随机过程. Browian 运动.
宏：扩散方程. / Fokker-Plank 方程.
微观：朗之万方程.

$$\begin{cases} dx = v\,dt \\ dv = -\alpha v\,dt + a\,dt + \sigma\,dW \end{cases}$$

数学基础.
Ito lemma
大数/中心极限

$\langle x^2 \rangle = 2Dt$

$f(x, v, t)$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial v}dv + \frac{1}{2}\frac{\partial^2 f}{\partial v^2}(dv)^2$$

Ito 1915-2008

1935-1938 本科

1939-1943 统计局 (1942-1943, Ito lemma)

1943-1954 副教授.

1945. 博士.

MC Simulation 随机数做计算.

量子耗散 (量子信息)

具体计算.

<1> ① $dx = f\,dt + \sigma\,dW$

② $\begin{cases} dx = v\,dt \\ dv = -\alpha v\,dt + f\,dt + \sigma\,dW \end{cases}$

<2>. $\int f(x_1, x_2, \cdots, x_n)\,dx_n \quad 3 \le n \le 10.$

量子 Langevin 方程

1> 推导 Langevin 方程

2> 更普遍的形式. Kernel $\beta(t)$ 函数

3> 量子化

4> $N\to\infty$ 什么东西影响物理.

$$m\ddot{x}+\int_{t_0}^{t}\beta(t-t')dt'+\beta(t)x(t_0)+\nabla U=\xi$$

$$\xi(t)=\sum_l \xi_l\Big(b_l e^{-i\underset{\substack{\uparrow\\ \text{频率}}}{\omega_l} t}+b_l^+ e^{i\omega_l t}\Big)$$

$$\langle\xi(t)\rangle=\sum_l \xi_l\langle b_l e^{-i\omega_l t}+b_l^+ e^{i\omega_l t}\rangle$$

$$=0$$

$$\langle O\rangle=\frac{\mathrm{Tr}(Oe^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$$

$\langle b_l\rangle=\langle b_l^+\rangle=0$ 对于平衡态热库.

$$\langle\xi(t)\xi(t')\rangle=\sum_{l l'}\xi_l^*\xi_{l'}\langle(b_l^+e^{i\omega_l t}+b_l e^{-i\omega_l t})(b_{l'}e^{-i\omega_{l'}t'}+b_{l'}^+e^{i\omega_{l'}t'})\rangle$$

$$=\sum_l|\xi_l|^2\left(n_l e^{i\omega_l(t-t')}+(n_l+1)e^{-i\omega_l(t-t')}\right)$$

1). 高温 $\beta \to 0$, $T \to \infty$

$n_l = \langle b_l^\dagger b_l \rangle$

$$n_l \approx n_l + 1 \approx \frac{k_B T}{\omega_l}$$

$n_l + 1 = \langle b_l b_l^\dagger \rangle$

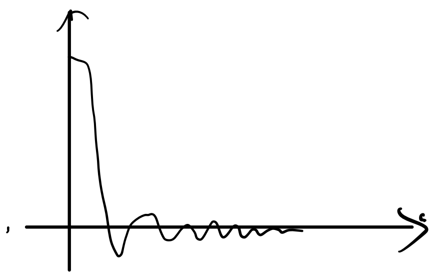
$$n_l = \frac{1}{e^{\beta \omega_l} - 1}$$

$\langle \xi(t) \xi(t') \rangle$

$$\approx \sum_l |\xi_l|^2 \frac{2k_B T}{\omega_l} \cos[\omega_l (t-t')]$$

无法证明上式 $\propto \delta(t-t')$

可以说明长时间平均 $= 0$



记忆时间 $T_l \approx \left(\frac{2\pi}{\omega_l}\right)$ 其中 ω_l 取最小值.

Spin-Boson model.

$$H = -\frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z + \sum_l \omega_l b_l^\dagger b_l + \sum_l g_l \sigma_z (b_l^\dagger + b_l)$$

① 每项意义.

② 精确求解 $\begin{cases} g_l = 0 \\ \Omega = 0 \end{cases}$ $H = H_\uparrow + H_\downarrow$.

$$H_\uparrow = \left[\frac{\delta}{2} + \sum_l \omega_l b_l^\dagger b_l + \sum_l g_l (b_l^\dagger + b_l)\right] |\uparrow\rangle\langle\uparrow|$$

$$H_\downarrow = \left[-\frac{\delta}{2} + \sum_l \omega_l b_l^\dagger b_l - \sum_l g_l (b_l^\dagger + b_l)\right] |\downarrow\rangle\langle\downarrow|$$

③极化.

