DATE 1 2 3 4 5 6 7 8 9 10 11 12 The month
Mon Tue Wed Thu Fri Sat Sun

随机过程.

Brownian motion.
1) 宏观 扩散 / Fokker - Plack eq.
微观 朗之万方程. $dx = v\,dt$.
$$dv = -\alpha v + f + \sigma dw.$$

数学基础 Ito lemma.
中心极限定理. 大数定理.

$\langle x^2 \rangle = 2Dt$.
⇓
涨落 ∝ 时.

$$df(x,t) = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x} dx + \underbrace{\frac{1}{2}\frac{\partial^2 f}{\partial x^2} dx^2}_{}$$

→ break chain rule.

Ito: 1915 - 2008

1935 - 1938 本科

~39 - 43 统计局.

43 - 54 副教授.

45 博士.

42 - 43 Ito lemma.

Runge

AAC solution 随机数值计算.

量子耗散. (量子信息)

具体计算: 1) $dx = f\,dt + \sigma dw$

2) $dx = v\,dt$

3) $dv = (-\alpha v + f)\,dt + \sigma dw$.

4) $\int f(x_1, \cdots, x_n)\, dx_1 \cdots dx_n$, $3 \le n \le 10$

5) $Z = Tr(e^{-\beta H}) = \int dx_1 \cdots dx_n\, e^{-\beta H(x_1, \cdots, x_n)}$

简单的问题.

Ising Model 和 MC 模拟!

Fokker - Planck

宏观 - 微观.

$PV = nRT$.

$\rho(x, p, t) \Rightarrow d\rho = 0 \Rightarrow$ 动态平衡.

$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0 \Rightarrow \vec{J} = (\vec{v}\rho, \vec{a}\rho)$

速度 / 力

只有一次导数.

$\frac{\partial \phi}{\partial t} = D \frac{\partial^2 \phi}{\partial x^2}$ 二次导数 (Ito lemma). 二阶

Coupled Langevin eq.

$\frac{dx}{dt} = F(x, y) + \xi_1$.

$\frac{dy}{dt} = G(x, y) + \xi_2$

drift term diffusion term

$\frac{\partial}{\partial t} P(x, y, t) =$ 线性 + 扩散项.

$-\frac{\partial}{\partial x}(FP)$ $\left(D_1 \frac{\partial^2 P}{\partial x^2} + D_2 \frac{\partial^2 P}{\partial y^2}\right)$

$-\frac{\partial}{\partial y}(GP)$.

两个极限.

1) $F=0$, $G=0$ 纯扩散方程.

2) $D_1=0$, $D_2=0$. Liouville eq 方程.

$$\frac{d}{dt}P(x,y,t)=0 \Rightarrow \frac{\partial P}{\partial t}+\frac{\partial}{\partial x}(FP)+\frac{\partial}{\partial y}(GP)=0.$$

量子 Langevin eq.

Caldeira-Leggett model (CL model).

一个振子与环境相互作用.

可解.

ref: 1) Calderia-leggett model wiki.

2) Calderia leggett PRL 1981.

3) Benguria, Kac, PRL, 1981. "Quantum Langevin eq"

$\boxed{4)}$ Ford, Kac. 1987. "On the Quantum Langevin eq"

数学: Feyman-Kac Integral.

$$m\ddot{x}=-\alpha\dot{x}+f+\xi.$$

α: 阻力
ξ: 随机力.

唯象的图像.

①从原理论推导该方程.

②量子化.

$$H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2=\hbar\omega(a^+a+\frac{1}{2})$$

a表示 x的函数.

$$x\propto(a+a^+)$$

$x\sim a+a^+$, f, ξ怎么表示?

$$a^+a+\frac{1}{2}=\frac{p^2}{2m\hbar\omega}+\frac{m\omega}{2\hbar}x^2.$$

$$c^2-d^2=(c+d)(c-d)$$

$$c^2+d^2=(c+id)(c-id).$$

$$+icd-idc.$$

$$=(c+id)(c-id)+i[c,d].$$

$$=a^+a+\frac{1}{2}.$$

$$c=\sqrt{\frac{m\omega}{2\hbar}}x.$$

$$d=\frac{p}{\sqrt{2m\hbar\omega}}$$

$$\begin{cases} a=\sqrt{\frac{m\omega}{2\hbar}}x-\frac{ip}{\sqrt{2m\hbar\omega}} \\ a^+=\sqrt{\frac{m\omega}{2\hbar}}x+\frac{ip}{\sqrt{2m\hbar\omega}}. \end{cases}$$

$$x=\sqrt{\frac{\hbar}{2m\omega}}(a+a^+).$$

$$m\ddot{x}=-\alpha\dot{x}-m\omega^2x+\xi. \quad U=\frac{1}{2}kx^2 \quad f=-\nabla U=kx$$

$$x=\sqrt{\frac{\hbar}{2m\omega}}(a+a^+).$$

$$\xi=\eta(b+b^+) \leftarrow \text{Bath 自由度.}$$

b 的运动方程在哪里?

