

离散化.

积分.
偏微分方程. 矩阵.
:

多项式插值.

$$f(x) = \sum_{i=1}^{n-1} e_i(x_i) y_i + O(x^N)$$

Runge 现象



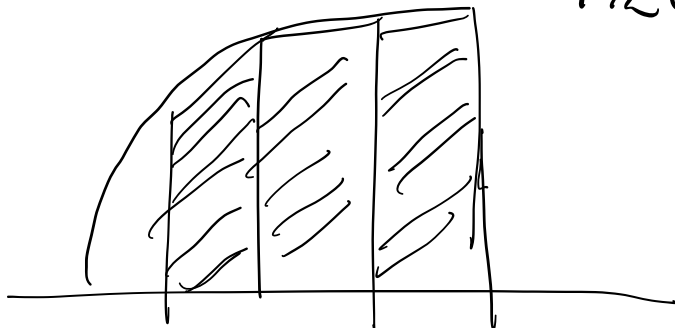
PPT (Integrate $[f, x]$,

Integrate $[f, \{x, a, b\}]$,

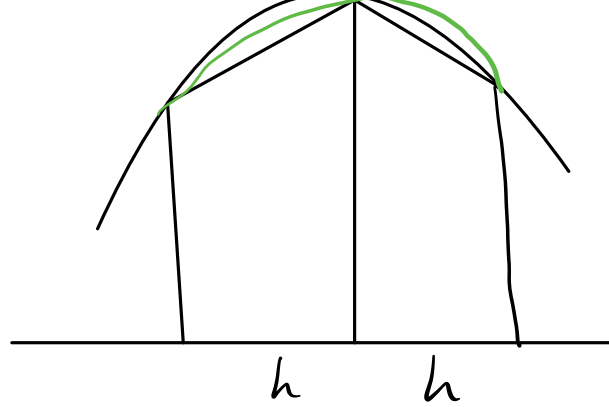
Interpolating Polynomials)

1) Simpson
2) 高阶

1次插值.



$$S_1 = \frac{h}{2} (y_1 + 2y_2 + y_3)$$



三次插值

$$S_2 = \frac{h}{3} (y_1 + 4y_2 + y_3)$$

$S_2 - S_1 = O(h^3)$. Taylor 展开

1D $S = h \sum C_i f_i$

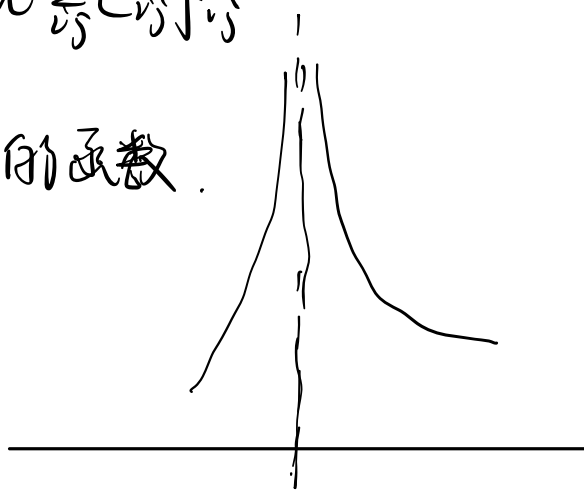
$C_i = (1, 2, 2, \dots, 2, 1)$

$C_i = (1, 4, 2, 4, 2, \dots, 4, 1)$

2D. $S = h^2 \sum C_{ij} f_{ij}$

变化剧烈的函数.

奇点



$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

or

$$\int_{-1}^1 \frac{1}{\sqrt{|x|}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{x}} dx$$

$$+ \int_{-1}^0 \frac{1}{\sqrt{-x}} dx.$$

等步长不合适,

矩阵

线性代数 程序

① $Ax = b$

$x = A^{-1}b$

$O(N^3)$

② 本征值问题

两个函数库 (BLAS, LAPACK)

(FFTW, ARPACK)

方阵 ($n=m$)

厄密 $A = A^+$

么正变换

$U^+ A U = \lambda$

非厄密 $A \neq A^+$ 相似变换

$P^{-1} A P = \lambda$

R $O(N)$

C $O(N^2)$

(类型)

矩阵

A_{mn}

非方阵 ($n \neq m$)

奇异值分解 (SVD)

$A = U D V^T$



$\sum u u^T = 1$ U

D

$\sum v v^T = 1$

V

Mathematica

Single Value Decomposition [m]

矩阵分解

① LU分解. (LU Decomposition)

$$\text{找到 } A = \begin{pmatrix} \text{///} \end{pmatrix} \times \begin{pmatrix} \text{///} \end{pmatrix}$$

L U

上三角 下三角.

