3.17 拓扑场论课程讲义 (助教整理)

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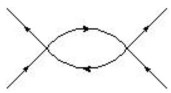
本节可供参考：Shankar 第 14 章

注：严格来说以下画的费曼图应该是虚线加上箭头，因为这里是玻色场

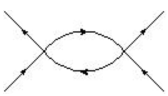


$$\sim \int dk \frac{1}{(k-q)^2 - m^2 + i\epsilon} \frac{1}{k^2 - m^2 + i\epsilon} \tag{1}$$

已知 $\int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m^2} = \frac{\Gamma(1-\frac{d}{2})}{(4\pi)^{d/2} m^{2-d}}$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2} \sim \int_0^{\bigwedge \to +\infty} \frac{dk k^3}{k^2 - m^2} \sim \int dk \cdot k \sim \bigwedge^2$$

$$\int \frac{d^d k}{(k^2 + m^2)^2} \sim \int_0^{+\infty} \frac{dk k^{d-1-\epsilon}}{(k^2 + m^2)^2} \sim ln \bigwedge (when\ d = 4)$$

故  $\sim ln \bigwedge (d = 4)$

注意 1：别纠结计算细节及最后精确结果，完全掌握需要半年到一年，做科研需要的时候再学，到时间有人跟你讲怎么做

注意 2：只需掌握整体图像和发散情况 $\sim ln \bigwedge, \sim \bigwedge^\alpha$ 即可。要将物理图像和细节分开，先掌握物理图像而不纠结于细节

1 Dyson Equation

上一节课我们提到了质量修正

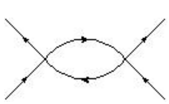
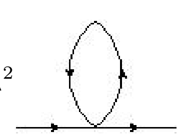
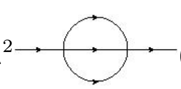
$$g = g_0 + g_0 \varSigma g_0 + \cdots, g = \frac{1}{p^2 - m^2 + i\epsilon}, g_0 = \frac{1}{p^2 - m_0^2 + i\epsilon}$$

$$m^2 = m_0^2 + \varSigma (\varSigma \text{为自由能})$$

连通定理 $< O >= \sum_{n=0}^{+\infty} \frac{i^n}{n!} < O S_I^n >_0^c$，详细证明见点击链接

一般设定 $V = 1$，因为 $\frac{1}{V} \sum_k$ 成对出现，最终 V 往往会约分掉

连通定理告诉我们要舍弃掉不连通的图。理解 Feynman Diagram 就一定要理解基本图

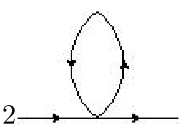
 $\propto \wedge^2$  $\propto \wedge^2$  $(sun - rise - diagram) :?$ (2)

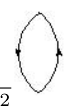
滑稽一下：企鹅费曼图

$$O = \phi^*(p)\phi(p)$$

只考虑一阶修正如下所示：

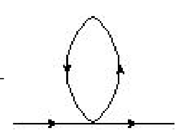
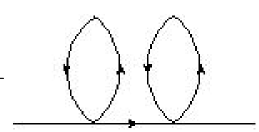
$$< O > = < O >_c + \frac{i}{1!}(-\frac{\lambda}{4!}) \sum_{k_1 k_2 k_3} < \phi^*(p)\phi(p)X >_0^c$$

$$= \frac{i}{p^2 - m^2} + \frac{i}{1!}(-\frac{\lambda}{4!}) * 12$$  [12 为简并度] (3)

$$= \frac{i}{p^2 - m^2} + \frac{i}{1!}(-\frac{\lambda}{4!}) * 12 \frac{i}{p^2 - m^2}$$  $$* \frac{i}{p^2 - m^2}$$

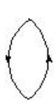
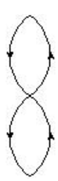
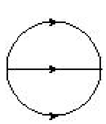
$$= \frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2} \Sigma \frac{i}{p^2 - m^2}$$

考虑更高阶修正：

单条腿修正 = +  +  $+ \cdots\cdots$ (4)

$$< O > = < O >_c + \frac{i}{1!}(-\frac{\lambda}{4!}) \sum_{k_1 k_2 k_3} < \phi^*(p)\phi(p)X >_0^c + \frac{i^2}{2!}(-\frac{\lambda}{4!})^2 \sum_{k_1 k_2 k_3 k_4 k_5 k_6} < \phi^*(p)\phi(p)XX >_0^c$$

$$\rightarrow g = g_0 + g_0 \Sigma g_0 + g_0 \Sigma g_0 \Sigma g_0 + \cdots \tag{5}$$

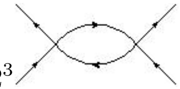
$\Sigma =$  +  +  $+ \cdots$

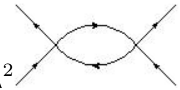
故我得到了 Dyson Equation，可进一步得到真实质量 m_0 和观测质量 m 的关系

$$m_0^2 + \Sigma^2 = m^2 \tag{6}$$

下面我们导出相互作用系数 λ 的修正，定义 $O =< \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4) >$，其中 $\sum p_i = 0$

$$O = -i\lambda X + \frac{i^2}{2!}(-\frac{\lambda}{4!})^2 \sum_{kq} < X_k X_q \phi\phi\phi\phi >_0^C$$

$$= -i\lambda X + \frac{i^2}{2!}(-\frac{\lambda}{4!})^2 C_4^2 C_4^2 C_4^2 * 2^3$$  (这里不考虑单条腿修正) (7)

$$-i\lambda_{eff} X = -i\lambda X - \frac{3}{2}\lambda^2$$  $+ \cdots$

$$\to \lambda_{eff} = \lambda_{eff}(\lambda, \wedge)$$

2 重整化方法初步 (可参考Shankar 第 14 章)

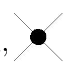
Wilson 发明，最早发在 PRB，拿了诺奖，也因此 tenured 了

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4$$

令 $\phi = \sqrt{Z}\phi_R$，有

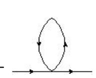
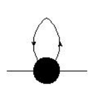
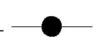
$$\begin{aligned} L &= L_0 + L_I \\ L_0 &= \frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{m_R^2}{2}\phi_R^2 \\ L_I &= -\frac{\lambda_R}{4!}\phi_R^4 + \frac{1}{2}[\delta_Z(\partial_\mu\phi_R)^2 - \delta m \phi_R^2] - \frac{\delta\lambda}{4!}\phi_R^4 \\ < \phi^*(p)\phi(p) > &= \frac{Z}{p^2 - m^2 + i\epsilon} + finite \end{aligned} \tag{8}$$

重新定义重整化后的费曼图：

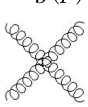
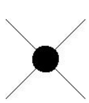
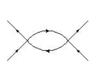
$< \phi_R^*(p)\phi_R(p) >=$ $= \dfrac{i}{p^2 - m_R^2}$  $= (\delta z p^2 - \delta m), X = -i\lambda,$  $= -i\delta\lambda$ (9)

定义修正后传播子如下所示：

注：这里定义的修正后传播子知识一种表示，严格来说这个螺旋线表示其他粒子

 $=$ $+$  $+$  $+$ 

$$g(p) = g_0(p) + \Sigma$$

 $= X +$  $+$  (10)

$$\lambda_{eff}(p) = \lambda_R + \delta\lambda + second - term(\text{要求求出相关参数去除掉相互作用，即:}\lambda_{eff}(p) = \lambda_R)$$

Wilson 重整化群：$\wedge \to 0.99\wedge \to 0.99^2\wedge \to \cdots$，最后出现 Wilson fixed point，附近有相变。