Casimir Effect and Vacuum Fluctuations

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Abstract

Vacuum fluctuations have observable consequences, like the Casimir force between two plates in vacuum. There now exists good agreement between theory and experimental measurements of this force. The role of the Casimir effect in diverse fields such as Quantum Field Theory, Condensed Matter Physics and Atomic Physics etc is discussed. A brief description of current applications to nanotechnology and nanomechanical devices is also given.

Introduction

The Casimir effect is a small attractive force which acts between two closed parallel uncharged conducting plates. It is due to quantum vacuum fluctuation of the electromagnetic field.

The effect was predicted by the Dutch physicist Hendrick Casimir in 1948. For many years the Casimir effect was little more than a theoretical curiosity. But interest in the phenomenon has blossomed in recent years. Experimental physicists have realized that the Casimir force affects the working of micromachined devices, while advances in instrumentation have enabled the force to be measured with ever-greater accuracy [1].

Vacuum and vacuum fluctuation

Although the Casimir force seems completely counterintuitive, it is actually well understood. In the old days of the classical mechanics the vacuum was what remained when emptied a container of all its particles and lowered temperature down to absolute zero. The arrival of quantum mechanics, however, completely changed our notion of a vacuum. All fields – in particular electromagnetic fields – have fluctuations. The vacuum is not really empty. It is filled with virtual particles, which are in a continuous state of fluctuation. Virtual particle-antiparticle pair can be created from vacuum and annihilated back to vacuum. These virtual particles exist for a time dictated by Heisenberg Uncertainty relation.

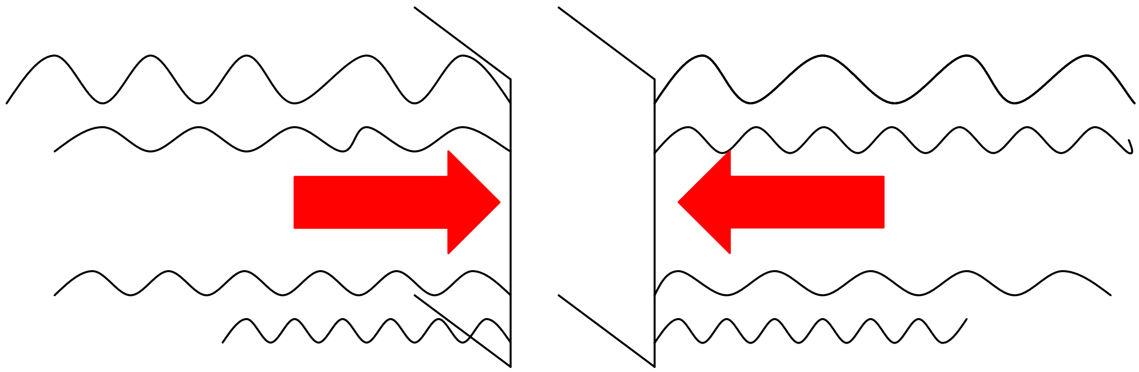
$$\Delta E.\Delta t \approx \hbar$$

Photons (quanta of electromagnetic waves) are the dominant virtual particles in vacuum fluctuations but other particles produced as well.
A vacuum is not simply 'nothing at all', but is best pictured as a superposition of many different states of electromagnetic field. Thus the creation and subsequent absorption of a photon by the vacuum implies the vacuum fluctuates.

However, vacuum fluctuations are not some abstraction of a physicist's mind. They have observable consequences that can be directly visualized in experiments on a microscopic scale. For example, an atom in an excited state will not remain there infinitely long, but will return to its ground state by spontaneously emitting a photon. This phenomenon is a consequence of vacuum fluctuations. The Casimir force is the most famous mechanical effect of vacuum fluctuation. The Casimir force appears between two mirrors in vacuum as a consequence of the radiation pressure of vacuum fluctuation [2].

The Casimir effect

Casimir calculated the force in a geometrical configuration where two plane mirrors are placed in vacuum.



Explanation of the Casimir effect

- The origin of the Casimir effect is essential geometrical
- Since the space between two plates is different from the space outside, the vacuum fluctuations are also different in the two regions.
- The fluctuations exert different forces on the plates from inside and outside, resulting in a net pressure.

Casimir force and particle statistics

- Particles other than photon also contribute a small effect but only the photon force is measurable.
- All bosons such as photons produce an attractive Casimir force while fermions make a repulsive contribution.