用处: 求解 $Ax = b$ $Ax = LUx = b$

$$\Rightarrow \begin{cases} Ly = b \\ y = Ux \end{cases}$$

② QR分解. (QR Decomposition)

$$\begin{pmatrix} \text{Unitary} \end{pmatrix} \times \begin{pmatrix} \text{///} \end{pmatrix} = A.$$

Gram-Schmidt 分解.

③ SVD分解. (非方阵, 方阵都可)

特殊矩阵

1. 范德龙行列式.

$$\begin{bmatrix} a_1 & a_1^2 & \dots & a_1^n \\ a_2 & a_2^2 & \dots & a_2^n \\ \vdots & \vdots & \dots & \vdots \\ a_n & a_n^2 & \dots & a_n^n \end{bmatrix}$$

2. Pfaffian $2n \times 2n$ skew matrix $A^T = -A$

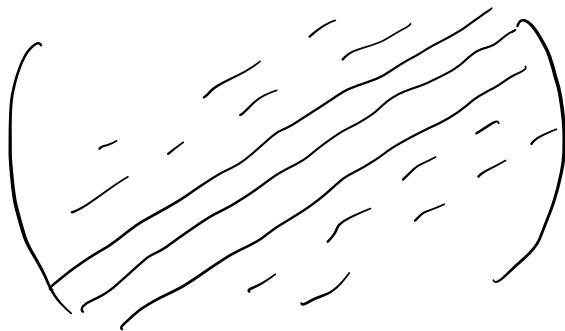
$$Pf^2(A) = \det(A)$$

3. 三对角矩阵. $O(N^2)$ 占用内存小, $N = 10^8$

4. Toeplitz matrix

$$A = \begin{pmatrix} t_0 & t_1 & t_2 & \dots & & \\ t_1 & t_0 & & & & \\ t_2 & & & & & \\ \vdots & & & & & \\ & & & & & t_1 \\ & & & & t_0 & \\ & & & & t_1 & \end{pmatrix}$$

Circular matrix.



5. 随机矩阵.

$$P(H) = A e^{-\text{tr}(H^2)/\sigma^2}$$

作业:

求解 google matrix
的本征值, 讨论其分布

$$G = CP + (I - C)E$$

随机矩阵. $n \times n$
rank-1

$N > 2000$ 矩阵

矩阵乘法的算法复杂度.

$$AB = \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{pmatrix} \begin{pmatrix} | \\ | \\ | \\ | \\ | \\ | \\ | \\ | \end{pmatrix}$$

$$O(n \times n \times n) = O(n^3)$$

乘法 + 10 加法.

1969 斯特拉森

$$\begin{pmatrix} \text{---} \\ | \\ \text{---} \end{pmatrix} \times \begin{pmatrix} \text{---} \\ | \\ \text{---} \end{pmatrix} = 8 \text{ 次乘法.}$$

2x2 A 2x2 B.

可以只做 7 次乘法

$$P_1 = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$P_2 = (A_{21} + A_{22})B_{11}$$

$$P_3 = A_{11}(B_{12} - B_{22})$$

$$P_4 = A_{22}(B_{21} - B_{22})$$

$$P_5 = (A_{11} + A_{12})B_{22}$$

$$P_6 = (A_{21} - A_{11})(B_{11} + B_{12})$$

$$P_7 = (A_{12} - A_{22})(B_{21} + B_{22})$$

$$\Rightarrow \begin{cases} C_{11} = P_1 + P_2 - P_5 + P_7 \\ C_{12} = P_3 + P_5 \\ C_{21} = P_2 + P_4 \\ C_{22} = P_1 - P_2 + P_3 + P_6 \end{cases}$$

$$O(n^{2.58})$$