a single spin; assuming the saddle-point is spin-rotation invariant (this is true in both the spin-fluid and spin-glass phases) we obtain the single-site coherent-state path integral $Z_0 = \int \mathcal{D}\hat{\mathcal{S}} \exp(\mathcal{L})$ with

$$\mathcal{L} = S_B + \frac{J^2}{2M} \int_0^{1/T} d\tau d\tau' Q^{ab}(\tau - \tau') \hat{\mathcal{S}}^a(\tau) \cdot \hat{\mathcal{S}}^b(\tau') \quad (3)$$

and the self-consistency condition

$$Q^{ab}(\tau - \tau') = \frac{1}{M^2} \langle \hat{\mathcal{S}}^a(\tau) \cdot \hat{\mathcal{S}}^b(\tau') \rangle_{Z_0} \quad (4)$$

Here $a, b = 1 \dots n$ are replica indices, τ, τ' are Matsubara times, and S_B is the single-spin kinematic Berry phase term [13]. The Edwards-Anderson order parameter [16] for the spin-glass phase is $q_{EA} = Q^{aa}(\tau \rightarrow \infty)$. Moreover, Q^{ab} , $a \neq b$, is τ -independent and non-zero only in the spin-glass phase [17,18].

An exact evaluation of Z_0 is clearly not possible. We therefore consider the large M limit, discussing first the limit (C) above. This is achieved by the Schwinger boson realization of $\hat{\mathcal{S}}$

$$\hat{\mathcal{S}}_\mu^{a\nu} = b_\mu^{a\dagger} b^{a\nu} \quad ; \quad \sum_\mu b_\mu^{a\dagger} b^{a\mu} = n_b \quad (5)$$

where b is a boson annihilation operator, $\mu, \nu = 1 \dots M$. In the large M limit, Eqns (3,4) reduce to the following equations for the boson Green's function $G_B^{ab}(\tau) = (1/M) \sum_\mu \langle T(b^{a\mu}(\tau) b_\mu^{b\dagger}(0)) \rangle$ and its Fourier transform $G_B^{ab}(i\omega_n)$

$$G_B(i\omega_n) = (-i\omega_n + \lambda - \Sigma_B(i\omega_n))^{-1} \quad (6)$$

$$\Sigma_B^{ab}(\tau) = J^2 G_B^{ab}(\tau) G_B^{ab}(\tau) G_B^{ba}(-\tau) \quad (7)$$

while $Q^{ab}(\tau) = G_B^{ab}(\tau) G_B^{ba}(-\tau)$. Here λ is a chemical potential set by the constraint $G^{aa}(\tau = 0^-) = \kappa$. These two equations can be combined into a single integro-differential equation for $G_B^{ab}(\tau)$. We also require that solutions satisfy conditions imposed by the spectral representation of a boson Green's function: $G_B^{aa}(z)$ is analytic for $\text{Im}(z) > 0$, $\omega \text{Im}(G_B^{aa}(\omega + i0^+)) \geq 0$ and $G_B^{aa}(z) \sim -1/z$ for large $|z|$. The replica-diagonal components of Eqns (6,7) also bear a formal resemblance to a perturbative solution of the infinite-dimensional Hubbard model [19]; however there are some significant differences which turn out to have dramatic consequences in the nature of the solution.

We will focus here only on the spin-fluid phase, whence all correlations are replica-diagonal, and replica indices will be dropped. An immediate consequence of (6,7) is that the zero-temperature boson spectrum must be gapless! For suppose that the spectral weight $\text{Im}(G_B(\omega + i0^+)) = 0$ for $|\omega| < \Delta$; then (7), expressed in real frequencies, implies that

$\text{Im}(\Sigma_B(\omega + i0^+)) = 0$ for $|\omega| < 3\Delta$ - this agrees with the real-frequency version of (6) only if $\Delta = 0$.

