二次量子化.

Dirac. 任意对偶量都可以量子化.

正则动量 p. 与正则坐标 q.

例. x, p : θ, N ; a^+, a ; $c^*, c \in \mathbb{C}$

单粒子

$$H = \langle \psi(x) | H | \psi(x) \rangle = \sum_{n,m} C_n^* C_m \langle \varphi_n | H | \varphi_m \rangle.$$

C数变成产生湮灭算子

$$H = \int \psi^+(x)\, H\, \psi(x)\, dx \Rightarrow \sum_{n,m} C_n^+ C_m \langle \varphi_n | H | \varphi_m \rangle$$

构建意义.

$$\psi(x) = \sum_n C_n | \varphi_n(x) \rangle$$

$$\psi^+(x) = \sum_n \langle \varphi_n(x) | C_n^+$$

今天的目标. 认识 $\psi^+(x)$.

结论: $\psi^+(x)$ 在 x 点产生一个粒子.

$$\psi^+(x) |0\rangle = |x\rangle.$$

$a^+ |0\rangle = |1\rangle$. → 谐振子(单个)

$a_i^+ |0\rangle = |i\rangle$ ⇒ 第 i 个谐振子.

可以认为第 i 个点上产生一个粒子.

$$|\cdots 0 \cdots 0\, 1 \cdots 0\rangle.$$

第 i 个.

$$a_x^+ |0\rangle = |x\rangle$$

$$\psi^+(x) |0\rangle = |x\rangle$$

$H=\int \langle x|H|x\rangle \, dx$.

完备基 $\int |x\rangle\langle x| \, dx = 1$.

插入一组本征值 $H|\varphi_\alpha\rangle = \varepsilon_\alpha |\varphi_\alpha\rangle$. $\sum_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha| = 1$.

$$H=\sum_\alpha \int \langle x|H|\varphi_\alpha\rangle\langle\varphi_\alpha|x\rangle \, dx$$

$$=\sum_\alpha \varepsilon_\alpha \int \langle x|\varphi_\alpha\rangle\langle\varphi_\alpha|x\rangle \, dx$$

$$=\sum_\alpha \varepsilon_\alpha \int$$

对 $\forall H$ 单粒子模型 $H=\int |x\rangle\langle x|H|y\rangle\langle y| \, dx\,dy$.

$$=\int \langle x|H|y\rangle \, |x\rangle\langle y| \, dx\,dy.$$

插 $\sum_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|$.

$$=\sum_\alpha \varepsilon_\alpha \int \langle x|\varphi_\alpha\rangle\langle\varphi_\alpha|y\rangle \, |x\rangle\langle y| \, dx\,dy$$

$$=\sum_\alpha \varepsilon_\alpha \int |x\rangle\langle x|\varphi_\alpha\rangle\langle\varphi_\alpha|y\rangle\langle y| \, dx\,dy$$

$$=\sum_\alpha \varepsilon_\alpha |\varphi_\alpha\rangle\langle\varphi_\alpha|.$$

OR: $H|\varphi_\alpha\rangle = \varepsilon_\alpha|\varphi_\alpha\rangle$

$$\sum_\alpha H|\varphi_\alpha\rangle\langle\varphi_\alpha| = \sum_\alpha \varepsilon_\alpha|\varphi_\alpha\rangle\langle\varphi_\alpha|.$$

$$\Rightarrow \quad H = \sum_\alpha \varepsilon_\alpha|\varphi_\alpha\rangle\langle\varphi_\alpha|.$$

令 $P_{\alpha\beta} = |\alpha\rangle\langle\beta|$.

$$H = \sum_\alpha \varepsilon_\alpha P_{\alpha\alpha}$$

讨论 $P_{\alpha\beta}$ 算子: $\begin{cases} |\alpha\rangle\langle\beta|\beta\rangle = |\alpha\rangle. \\ |\alpha\rangle = a_\alpha^+|0\rangle. \end{cases}$

物理上: $|\alpha\rangle\langle\beta|\beta\rangle = |\alpha\rangle.$

$$a_\alpha^+ a_\beta|\beta\rangle = a_\alpha^+ a_\beta|0_\alpha 1_\beta\rangle.$$
$$= a_\alpha^+|0_\alpha 0_\beta\rangle.$$
$$= |\alpha\rangle.$$

$H = \sum_\alpha \varepsilon_\alpha|\varphi_\alpha\rangle\langle\varphi_\alpha|$ 对角基矢.

一般基矢. $|n\rangle\langle n|$

$$H = \sum_{n,m} |n\rangle\langle n|H|m\rangle\langle m|.$$

$$= \sum_{n,m} \langle n|H|m\rangle\, |n\rangle\langle m|.$$

$$= \sum_{n,m} \langle n|H|m\rangle\, c_n^+ c_m.$$

单粒子.

总结: 1) 基(完备).