正确的做法:

环境中有无穷多小振子:

↳ Environment. / Bath.

$$H = \frac{p^2}{2m} + V(x) + \sum_{j=1}^{n}\left[\frac{p_j^2}{2m_j} + \frac{1}{2}k_j (q_j - x)^2\right].$$

| 可解析.

| 可数值计算.

> 作业：模拟以上过程：$N=100$. m_i, k_i 随意取.

$$\begin{cases}\dot{p} = -\dfrac{\partial H}{\partial q} \\ \dot{q} = \dfrac{\partial H}{\partial p}\end{cases} \Rightarrow \begin{cases}\dot{p}_i = -\dfrac{\partial H}{\partial q_i} = -k_i (q_i - x) \\ \dot{q}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{p_i}{m_i} \\ \dot{x} = \dfrac{\partial H}{\partial p} = \dfrac{p}{m}. \\ \dot{p} = -\dfrac{\partial H}{\partial x} = -V'(x) - \sum_j k_j (x - q_j).\end{cases}$$

$\Rightarrow$ 矩阵

$$\dot{p}_i = -k_i (q_i - x)$$

$$\dot{q}_i = \frac{p_i}{m_i}$$

$$m_i \ddot{q}_i = -k_i (q_i - x)$$

$$\boxed{m_i \ddot{q}_i + k_i q_i = k_i x}.$$

外力

Langevin eq: $m\ddot{x} = -\alpha \dot{x} + \xi$.

$1/\gamma$.

线性，二阶.

$L u = f$.

$$u = u_0 + \int g(x - x') f(x')\, dx'$$

$L u_0 = 0$. 平解 / 齐次解.

$$m_i \ddot{q}_i + k_i q_i = 0.$$

奇次通解: $q_i = A\cos(w_i t) + B\sin(w_i t)$.

$$q_i \propto e^{i w_i t}$$

$$m_i w_i^2 = k_i.$$

$$q_i(0) = A.$$

$$m_i q_i'(0) = p_i 0.$$

$$q_j(t) = \underbrace{q_j(0)\cos(w_j t) + p_j(0)\sin(w_j t)/m_j w_j}_{\text{和x无关、随机力}\ \xi}$$

$$+ \underbrace{x(t) - x(0)\cos(w_j t)}_{\text{x有关力.}} - \underbrace{\int_0^t \cos(w_j(t-t'))\dot{x}(t')\,dt'}_{\dot{x}\text{有关.}}$$

Langevin $m\ddot{x} = -\alpha\dot{x} + f + \xi$.

$$\dot{x} = \frac{p}{m}.$$

$$\dot{p} = -V'(x) - \sum_j k_j\left[-q_j^0 + x(0)\cos(w_j t) + \int_0^t \cos(w_j(t-t'))\dot{x}(t')\,dt'\right]$$

$$\dot{p} = -V'(x) + \sum_j k_j q_j^0 - \left[\sum_j k_j \cos(w_j t)\right] x(0)$$

$$- \int_0^t \sum_j k_j \cos(w_j(t-t'))\dot{x}(t')\,dt'$$

$$\dot{p} = -V'(x) + \xi - B(t)x(0) - \int_0^t B(t-t')\dot{x}(t')\,dt'$$

$$\dot{x} = \frac{p}{m}.$$

(*) 卷积 $\alpha\delta(t-t') \Rightarrow \alpha\dot{x}$

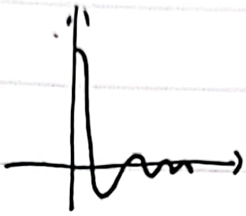
$$B(t) = \sum_j k_j \cos(w_j t)$$

$$= \sum_j m_j w_j^2 \cos(w_j t)$$

$$m\ddot{x} + \int_0^t B(t-t')\dot{x}(t')dt' + V'(x) + B(t)x(0) = F.$$

$F = \sum_i k_i q_i(0)$ 随机力.

如果 ~~...~~)



$m\ddot{x} + \alpha\dot{x} + V'(x) = F.$

(满足量子化.

$$F = \sum_j k_j q_j^0$$

$$= \sum_j k_j \left[q_j(0) \cos(w_j t) + \frac{p_j(0)}{m_j w_j} \sin(w_j t) \right].$$

其中 $[q_j, p_j] = i\hbar \delta_{ij}$.

$$\begin{cases} q_j = \sqrt{\dfrac{\hbar}{2m_j w_j}} (b_j + b_j^+) \\ p_j = i\sqrt{\dfrac{\hbar m_j w_j}{2}} (b_j - b_j^+) \end{cases}$$

$$F = \sum_j f_j (b_j e^{-iw_j t} + b_j^+ e^{iw_j t})$$

$$\langle F^2(t) \rangle \sim \eta^2 \langle (b+b^+)^2 \rangle$$

$$\propto \eta^2 \langle bb^+ + b^+b + b^2 + b^{+2} \rangle$$

$$\langle F(t)\, F(t') \rangle = D\,\delta(t - t')$$

$\propto k_B T$.