一个简例.

$$H = \omega b^\dagger b + g(b + b^\dagger)$$

令 $a^\dagger = b^\dagger + \frac{g}{\omega}$, $a = b + \frac{g}{\omega}$.

则 $\omega a^\dagger a = \omega b^\dagger b + g(b + b^\dagger) + \frac{g^2}{\omega^2}$

故 $H = \omega a^\dagger a - \frac{g^2}{\omega^2}$

平移算子.

$$e^{a\frac{\partial}{\partial x}} f(x) = \left(1 + a\frac{\partial}{\partial x} + \frac{a^2}{2}\frac{\partial^2}{\partial x^2} + \cdots\right) f(x)$$

$$= f(x+a)$$

QM中 $\hat{u} = e^{-i\alpha\hat{p}}$ $\quad \hat{u}^\dagger x \hat{u} = e^{i\alpha\hat{p}} x e^{-i\alpha\hat{p}}$

$$= x + [\hat{p}, \hat{x}](i\alpha)$$

$$= x + \alpha$$

$$H = \omega b^\dagger b + g(b + b^\dagger)$$

$u^\dagger H u = \omega b^{\mu\dagger} b^{\mu} + \text{const}$

H本征态. $u|n\rangle = e^{-\alpha_i(b_i - b_i^\dagger)}|n\rangle$, $\alpha_i \in \mathbb{R}$

数值方法. 变分法.

$$|\Psi\rangle = |\uparrow\rangle \varphi_\uparrow + |\downarrow\rangle|\varphi_\downarrow\rangle$$

$$\varphi_\uparrow = \sum_n A_n e^{-\sum_l f_{nl}(b_l^+ - b_l)} |0\rangle_{Bath}.$$

$$\varphi_\downarrow = \sum_n B_n e^{-\sum_l g_{nl}(b_l^+ - b_l)} |0\rangle_{Bath}.$$

定义 $\mathcal{L} = \frac{i}{2}\langle\psi|\overleftrightarrow{\partial_t}|\psi\rangle - \langle\psi|H|\psi\rangle$

$$= \mathcal{L}(u, u^*, t) \qquad u = A_n, B_n, f, g.$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial u_n^*}\right) - \frac{\partial \mathcal{L}}{\partial u_n} = 0 \qquad \text{Equation of Motion.}$$

$$\left.\begin{aligned} &i\frac{\partial}{\partial t}|\Psi\rangle = H|\psi\rangle \\ &|\psi\rangle = \sum_n C_n|n\rangle \end{aligned}\right\} \Rightarrow \quad i\dot{C}_n = \sum_m t_{nm} C_m$$

$$t_{nm} = \langle m|H|n\rangle$$

$$\bar{E} = \langle\psi|H|\psi\rangle = \sum_{mn} t_{nm} C_n^* C_m \quad \begin{cases} i\dot{C}_n = \dfrac{\partial \bar{E}}{\partial C_n^*} \\ i\dot{C}_n^* = -\dfrac{\partial \bar{E}}{\partial C_n} \end{cases} \quad \text{正则方程.}$$

$$\mathcal{L} = \frac{i}{2}\langle\psi|\overleftrightarrow{\partial}|\psi\rangle - \langle\psi|H|\psi\rangle$$

$$\langle\psi|\overleftrightarrow{\partial}|\psi\rangle = \langle\psi|\partial_t|\psi\rangle - \langle\partial_t\psi|\psi\rangle$$

$$= \sum_n C_n^*\dot{C}_n - \dot{C}_n^* C_n$$

$$\Rightarrow L = \frac{i}{2}\sum_n \left(C_n^* \dot{C}_n - \dot{C}_n^* C_n\right) - t_{nm} C_n^* C_m$$

$$= \frac{i}{2}\frac{d}{dt}\left(\sum_n C_n^* C_n\right) - i\sum_n \dot{C}_n^* C_n - t_{nm} C_n^* C_m.$$

$$= -i\sum_n \dot{C}_n^* C_n - t_{nm} C_n^* C_m$$

$$\left.\begin{aligned} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{C}_n^*}\right) &= -i\dot{C}_n \\ \frac{\partial L}{\partial C_n^*} &= -\sum_m t_{nm} C_m \end{aligned}\right\} \Rightarrow i\dot{C}_n = -\sum_n t_{nm} C_m$$

Buffon. 投针计算π. .

N是总的个数.

Nc是相交的针的个数.

$$\pi = \frac{2l}{d}\left(\frac{N}{N_c}\right)$$