二次量子化：正则量子化.

H: $H(x, p) \to H(a, a^+)$

$$\begin{cases} x = \sqrt{\dfrac{\hbar}{2m\omega}}\,(a+a^+) \\ p = i\sqrt{\dfrac{\hbar m\omega}{2}}\,(a^+-a) \end{cases}$$

x: $\int dx\, |x\rangle\langle x| = 1$

p: $\int dp\, |p\rangle\langle p| = 1$.

$\left.\begin{matrix} a \\ a^+ \end{matrix}\right\} \sum_n |n\rangle\langle n| = 1$.

可以直接用于不同的统计

缺点. 特殊 ⇒ EM, Bose, fermion

优点：算子

1) 避免复杂波函数表示.

2) 处理多体系统.

体现物理上的量子化.

谐振子 ⇒ 基态 ⇒ 应用精确测量.

1) model → 二次量子化.

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

x：抽象.

$$\begin{cases} x = \sqrt{\dfrac{\hbar}{2m\omega}}\,(a+a^+) \\ p = i\sqrt{\dfrac{\hbar m\omega}{2}}\,(a^+-a) \end{cases}$$

$$H = \hbar\omega\left(a^+a + \frac{1}{2}\right) + g_{eff}(a+a^+)^4$$

$$(a+a^+)^2 = a^2 + aa^+ + a^+a + a^{+2}$$
$$= a^2 + 2a^+a + a^{+2} + 1.$$

	a^2	$2a^+a$	a^{+2}	1
a^2	a^4	$2a^2a^+a$	a^2a^{+2}	a^2
$2a^+a$	$2a^+a^3$	$4(a^+a)^2$	$2a^+aa^{+2}$	$2a^+a$
a^{+2}	$a^{+2}a^2$	$2a^{+3}a$	a^{+4}	a^{+2}
1	a^2	$2a^+a$	a^{+2}	1

For $H = \hbar\omega(a^+a+1)$

$$i\dot{a} = [a, H]$$
$$= \hbar\omega a$$
$$a(t) = e^{-i\hbar\omega t} a(0)$$

$[a, a^+] = 1$

$i\dot{a} = \hbar\omega a + \underbrace{[a, a^{+4}]}_{a^{+3}\ \overline{e^{-i3\hbar\omega t}}} + [a, 2a^+aa^+] \sim aa^+ \text{ or } a^+a$

$[a, a^+a^2] \sim a^2 \quad \sim a \quad \sim a^+$

Rotating wave 近似.(旋波近似)

缓变 快变

$$\dot{x} = -ax + f(t)x$$
$$x = e^{-at} + \int_0^t f(\tau)d\tau\, X_0$$

多体/固体理论 RPA → Random phase (无规相位近似)、保能量守恒

$$\Rightarrow H = \hbar\omega(a^+a + \tfrac{1}{2}) + 4g_{eff}\, a^+a + 4g_{eff}(a^+a)^2 + g_{eff}(a^{+2}a^2 + a^2a^{+2})$$

$(a^2 a^{+2} + a^{+2} a^2)|n\rangle$.

$a^2 a^{+2}|n\rangle = \sqrt{n+1}\sqrt{n+2}\ a^2|n+2\rangle$

$= (n+2)(n+1)|n\rangle$.

$a^{+2} a^2|n\rangle = n(n-1)|n\rangle$.

$(n^2 - n + n^2 + 3n + 2)|n\rangle$

$= (n^2 + 2n + 2)|n\rangle$.

$\Rightarrow (a^2 a^{+2} + a^{+2} a^2) = (a^+ a)^2 + 2a^+ a + 2$.

存在 $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + (B_0 + B_1 x)\sigma_x + \Omega\sigma_z$.

$= \hbar\omega(a^+ a + \frac{1}{2}) + B_0\sigma_x + \Omega\sigma_z + B_1 x\sigma_x$.

$x \to \sqrt{\frac{\hbar}{2m\omega}}(a + a^+)$, $\sigma_x \to (\sigma_+ + \sigma_-)$

$\sigma_x = \sigma_+ + \sigma_-$.

$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y)$

$\sigma_- = \frac{1}{2}(\sigma_x - i\sigma_y)$.

$B_1\sqrt{\frac{\hbar}{2m\omega}}(a + a^+)(\sigma^+ + \sigma^-)$

$= B_1\sqrt{\frac{\hbar}{2m\omega}}(a^+\sigma^- + a^-\sigma^+)$

+ 反旋波

$a\sigma + a^+\sigma^+$

Rotating wave.

数值计算

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + (B_0 + B_1 x)\sigma_x + \Omega\sigma_z$

$\Rightarrow |\psi\rangle = \sum_{n\sigma} C_{n\sigma}|\varphi_n(x)\rangle\chi_\sigma$

其中 $\chi_\downarrow = \begin{pmatrix}0\\1\end{pmatrix}$ $\chi_\uparrow = \begin{pmatrix}1\\0\end{pmatrix}$

$H_0|\varphi_n(x)\rangle = \hbar\omega(n + \frac{1}{2})|\varphi_n(x)\rangle$

重新编号：

$|\varphi_{0\downarrow}\rangle$ $|\varphi_{0\uparrow}\rangle$ $|\varphi_{1\downarrow}\rangle$ $|\varphi_{1\uparrow}\rangle$ ...

