$C_V \sim t^{-\alpha}$

$m \sim |t|^{\beta}$

$\chi = \frac{\partial m}{\partial h} \sim |t|^{-\gamma}$

$m \sim h^{\frac{1}{\delta}}$

$g(r) \sim \frac{e^{-r/\xi}}{r^{2-d+\eta}}$

$\xi \sim t^{-\nu}$

η.

6个临界指数.实际上只有2个独立变量.

相变 (关联长度发散) =
① 平均场
② 数值(MC, DMRG, ED ....)
③ 标度假设.

自由能是 t, h 的广义齐次函数.

$f(t,h) = \lambda^{-1} f(t\lambda^{a}, h\lambda^{b})$

标度假设.

$t = \frac{T-T_c}{T_c}$, $h = \frac{h-h_c}{h_c}$

$$\begin{cases} C_V \sim |t|^{-\alpha} \\ m \sim (-t)^{\beta} \\ \chi \sim |t|^{-\gamma} \\ m \sim h^{\frac{1}{\delta}} \quad \big|_{t=0} \\ \xi \sim |t|^{-\nu d} \end{cases}$$

t=0 时

$G(r) = \frac{1}{r^{d-2+\eta}}$

$\{\alpha, \beta, \gamma, \delta, \nu, \eta\}$ 是否有联系?

Ref: Kardar 书.

The scaling Hypothesis

出发点: 相变点 → ξ → 发散

$$\begin{cases} f = \min\left(\frac{t}{2}m^2 + um^4 - hm\right) \\ e^{-\beta F} = Z \end{cases}$$

$$\begin{cases} t \to 0 \\ u \to 0 \\ h \to 0 \end{cases}$$

$$f(\lambda t, \lambda^{3/2} h) = \lambda^2 f(t, h)$$

$$\Downarrow$$

$$f(t, h) = t^2 f\left(1, \frac{h}{t^{-3/2}}\right)$$

$$f(\lambda t, \lambda^{\frac{3}{2}} h) = \min\left(\frac{t\lambda}{2}m^2 + um^4 - \lambda^{\Delta} hm\right) = \lambda^{\nu} \min\left(\frac{t}{2}m^2 + um^4 - hm\right)$$

回到 f

① $h=0$, $t \neq 0$. $\quad f = t^{\alpha} f\left(\frac{h}{t^{\Delta}}\right) = t^2 f(0)$, $\quad f = t^{2-\alpha} f\left(\frac{h}{t^{\Delta}}\right)$

② $t=0$, $h \neq 0$. $\quad f = t^{2-\frac{4}{3}\Delta} h^{4/3}$

① $h=0$, $f = \frac{t}{2}m^2 + um^4$. $\Rightarrow$ $f = \frac{t}{2}\left(-\frac{t}{4u}\right) + u\left(\frac{-t}{4u}\right)^2 \propto t^2/u$

$$= -\frac{1}{16}\frac{t^2}{u} \Rightarrow f(0) = -\frac{1}{16u}$$

② $t=0$. $h \neq 0$.

$$f = um^4 - hm \Rightarrow \frac{\partial f}{\partial m} = 0 \Rightarrow m = \left(\frac{h}{4u}\right)^{1/3}$$

$$f = t^2 \left(\frac{h}{t^\Delta}\right)^{4/3} \qquad \Delta = 3/2.$$

$$f = |t|^{2-\alpha} g\left(\frac{h}{t^\Delta}\right)$$

热容： $C_v \sim \left.\frac{\partial^2 f}{\partial t^2}\right|_{h=0} = t^{-\alpha}$

磁化强度.

$$m \sim \left.\frac{\partial f}{\partial h}\right|_{h=0} = |t|^{2-\alpha-\Delta} \left.\frac{\partial g\left(\frac{h}{t^\Delta}\right)}{\partial h}\right|_{h=0}.$$

$$\propto |t|^{2-\alpha-\Delta}$$

$$\Rightarrow \quad \beta = 2-\alpha-\Delta$$

磁化率. $\chi = \left.\left(\frac{\partial m}{\partial h}\right)\right|_{h=0} = t^{2-\alpha-2\Delta}$

$$\Rightarrow \quad \gamma = 2\Delta + \alpha - 2.$$

Rushbroke. $\underline{\alpha + 2\beta + \gamma} = \alpha + (4 - 2\alpha - 2\Delta) + 2\Delta + \alpha - 2$

$$= \underline{2}$$

$$m \sim h^{\frac{1}{\delta}} = h^{\frac{2-\alpha-\Delta}{\Delta}} \Rightarrow \delta = \frac{\Delta}{2-\alpha-\Delta}$$

$$\underline{\delta - 1 = \gamma/\beta}.$$ widom 关系

	α	β	γ	δ	ν
$d=2$ Ising	0	$\frac{1}{8}$	$\frac{7}{4}$	15	1
$d=3$ Ising	0.12	0.31	1.25	5	0.64
$d=3$ XY	0.00	0.33	1.33	5	0.66
$d=3$. Heisenberg	-0.14	0.35	1.4	5	0.7

Josephson relation

$$2-\alpha = \nu d.$$

$$-\beta F = \ln Z = \left(\frac{L}{\xi}\right)^d g_s + \underbrace{\left(\frac{L}{a}\right)^d g_a}_{\text{有限值}}$$

$$\xi \sim t^{-\nu}$$

$$f \sim \xi^{-d} = t^{\nu d} f(0)$$

$$\because f \sim t^{2-\alpha} f(0) \quad \Rightarrow \quad 2-\alpha = \nu d$$

Josephson relation

$$f \propto t^{2-\alpha} f\left(\frac{h}{t^{\Delta}}\right)$$

$$G(x) = \langle m(x) m(0) \rangle - \langle m(x) \rangle \langle m(0) \rangle \propto \frac{1}{|x|^{d-2+\eta}}$$

1) 空间涨落

$$F = -\frac{1}{2} |\nabla \phi(x)|^2 + \frac{t}{2} m^2 + u m^4 - h m$$