Let us focus on the low-frequency behavior of G_B : assume that $G_B(\omega) \sim \omega^\mu$. Then from (7) we get $\text{Im}(\Sigma_B(\omega)) \sim \omega^{2+3\mu}$. This can be consistent with (6) only if $\lambda = \Sigma_B(\omega = 0)$ and $\mu = -1/2$. As G_B is analytic in the upper-half frequency plane, we write

$$G_B(z) = \frac{i\Lambda e^{-i\theta}}{\sqrt{z}} + \dots \quad \text{Im}(z) > 0 \quad (8)$$

where $\Lambda > 0$. The positivity conditions on the spectral weight require $0 < \theta < \pi/2$. Inserting this into (7) we find for $\text{Im}(z) > 0$ that

$$\Sigma_B(z) = \Sigma_B(0) + i \frac{J^2 \Lambda^3 \sin(2\theta)}{\pi} e^{i\theta} \sqrt{z} + \dots \quad (9)$$

Finally, this is consistent with (6) if $\lambda = \Sigma_B(0)$ and

$$\Lambda = \left(\frac{\pi}{J^2 \sin(2\theta)} \right)^{1/4} \quad (10)$$

The parameter θ remains undetermined. This is fortunate, as we need a single degree of freedom to satisfy the boson-number constraint $G_B(\tau = 0^-) = \kappa$. We will treat θ as the independent parameter, with $\kappa(\theta)$ a function to be determined. We expect $\kappa \rightarrow 0$, as $\theta \rightarrow 0$; increasing θ therefore corresponds to increasing ‘spin’. We can also determine the low-frequency behavior of the spin-susceptibility $\chi(\tau) = Q^{aa}(\tau)$; we find that it has the form (1) with the constant X given by

$$X = \frac{(\pi \sin(2\theta))^{1/2}}{2J} \quad (11)$$

We expect the low-frequency susceptibility to increase monotonically with increasing ‘spin’ κ , and therefore increasing θ . However, X has a maximum at $\theta = \pi/4$. This leads us to conjecture that the transition to the spin-glass phase occurs at $\theta = \pi/4$ and only the range of values $0 < \theta < \pi/4$ correspond to the spin-fluid phase. A second possibility, which cannot be ruled out, is that there is a first-order transition to a spin-glass phase at a value of $\theta < \pi/4$.

We have performed a detailed numerical study of Eqns (6,7) to determine the complete frequency dependence of Green's function. We chose a value of θ , and a trial form for $\text{Im}(G_B(\omega + i0^+))$ whose low-frequency limit satisfies Eqn (8). The real-frequency version of (7) expresses $\text{Im}(\Sigma_B(\omega + i0^+))$ as a double convolution of $\text{Im}(G_B(\omega + i0^+))$; these convolutions were performed by direct numerical integration. The real part $\text{Re}(\Sigma(\omega + i0^+))$ was determined by a Kramers-Kronig transform, and λ was set at $\lambda = \Sigma_B(\omega = 0)$. Finally $\text{Im}(G_B(\omega + i0^+))$ was determined from (6) and the whole procedure was iterated, until the solution converged. The singularities in G_B and Σ_B at low frequencies were accounted for by performing the numerical integration in a variable $x \sim \sqrt{\omega}$ at the integration end-points - this absorbed the leading singularity. Subleading singularities were treated by using a dual mesh-size in the integration - a very fine mesh (x spacing = $0.0003\sqrt{J}$) was used at the end-points and a coarse mesh elsewhere. Upto 1700 points were used in the numerical integration. There was little difficulty in converging to a solution for values of θ less than approximately $\pi/6$; we are reasonably certain that there are physically sensible solutions of (6,7) for this range values of θ . One such solution, at $\theta = \pi/12$ is shown in Fig 1 which was found to have $\kappa = 0.051$. The numerical iteration became increasingly unstable with increasing θ and did not converge to any smooth solution for large θ . Our numerical experience is consistent with the conjecture that there are no physically sensible solutions for $\theta > \pi/4$ - this is the range of values of θ where we expect a spin glass phase.