2) 对应关系. $c_n^+ c_m \Rightarrow |n\rangle\langle m|.$

$c_n^+|0\rangle = |n\rangle$

有相互作用(2体) 改基?

H_1 相互 H_2

$|n\rangle$ $|m\rangle$.

空间 $H_1 \otimes H_2$.

单体 $H \Rightarrow$ 基 $\sum_n |n\rangle\langle n| = 1$.

两粒子 $H = H_1 + H_2 \Rightarrow \sum_{n,m} |n,m\rangle\langle n,m| = 1$.

$$V = I\, V(x-y)\, I$$

$$= \sum_{\alpha,\beta} |\alpha,\beta\rangle\langle\alpha,\beta| V |\gamma,\delta\rangle\langle\gamma,\delta|.$$

$$= \sum \cancel{|\alpha,\beta\rangle} \langle\alpha,\beta| V |\gamma,\delta\rangle\, |\alpha,\beta\rangle\langle\gamma,\delta|.$$

$$= \sum_{\alpha} V_{\alpha\beta,\gamma\delta}\, |\alpha,\beta\rangle\langle\gamma,\delta|.$$

$$= \sum V_{\alpha\beta,\gamma\delta}\, a_\alpha^+ a_\beta^+ a_\gamma a_\delta.$$

$$H = \int \psi^+(x)\, H\, \psi(x)\, dx$$

$$V = \int \psi^+(x)\, \psi^+(y)\, V(x-y)\, \psi(y)\, \psi(x)\, dx\, dy.$$

$$\psi_\alpha(x) = \sum_\alpha a_\alpha \varphi_\alpha(x)$$

$$\Leftrightarrow \int a_\alpha^+ a_\beta^+ a_\gamma a_\delta \langle \varphi_\alpha(x)\, \varphi_\beta(y) | V(x-y) | \varphi_\gamma(y)\, \varphi_\delta(x)\rangle$$

$$= \sum a_\alpha^+ a_\beta^+ a_\gamma a_\delta \langle \alpha\beta | v | \gamma\delta \rangle .$$

理解 $\psi^+(x) \Rightarrow a_x^+ \Rightarrow c_x^+$

表示在x处产生一个粒子 $|x\rangle = |1_x\rangle$.

$$|\alpha\rangle = \sum_{\beta n} |\overset{n}{\beta}\rangle \langle \overset{n}{\beta} | \alpha \rangle .$$

$$= \sum_{n\beta} \langle n | \alpha \rangle \, |n\rangle .$$

$$c_\alpha^+ |0\rangle = |\alpha\rangle \qquad |n\rangle = c_n^+ |0\rangle .$$

$$\Rightarrow c_\alpha^+ |0\rangle = \sum_n \langle n | \alpha \rangle \, c_n^+ |0\rangle .$$

$$\alpha = x$$

$$c_x^+ = \sum_n \langle n | x \rangle c_n^+ .$$

$$\left(c_n^+ = \sum_x \langle x | n \rangle c_x^+ \right)$$

$$\psi^+(x) = \sum_n \langle n | x \rangle c_n^+$$

$$\psi^+(x) |0\rangle = \sum_n \langle n | x \rangle c_n^+ |0\rangle .$$

$$= \sum_n \langle n | x \rangle |n\rangle .$$

$$= |x\rangle .$$

二次量子化的标准方法：

$$H = h(x) + V(x-y)$$

$$H = \int \psi^+(x) h(x) \psi(x) + \int \psi^+(x) \psi^+(y) V(x-y) \psi(y) \psi(x) \, dx dy .$$

$$\psi(x) = \sum_n c_n \varphi_n(x) .$$

$$\psi^+(x) \psi^+(y) = \sum_{n,m} \varphi_n(x) \varphi_m(y) \, c_n^+ c_m^+ |0\rangle$$

$$H=\sum_{n,m} C_n^+ C_m \langle n|H|m\rangle + \sum C_n^+ C_m^+ C_k C_l \langle nm|V|kl\rangle$$

如果取 h 的本征值.

$\Rightarrow \sum \varepsilon_n C_n^+ C_n$ + Interaction.

相容(多体). MC 方法.
平均场方法. (Mean ~~faild~~ field Theory
的一级近似.

回顾:

$$x^3=2x^2+5 \qquad x^3=m^2x.$$

$$\Rightarrow 2x^2-m^2x+5=0.$$

$$x=\frac{m^2\pm\sqrt{m^4-40}}{4}=m.$$

自洽 ⇒ 找值.
递归.

Bose-Hubbard Model.