Brief theoretical derivation

The Hamiltonian of the electromagnetic field can be written in a normal-mode decomposition as:

$$\hat{H} = \sum_k \sum_\lambda \hbar\omega_k (\hat{a}_{k\lambda}^+ \hat{a}_{k\lambda} + \frac{1}{2})$$

Define the vacuum state as the state with no photons in any mode. Thus the vacuum energy is

$$E_0 = \tfrac{1}{2} \sum_k \sum_\lambda \hbar\omega_k = \sum_k \hbar\omega_k$$

First consider a one-dimensional system where two conducting reflecting mirrors are placed a distance L apart. The presence of the cavity allows only discrete modes, with a density of modes $k = \frac{\nu\pi}{L}$.

Thus, we can write for the energy inside the cavity

$$E_0 = \sum_k \hbar\omega_k = \sum_k \hbar ck = \frac{\pi\hbar c}{L}\sum_{\nu=1}^{\infty}\nu$$

The vacuum energy in the same space without the mirrors is

$$E_{free} = \frac{\pi\hbar c}{L}\int_0^{\infty}\nu d\nu$$

Both of these energies are infinite. Thus, the change in energy produced by the presence of the cavity is

$$\Delta E = E_0 - E_{free} = \frac{\pi\hbar c}{L}\left[\sum_{\nu=1}^{\infty}\nu - \int_0^{\infty}\nu d\nu\right].$$

Using the Euler-Maclaurin summation formula and conversion factor, $\lim_{\varepsilon\to\infty} e^{-\varepsilon\nu}$, we have

$$\Delta E = -\frac{\pi\hbar c}{12L}$$

Therefore, there is an attractive force between the two mirrors

$$F = \frac{\partial\Delta E}{\partial L} - \frac{\pi\hbar c}{12L^2}.$$

We now consider three dimensions. If we consider a box with two sides (x and y) of length D, and the third (z) of length L, where L<<D, the sums for x and y can be replaced by integrals, and the energy difference can be written by:

$$\Delta E = \frac{D^2\hbar c}{\pi^2}\left[\sum_{\nu=1}^{\infty}\int_0^{\infty}dk_x\int_0^{\infty}dk_y\left(k_x^2 + k_y^2 + \frac{\nu^2\pi^2}{L^2}\right)^{1/2} - \frac{L}{\pi}\int_0^{\infty}dk_x\int_0^{\infty}dk_y\int_0^{\infty}dk_z\left(k_x^2 + k_y^2 + k_z^2\right)^{1/2}\right].$$

Using the third derivative in the Euler-Maclaurin summation formula, we have

$$\Delta E = -\left(\frac{\pi^2\hbar c}{720L^3}\right)D^2.$$

Therefore, the force F_{cas} and energy E_{cas} can be written as:

$$F_{cas} = \frac{\pi^2}{240}\frac{\hbar c}{L^4}A$$

$$E_{cas} = \frac{\pi^2}{720}\frac{\hbar c}{L^3}A$$

Where $\hbar$ is Planck's constant, c is speed of light and A is area of the mirrors. The signs correspond to a convention opposite to the standard convention of thermodynamics: the force is attractive and corresponds to a negative pressure; meanwhile, the energy is binding energy corresponding to a mean energy density slightly smaller inside the cavity than in the outside vacuum. Note that the energy density and pressure obey the equation of state of pure radiation.

An important feature of the Casimir effect is that even though it is quantum in nature, it predicts a force between macroscopic bodies. For two plane-parallel metallic plates of area $A = 1cm^2$ separated by large distance (on the atomic scale) of $L = 1\mu m$ the value of the attractive force is $F_{cas} \approx 1.3 \times 10^{-7}\,\mathrm{N}$. This force while small, is now within the range of modern laboratory force measurement technique. Unique to the Casimir force is its strong dependence on shape, switching from attractive to repulsive as a function of the geometry and topology of quantization manifold [3,4]. This makes the Casimir effect a likely candidate for applications in nanotechnologies and nanoelectromechanical devices. The attraction between neutral metallic plates in a vacuum was first observed experimentally in [5]

The role of the Casimir effect in different field of physics

The Casimir effect is an interdisciplinary subject. It plays an important role in a variety of fields of physics.

In quantum field theory, the Casimir effect finds three main applications. In the bag model of hadrons in quantum chomodynamics the Casimir energy of quark and gluon fields makes essential contributions to the total nucleon energy. In Kaluza-Klein field theories Casimir effect offers one of the most effective mechanisms for spontaneous compactification of extra spatial opportunities to obtain more strong constraints for the parameters of long-range interactions and light elementary particles predicted by unified gauge theories, supersymmetry, supergravity, and string theory.

