多体 {
- 平均场 (spin, B-H, F-H) ⇒ 多体 ⇒ 单体+自洽.
- 严格对角化: {spin, Bose, Fermi}
  1) 数 Hilbert 空间. 给定守恒量.
  2) 求矩阵元, 务必处理
  母技巧, 字引表 $|i\rangle \Leftrightarrow |n_1, \cdots, n_L\rangle$
  $\langle i|\hat{H}|j\rangle$
- MC ⇒ 高维积分...

a—o—o—o—o—o—o—o

trick: $L=7$, $N=5$. Bose.

去掉不太可能出现在态, 使子空间减小.

例如 5个粒子全在一个site上, 这种过于集中的态

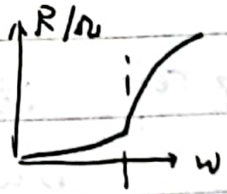
Anderson Localization (AL)

单体问题

Motivation: (1) 无序在各种材料中存在, 半导体.

AL P.W. Anderson 1957年最早提出 Anderson Localization 相变.

{ $W > W_c$ state localized.
{ $W < W_c$ extended.
for 3D.



扩展意味着波函数会传播, 往往意味着金属.

{ localized ⇒ insulator.

(2) 比较容易计算.

(3) 应用广泛.

Ref: (1) Hoffmann 计算物理. 讲转移矩阵一章.

(2) Thouless: J. phys. C. 1970

(3) Abrahams. Anderson. Licciardello ~~Ramar~~ Ramakrishnan PRL: 1979.

梯度.

(4) 李正中《固体理论》第12章.

方法: 转移矩阵

Ising model.

$$H = -J\sum_i \sigma_i \cdot \sigma_{i+1} + \underbrace{B}_{} \sum_i \sigma_i$$

$\sigma_i = \pm 1$. 磁场.

Lenz.

$$H = -t\sum_i (c_i^{+} c_{i+1} + h.c.) + V_i c_i^{+} c_i$$

$$H_{N\times N} = \begin{pmatrix} 0 & V_1 & t & & 0 \\ t & V_2 & t & & \\ & t & \ddots & \ddots & \\ 0 & & t & \ddots & \ddots \end{pmatrix}$$

☆ ref. 讨论 V_i 下的物理.

① white noise

② $V_i = V\cos\alpha i$ α 无理数.

③ correlated disorder.

1D → 推广高维.

$$H = -t\sum_i (c_i^+ c_{i+1} + h.c) + V_i c_i^+ c_i$$

1d.

V_i 无序 white noise $\begin{cases} \langle V_i \rangle = 0 \\ \langle V_i V_j \rangle = \sigma^2 \delta_{ij} \end{cases}$

数空间 $N=1$. (one particle)

$$K = \{ |i\rangle \} \equiv \{ c_i^+ |0\rangle \mid i \in \mathbb{Z} \}$$
$$= |0\,0 \cdots 0\,1 \cdots 0\rangle$$

$|n\rangle = \frac{1}{\sqrt{n!}} (a^+)^n |0\rangle$. 回忆

$\psi = \sum_i \alpha_i |i\rangle = \sum_i \alpha_i c_i^+ |0\rangle$.

$$\left(\begin{aligned} i\dot{c}_i &= [c_i, H] = \left(\frac{\partial H}{\partial c_i^+}\right) \\ &= -t(c_{i+1} + c_{i-1}) + V_i c_i \\ \text{use } [a, a^+] = 1 &\quad \text{or } [a, f] = \left(\frac{\partial f}{\partial a^+}\right) \end{aligned} \right).$$

$$E\alpha_i = -t(\alpha_{i+1} + \alpha_{i-1}) + V_i \alpha_i$$

$E\psi = H\psi \qquad E\alpha_i |i\rangle = \alpha_j H |j\rangle$.

$$E\alpha_i = \alpha_j \langle i | H | j \rangle$$
$$= V_i \alpha_i - t(\alpha_{j+1} + \alpha_{j-1})$$

IF $V_j = V = \text{const}$

$$\alpha_{j+1} = \alpha_{j-1} + \left(\frac{V-E}{t}\right)\alpha_j$$
$$\Rightarrow x^2 = 1 + \frac{V-E}{t} x \quad \Rightarrow \text{数列求和}.$$