Ising Model

$$H=h(\sigma_1+\sigma_2)+J\sigma_1\sigma_2$$

$$\sigma_1,\sigma_2=\pm 1$$

平均场: 多体 ⇒ 单体. ⇒ x, y.

$$x=dx+\bar{x}$$

$$y=dy+\bar{y}$$

$$xy=\bar{x}\bar{y}+\bar{x}dy+\bar{y}dx+\underbrace{dxdy}_{\text{要忽略.}}$$

忽略了关联

$$xy \simeq \bar{x}(\bar{y}+dy)+\bar{y}(\bar{x}+dx)-\bar{x}\bar{y}$$
$$= \bar{x}y+\bar{y}x-\bar{x}\bar{y}.$$

$$H = h(\sigma_1+\sigma_2)+Jm(\sigma_1+\sigma_2)-m^2J.$$
$$m = \langle\sigma_1\rangle = \langle\sigma_2\rangle.$$

$$H = h(\sigma_1+\sigma_2+\sigma_3)+J\sigma_1\sigma_2+J\sigma_2\sigma_3+J\sigma_3\sigma_1$$
$$+k\sigma_1\sigma_2\sigma_3.$$
$$= h(\sigma_1+\sigma_2+\sigma_3)+Jm(\sigma_1+\sigma_2+\sigma_2+\sigma_3+\sigma_3+\sigma_1)-3m^2J$$
$$+km^2(\sigma_1+\sigma_2+\sigma_3)-2km^3.$$
$$= (h+2mJ+km^2)(\sigma_1+\sigma_2+\sigma_3)-3m^2J-2km^3$$

单 spin. $H = A\cdot\sigma$.

$$Z = e^{-A\beta}+e^{A\beta} = e^{-\beta F}$$
$$F = -\frac{1}{\beta}\ln(e^{A\beta}+e^{-A\beta})$$

$$F = -m^2J+2\left(-\frac{1}{\beta}\right)\ln\left[e^{\beta(h+mJ)}+e^{-\beta(h+mJ)}\right]$$
$$\frac{\partial F}{\partial m}=0.$$

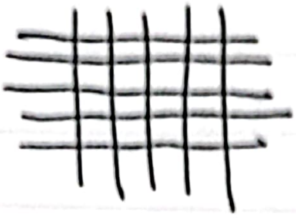
$H = A\sigma$ $\quad\uparrow\ A \qquad P_\uparrow = e^{-\beta A}/Z$ $\qquad Z = e^{A\beta}+e^{-A\beta}$

$\quad\downarrow\ -A. \qquad P_\downarrow = e^{\beta A}/Z.$

$$m = P_\downarrow(-1)+P_\uparrow(1) = \frac{e^{-\beta(h+mJ)}-e^{\beta(h+mJ)}}{e^{-\beta(h+mJ)}+e^{\beta(h+mJ)}}$$

2d Ising Model (1阶).

n个之间的 model



$$H = -J\sum_{\langle ij\rangle} \sigma_i\sigma_j + h\sum_i \sigma_i + k_{\square}\sum \cancel{\sigma_i\sigma_j\sigma_k\sigma_l}$$

$\sigma_i\sigma_j\sigma_k\sigma_l$

$\sigma_i = \pm 1$

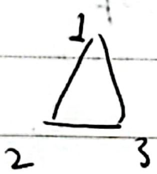
( $k_{\diamond}\sum \sigma_i\sigma_j\sigma_k\sigma_l$. 不讨论.)

$$H = -\sum_{ij} J\sigma_i\sigma_j + h\sum_i \sigma_i$$

西己位数

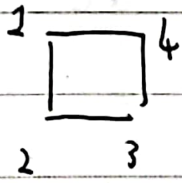
$\sigma_1\sigma_2$

$\sigma_1 \quad \sigma_2$ $\quad Jm(\sigma_1+\sigma_2) - m^2 J$.



$$Jm(\sigma_1+\sigma_2+\sigma_2+\sigma_3+\sigma_3+\sigma_1) - 3m^2 J.$$

$$= 2Jm(\sigma_1+\sigma_2+\sigma_3) - 3m^2 J$$

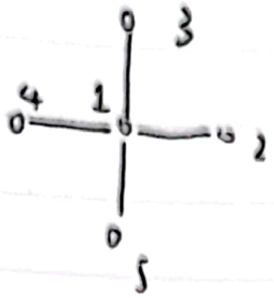


$$\Rightarrow Jm(\sigma_1+\sigma_2+\sigma_2+\sigma_3+\sigma_3+\sigma_4+\sigma_4+\sigma_1)$$

$$-4m^2 J$$

$$= 2Jm(\sigma_1+\sigma_2+\sigma_3+\sigma_4) - 4m^2 J.$$

前面的系数是该点的连接数.　　键的个数



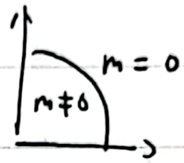
$4Jm\sigma_1 + \cdots$

配位数：4

目的：① $\frac{\partial F}{\partial m} = 0$

② 相图

铁磁相变



平均场 { 缺点
单粒子