In condensed matter physics, the Casimir effect leads to attractive and repulsive forces between the close material boundaries which depend on the configuration geometry, on temperature, and on the electrical and mechanical properties of the boundary surface. It is responsible for some properties of thin films and should be taken into account in investigations of surface tension and latent heat. The Casimir effect plays an important role in both bulk and surface critical phenomena.

In gravitation, astrophysics and cosmology, the Casimir effect arises in space-times with non-trivial topology. The vacuum polarization resulting from the Casimir effect can drive the inflation process. In the theory of structure formation of the Universe due to topological defects, the Casimir vacuum polarization near the cosmic string may play an important role.

In atomic physics, the long-range Casimir interaction leads to corrections to energy levels of Rydberg states. A number of the Casimir-type effects arise in cavity quantum electrodynamics when the radiative processes and associated energy shifts are modified by the presence of the cavity walls.

In Mathematical physics, the investigation of the Casimir effect has stimulated the development of powerful regularization and renormalization techniques based on the use of zeta function and heat Kernel expansion.

Measurements of the Casimir force

The first experiments dealing with the measurements of the Casimir force were done by M. J. Sparnaay [5]**.** The experimental technique based on a spring balance and parallel plates served to set the benchmarks. They also clarified the problems associated with other Casimir force measurements. The measurement was the first indication of an attractive Casimir force between metallic surfaces, approximately in line with the expectations. Most importantly from the experiments, M. J. Sparnaay clearly elucidated the problems that need to be overcome for a rigorous and conclusive measurement of the Casimir force. These fundamental requirements are:

(i) Clean plate surfaces completely free of chemical imparities and dust particles.

(ii) Precise and reproducible measurement of the separation between the two surfaces. In particular a measurement of the average distance on contact of the two surfaces which is nonzero due to the roughness of the metal surfaces and the presence of dust.

(iii) Low electrostatic changes on surface and low potential differences between the surfaces.

Each of the above instrumental or material requirements is difficult to obtain in practice, and certainly very difficult to obtain together. They have bedeviled the field because at least one or more of the above were neglected in the earlier force measurements. But modern experimental techniques have made it possible to satisfy all three requirements simultaneously. A comprehensive review of experiments is given in [6]. While Sparnaay's results were consistent with Casimir's theory, the uncertainty was about 100%.

Several experiments, performed over the years, have reached an experimental precision at the % level by using an atomic force microscope (AFM) [7] or micro-electromechanical system (MEMS) [8]. Furthermore, the measurements agree with theory also at the % level provided that deviations from the ideal situation considered by Casimir are properly accounted for. Some of the measurements were made using a sphere and a plate, instead of two parallel plates. Lamoreaux, in 1997, used a flat plate and a spherical lens and achieved an accuracy of 5% with theory. In the case of mirrors, the real optical properties of the mirrors have to be taken into account. A key experiment was performed by Mohideen and Roy, using an aluminum–coated sphere and a flat plate. They were able to obtain agreement within 1% of theory.

Application of the Casimir force in nanotechnology

Casimir force fundamentally influences the performance and yield of nanodevices. Most present day nanomechanical devices are based on thin cantilever beams above a silicon substrate fabricated by photolithography followed by dry and wet chemical etching. The cantilever's motion is greatly influenced by the Casimir force which dominates over other forces at distance of a few nanometers. Thus, movable components in nanoscale devices fabricated at distances less than 100nm between each other often stick together due to the strong Casimir force, leading to the collapse of movable element to the substrate or the collapse of neighboring components during nanoscale device operation. Therefore this phenomenon severely restricts the yield and operation of the devices, and the Casimir forces might well set fundamental limits on the performance and the possible density of devices that can be optimized on a single chip.

Conclusion

The Casimir effect has become the subject of diverse studies of general physical interest in variety of fields. It is equally interesting and important for Quantum Field Theory, Condensed Matter Physics, Gravitation, Astrophysics and cosmology, Atomic physics and Mathematical Physics. Currently the Casimir effect has been advanced as a new powerful test for hypothetical long-range interactions, including corrections to Newtonian gravitational law at small distances, predicted by the unified gauge theories, supersymmetry, supergravity and string theory. It is also gaining in technological importance in vital applications such as in nanoelectromechanical devices. Precision measurements of the Casimir force have led to an excellent agreement between the experiments by means of atomic force microscope and theory.

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