A complementary picture of the spin-fluid phase can be obtained by studying the large M limit (B). This takes $M \rightarrow \infty$ at fixed spin n_b - one is then in a region of the phase diagram well away from the transition to the spin-glass phase. For technical reasons it is also necessary to introduce of order M rows in the Young tableau of the spin representation; this is discussed in some detail in Ref. [13]. We will focus on the particle-hole symmetric representations which have n_b columns and $M/2$ rows as realized by the following operator decomposition

$$\hat{S}_\mu^{a\nu} = \sum_\alpha f_{\alpha\mu}^{a\dagger} f^{a\alpha\nu} \quad ; \quad \sum_\mu f_{\alpha\mu}^{a\dagger} f^{a\beta\mu} = \delta_\alpha^\beta M/2 \quad (12)$$

where f is a fermion annihilation operator. The fermions carry replica, spin, and ‘color’ indices $\alpha, \beta = 1 \dots n_b$. The subsequent analysis parallels closely that for the bosons. The fermion Green’s function is replica and color diagonal and its only non-zero component is $G_F(\tau) = (1/M) \sum_{\mu} \langle T(f^{\alpha\mu}(\tau) f_{\alpha\mu}^{\dagger}(0)) \rangle$. The only changes are that Eqn (7) is replaced by

$$\Sigma_F(\tau) = -J^2 n_b G_F^2(\tau) G_F(-\tau) \quad (13)$$

and the positivity constraints on the fermion spectral weight is $\text{Im}(G_F(\omega + i0^+)) > 0$. The presence of particle-hole symmetry requires that $\text{Im}(G_F(\omega + i0^+))$ is an even function of ω - this simplifies the analysis considerably. The low-frequency limit of G_F can be determined completely:

$$G_F(z) = \left(\frac{\pi}{4J^2 n_b} \right)^{1/4} \frac{(-1+i)}{\sqrt{z}} + \dots \quad \text{Im}(z) > 0$$

The dynamical susceptibility is found to have the same low-frequency dependence as in (1), with the constant X now given by

$$X = \frac{\sqrt{\pi n_b}}{2J} \quad (14)$$

As expected, X is a monotonically increasing function of n_b . A complete solution was obtained numerically and the results are shown in Fig 2.

The key unresolved issue in this work is of course the range of validity of the dynamic susceptibility in Eqn (1) - this is important in determining the significance, if any, of our results for dynamic neutron scattering experiments on random antiferromagnets [1–3]: (a) What are the consequences of $1/M$ fluctuations in the infinite-range model Z_0 (Eqns (3,4)) ? This question has been answered for a simpler infinite-range quantum spin-glass [18] where it was found that $1/M$ corrections *did not modify* the low-frequency behavior of the spectral weight. The structure of the fluctuations about the present mean-field theory is much more involved, but it is reasonable to expect that a similar phenomenon will occur here. (b) Is there an upper-critical dimension above which the properties of Z_0 describe the spin-fluid phase or its phase transition to spin-glass order in antiferromagnets with finite-range interactions ? (c) How are these results modified in ensembles with a nonzero average J_{ij} ?

We thank A. Georges, C.M. Varma, and A.P. Young for helpful discussions. This research was supported by NSF Grant No. DMR 8857228 and the A.P. Sloan Foundation.

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FIGURES

FIG. 1. Spectral weights of G_B and χ for the bosonic representation (5) at $\theta = \pi/12$. The ‘spin’ is n_b is order M : $n_b = \kappa M$, and for this value of θ , we found $\kappa = 0.051$. The sum-rule for χ is $\int_0^\infty d\omega \text{Im}(\chi(\omega)) = \pi\kappa(1 + \kappa)$

FIG. 2. Spectral weights of G_F and χ for the fermionic representation (12). Now the spin n_b is of order unity, and upto rescaling, the solution has the same form for all n_b . The sum-rule for χ is $\int_0^\infty d\omega \text{Im}(\chi(\omega)) = \pi n_b/4$