$|\varphi_1\rangle$ $|\varphi_2\rangle$ $|\varphi_3\rangle$ $|\varphi_4\rangle$

$\to \sum C_i|\varphi_i\rangle$

QM: $H\phi = E\phi$.

$$\begin{cases} |\phi\rangle = \sum_n c_n |\varphi_n\rangle \\ \langle\varphi_n|\varphi_m\rangle = \delta_{nm}. \end{cases}$$

$$\sum_n c_n H|\varphi_n\rangle = \sum_m E\, c_m |\varphi_m\rangle.$$

$$\rightarrow \sum_n c_n \langle\varphi_m|H|\varphi_n\rangle = E c_m.$$

$$([H])_{mn} = \langle\varphi_m|H|\varphi_n\rangle \quad [c] = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}.$$

$$[H][c] = E[c].$$

二次量子化:

$$|\psi\rangle = \sum_{n\sigma} c_{n\sigma} |n\sigma\rangle.$$

$$a^+|n\sigma\rangle = \sqrt{n+1}\,|n+1,\sigma\rangle.$$

$$\sigma_z|n\sigma\rangle = \sigma|n\sigma\rangle.$$

$$\sigma^+|n\downarrow\rangle = |n\uparrow\rangle.$$

$$\sigma^+|n\uparrow\rangle = 0.$$

模型3, $-o\!\sim\!o\!\sim\!o\!\sim\!o\!\sim\!o\!\sim\!o\!\sim\!o\!\sim\!o-$

$$H = \sum_i \frac{p_i^2}{2m} + \frac{k}{2}(x_{i+1}-x_i)^2.$$

$$x_i = \sqrt{\frac{\hbar}{2m\omega}}\,(a_i + a_i^+) \qquad \omega \text{待定}.$$

$$p_i = i\sqrt{\frac{m\hbar\omega}{2}}\,(a_i^+ - a_i)$$

$(a_{i+1}^+ + a_{i+1} - a_i^+ - a_i)^2$.

	a_{i+1}^+	a_{i+1}	$-a_i^+$	$-a_i$
a_{i+1}^+	$(a_{i+1}^+)^2$	$a_{i+1}^+ a_{i+1}$	$-a_{i+1}^+ a_i^+$	$-a_{i+1}^+ a_i$
a_{i+1}	$a_{i+1} a_{i+1}^+$	$(a_{i+1})^2$	$-a_{i+1} a_i^+$	
$-a_i^+$		$-a_i^+ a_{i+1}$	$(a_i^+)^2$	$+a_i^+ a_i$
$-a_i$	$-a_i a_{i+1}^+$		$+a_i a_i^+$	a_i^2

$$(a_i + a_i^+)(a_{i+1} + a_{i+1}^+)$$
$$= (a_i a_{i+1} + a_i a_{i+1}^+ + a_i^+ a_{i+1} + a_i^+ a_{i+1}^+)$$

$$H = \sum_i \left(\frac{p_i^2}{2m} + k x_i^2\right) - \sum_i k\, x_i x_{i+1} = \sum_i \hbar\omega\left(a_i^+ a_i + \tfrac{1}{2}\right) - k\frac{\hbar}{2m\omega}(a_i^+ a_{i+1} + a_{i+1}^+ a_i)$$

$\hookrightarrow \frac{1}{2}\cancel{k}\, m\omega^2$.

$$= \sum_i \hbar\omega\, a_i^+ a_i - \cancel{k}\,\hbar\omega \sum_i (a_i^+ a_{i+1} + h.c.)$$

$\cancel{\tfrac{1}{2}}\ \frac{1}{4}$

$$i\dot{a}_i = [a_i, H] = \hbar\omega\, a_i - \frac{\hbar\omega}{4}(a_{i+1} + a_{i-1})$$

$\hookrightarrow a_n(t) \sim e^{i(kn - \omega_n t)} a_0$

$$i\dot{x}_i = \hbar\omega x_i - \frac{\hbar\omega}{4}(x_{i+1} + x_{i-1})$$

$$x_n \sim e^{i(kn - \omega t)}.$$

$$\omega_k = \hbar\omega - \frac{\hbar\omega}{4}(e^{ik} + e^{-ik}) = \hbar\omega - \hbar\omega\cos k,$$

$$\omega_k = \hbar\omega(1 - \cos k)$$

$$= \hbar\omega\frac{k^2}{2} \quad \propto k^2 \text{ 声子}.$$

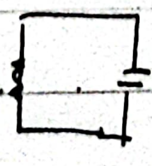
(一)电学谐振.

$U = \left(\frac{q^2}{2c}\right)$

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$

LC 电路

Ripple.



$p = I^2 R$.

$\omega \sim \frac{1}{\sqrt{LC}}$.

$$\left\{\begin{array}{l} \dfrac{d^2 q}{dt^2} = -\dfrac{q}{LC}. \\ \dfrac{dx^2}{dt^2} = -\omega_0^2 x = -\dfrac{1}{m}\nabla U. \end{array}\right.$$

$U = \frac{1}{2}m\omega^2 x^2$.

$$\begin{cases} H = \frac{L}{2}\dot{q}^2 + \frac{q^2}{2C} \\ H = \frac{m}{2}\dot{x}^2 + \frac{m\omega^2}{2}x^2 \end{cases}$$

$L \leftrightarrow m$

$x \leftrightarrow q$

$\dot{x}m = p \to I \sim \dot{q}$

R 耗能 $\sim \eta\dot{x}$