MC → 相变 | 平均场 Landau 理论 | 1 1-2 实验不一致
| 2D Ising Onsager (1944), Tang $\beta = \frac{1}{8}$

实验: C_v, χ 发散. ⇐ 原因 $\xi \to \infty$

数值(MC): 经典 $L \sim 10^4 \sim 10^5$. 量子: ...

标度假设. 标度不变性假设 $f(\lambda x) = \lambda^\nu f(x) \Rightarrow f(x) \sim x^\nu$

$f(1) = (\frac{1}{x})^\nu f(x)$ $f(x) = f(1) \cdot x^\nu$

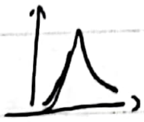
标度率假设.

向下决问题.

free energy: $F = f V$. f: 密度.

$C_v \sim |t|^{-\alpha}$ $\langle m \rangle \sim |t|^{-\beta}$ $\chi \sim |t|^{-\gamma}$ $\xi \sim |t|^{-\nu}$

$t = T - T_c$. or $t = \frac{T - T_c}{T_c}$.

C_v: 

m: 

χ: 

ξ: 

α. β. γ. ν 之间的关系.

1) 有: 来自同一体. ⇒ 推导, 实验一致.

2) 无. 不同的物理.

Kardar 书. (MIT) The scaling Hypothesis.

出发点: 临界点 ⇒ $\xi \to$ 发散.

$f = \frac{\beta F}{V} = \min(\underbrace{\frac{t}{2} m^2 + u m^4 - h m}_{t \to 0,\ h \to 0,\ m \to 0})$ } 唯一可用的理论.

$e^{-\beta F} = Z$

$f(t,h)$ 放弃严格计算，用标度假设来做.

$$f(\lambda t, \lambda^{3/2} h) = \lambda^2 f(t,h)$$

$$f(\lambda t, \lambda^{3/2} h) = \min\left(\frac{t}{2}\lambda m^2 + u m^4 - \lambda^{3/2} h m\right) = \lambda^2 \min\left(\frac{t}{2} m^2 + u m^4 - hm\right)$$

取 $\lambda t = 1$. $\lambda = \frac{1}{t}$. $\Rightarrow f(t,h) = t^2 f\left(1, \frac{h}{|t|^{3/2}}\right)$

$$= t^2 f\left(\frac{h}{|t|^{3/2}}\right)$$

回到 f: ① $h=0$. $t \neq 0$. $\Rightarrow f \sim t^{2-\alpha} f(0)$

② $t=0$, $h \neq 0$. $t^{2-\alpha} f(\infty)$

$$f = t^{2-\alpha} f\left(\frac{h}{t^{\Delta}}\right)$$

$h=0$ $\quad f = \frac{t}{2} m^2 + u m^4 \quad \frac{\partial f}{\partial m} = 0 \Rightarrow tm + 4um^3 = 0$

$$\Rightarrow m = 0 \text{ or } m = \pm\sqrt{\frac{-t}{4u}}$$

$$f = \frac{t}{2}\left(-\frac{t}{4u}\right) + u\left(-\frac{t}{4u}\right)^2 \propto t^2/u.$$

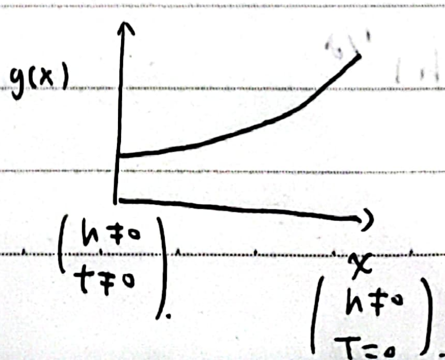
假设 $f = t^2 f\left(\frac{h}{t^{\Delta}}\right) \quad f(0) \to \left(-\frac{1}{16}\right)u.$

② $t=0$ $h \neq 0$. $\quad f = um^4 - hm$.

$$f' = 4um^3 - h = 0 \quad m = \left(\frac{h}{4u}\right)^{1/3}$$

$$f = u\left(\frac{h}{4u}\right)^{\frac{4}{3}} - h\left(\frac{h}{4u}\right)^{1/3} \propto h^{\frac{4}{3}}$$

$$f = |t|^{2-\alpha} g\left(\frac{h}{t^{\Delta}}\right)$$



$E \sim \frac{\partial f}{\partial t}\Big|_{h=0}$

$C_v \sim \frac{\partial^2 f}{\partial t^2}\Big|_{h=0}$

Heat capacity $\propto |t|^{-\alpha}$

磁性 : $m \sim \frac{\partial f}{\partial h} \sim t^{2-\alpha-\Delta}\, g'\!\left(\frac{h}{t^\Delta}\right)$

$$\overset{h\to 0}{\sim} t^{2-\alpha-\Delta} = t^\beta$$

$$\beta = 2-\alpha-\Delta$$

$$\chi \propto \left(\frac{\partial m}{\partial h}\right) \propto t^{2-\alpha-2\Delta}\, g''\!\left(\frac{h}{t^\Delta}\right) \propto t^{-\gamma}$$

$\gamma = 2\Delta+\alpha-2$ ∴ Ruchbrooke Identity.

$\chi \to \infty$

$m \sim t^{2-\alpha-\Delta}\, g_m\!\left(\frac{h}{t^\Delta}\right)$ $\quad g_m(x) = g''(x)$

If $h \gg |t|$ $\quad g_m(x) \sim x^p$

$m \sim t^{2-\alpha-\Delta}\left(\frac{h}{t^\Delta}\right)^p$

$\sim 2-\alpha-\Delta = \Delta p \Rightarrow p = \frac{2-\alpha-\Delta}{\Delta}$.

$m \propto h^p \propto h^{\frac{2-\alpha-\Delta}{\Delta}} = h^{(\frac{1}{\delta})}$

$\boxed{\delta - 1 = \gamma/\beta}$ Widom Identity.

$m \propto |h|^{1/\delta}$

$\beta = \frac{1}{2}$ for Landau.

$\boxed{f \sim t^{2-\alpha} f(\frac{h}{t^{\Delta}})}$ [illegible]

$\leftarrow f \sim t^{-\nu} g(\frac{h}{t^{\Delta}})$

	α	β	γ	δ	ν
d=2/Ising	0	1/8	7/4	5	1
d=3/Ising	0.12	0.31	1.25	5	0.64
d=3 XY	0.00	0.33	1.33	5	0.66
d=3 Heisenberg	−0.14	0.35	1.4	5	0.7

Josephson relation

$$2-\alpha = d\nu$$

$\xi \propto t^{-\nu}$.

$$\beta F = \ln Z = \left(\frac{L}{\xi}\right)^d g_s + \left(\frac{L}{a}\right)^d g_a \leftarrow \text{解析}.$$

a : lattice constant.

$$E = \int \varepsilon(x)\,dx$$

$$f \sim \xi^{-d} + \text{const} = t^{d\nu} g_f\left(\frac{h}{t^{\Delta}}\right)$$

$$G(x) = \langle m(x) m(0) \rangle - \langle m(0) \rangle \langle m(x) \rangle.$$

$$\propto \frac{1}{|x|^{d-2+\eta}}$$

| gapless
| critical

$\Rightarrow$ 讨论了. [illegible]

$$G(x) \sim e^{-|x|/\xi}$$

1) 空间涨落.

$-\frac{\partial^2}{\partial x^2}\phi$ ① $H = \left(\frac{\partial}{\partial x} m\right)^2 + \frac{t}{2} m^2 + u m^4 - h m$.

② 空间.