当 V_i 都满相数……

$$\alpha_{i+1} = -\alpha_{i-1} + (V_i - E)\alpha_i \quad (t=1)$$

$$\begin{pmatrix} \alpha_{i+1} \\ \alpha_i \end{pmatrix} = \begin{pmatrix} V_i - E & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \alpha_{i-1} \end{pmatrix} = \hat{T}_i X_i .$$

$$\Rightarrow X_{i+1} = \hat{T}_i X_i = T_i T_{i-1} X_{i-1}$$

$$= T_i T_{i-1} T_{i-2} \cdots T_1 X_1$$

$$X_{N+1} = T_N T_{N-1} \cdots T_1 X_1 \sim e^{-N/\xi}.$$

QR 分解：R：上三角阵.

$R_1 R_2 \Rightarrow$ 上三角.

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \begin{pmatrix} x & y \\ 0 & z \end{pmatrix} = \begin{pmatrix} ax & ay+bz \\ 0 & cz \end{pmatrix}.$$

$$T_1 X_1$$
$$\downarrow$$
$$T_2 Q_1 R_1 X_1$$
$$\downarrow$$
$$T_3 Q_2 R_2 R_1 X_1$$
$$\downarrow$$
$$Q_3 R_3 R_2 R_1 X_1$$
$$\vdots$$
$$Q_N R_N R_{N-1} \cdots R_1 X_1$$

求 $X_{N+1}^T X_{N+1}$

$$= (X_1)^+ R_1^+ R_2^+ \cdots R_N^+ Q_N^+ Q_N R_N \cdots R_1 (X_1).$$

$$= (X_1)^+ R_1^+ R_2^+ \cdots R_N^+ R_N \cdots R_1 (X_1).$$

$$\sim e^{-N/\xi}$$

$$(R_3 R_2 R_1)_{kk} = R_3^{kk} R_2^{kk} R_1^{kk}.$$

$$X_{N+1}^+ X_{N+1} \sim e^{-2(N+1)/\xi} = f(N)\, e^{-2(N+1)/\xi}.$$

$$\ln f(N) - \frac{2(N+1)}{\xi} = \ln(X_1^+ R^+ R X_1).$$

$$\Rightarrow \frac{\ln f(N)}{N+1} = +\frac{2}{\xi} + \frac{1}{N+1}(X_1^+ R^+ R X_1).$$

$$\frac{2}{\xi} = -\lim_{N\to\infty} \frac{1}{N+1} \ln(X_1^+ R^+ R X_1)$$

ξ 关联长度.

定理: Oseledec 遍历定理. (动力学系统)

$$T = \lim_{N\to\infty} (T_N T_N^+)^{1/2N} \text{ 存在, 唯一.}$$

极限存在 ξ 存在.

上三角阵的本征值:

$$\det\left[\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} - x\right] = (a-x)(d-x)(f-x) = 0.$$

上三角阵的本征值为对角元.

$R = R_N \cdots R_1$

$\downarrow \ \lambda_k = (R_N)_{kk} (R_{N-1})_{kk} \cdots (R_1)_{kk}$.

$(TT^+) = P^+\lambda^+ P P^+ \lambda P$

$= P^+\lambda^+\lambda P$

$(\lambda^+\lambda)_{kk} = \prod_j (R_j)_{kk} (R_j^+)_{kk}$

$= \prod_j |(R_j)_{kk}|^2$

$f(N) e^{-\frac{2N}{\xi_k}} = \prod_{j=1}^{N} |(R_j)_{kk}|^2$

~~$\Rightarrow \frac{1}{\xi_k} = \lim_{N\to\infty} \frac{1}{2N} |(R_j)_{kk}|^2$~~

$\frac{\ln f(N)}{2N} - \frac{1}{\xi_k} = \left(\sum_{j=1}^{N} \frac{1}{2N} \ln |(R_j)_{kk}|^2 \right)$

$\Rightarrow \frac{1}{\xi_k} = -\lim_{N\to\infty} \frac{1}{2N} \sum_j \ln |(R_j)_{kk}|^2$

取最大的 ξ_k（衰减最慢）.