1 2 3 4 5 6 7 8 9 10 11 12 The month

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网页

罗列重点.
作业.
阅读材料.
Project 题目.
去年的考题.

前面一门. 数值方法. Mathematica.

Mathematica for ~~physicist~~ ~~physics~~ physics. M. Stone p20页.

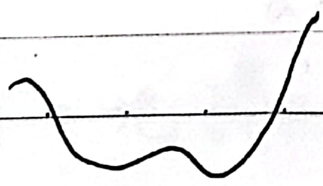
~~手册~~. 作为手册.

思想: 从简单到复杂

内容: 计算方法 (插值, 积分, 本征值, 方程求根, 极值)
随机数. FFT.

物理: 确定性.
统计.
动力学. 本征值. 随机. 极值.

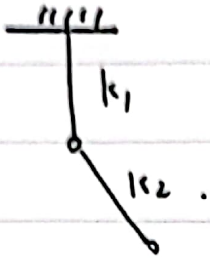
$$\sum_{i=0}^{n} i C_m^i \sum_{j=0}^{i} (-1)^j C_i^j (i-j)^n = (a_1 m+b_1)\left[(a_2 m+b_2)^n-(a_3 m+b_3)^n\right]$$



$$H=-\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)$$ 本征值

$\dot{v} = -(1+t)v + \xi(t)$

$\langle v \rangle (t=5) = ?$



$$H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_1 \cos x + V_2 \cos(2x+\theta)$$

$$H = \omega \sigma_z + \Omega a^+ a + g(a^+ \sigma^- + h.c.)$$

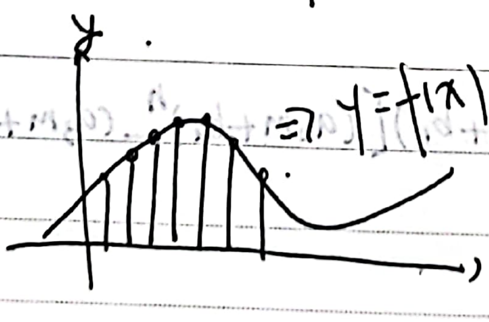
Interpolation 插值

一). 目的.

二). 缺点

三). $[a,b]$ ⇒ 积分. 面积. 体积 ...

$(x_i, f(x_i) = y_i)$



⇒ $y=f(x)$ 函数. ⇒ 预言能力

$$f(x) = \sum_n c_n \varphi_n(x)$$

多项式.

缺点：Runge 现象.

多项式：已知：$(x_1, y_1) \cdots (x_n, y_n)$

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1}$$

令 $y(x_i) = y_i$

$$a_0 + a_1 x_i + a_2 x_i^2 + \cdots + a_{n-1} x_i^{n-1} = y_i$$

线性方程组. 简单:

$$\begin{pmatrix} 1 & x_1 & x_1^2 \cdots & x_1^{n-1} \\ 1 & & & \\ \vdots & & & \\ 1 & x_n & x_n^2 \cdots & x_n^{n-1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$\det(A) = \prod_{i<j} (x_i - y_j) \neq 0.$$

$$Aa = Y$$

$$a = A^{-1} Y.$$

唯一地确定系数 a.

$$\left.\begin{array}{l}\text{误差}: R_n(x) \\ e_n(x)\end{array}\right\}$$ 所有问题的误差估计.

$R_n(x_i)=0$.

$R_n(x) \propto \prod_i (x-x_i)$

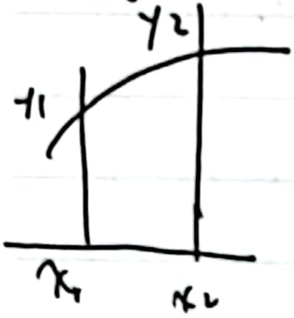
$$f(x)=a_0+a_1x+\cdots+a_{n-1}x^{n-1}+\underbrace{f\prod_{i=1}^{n}(x-x_i)}_{\text{最高次 } x^n}.$$

Lagrange.

两个重要插值 ~~bon~~ 拉格朗日. Newton.

特点: 闭合表达式
讨论+理解.

Lagange Interpolation. 几何图像.



$$y=\frac{y_2-y_1}{x_2-x_1}(x-x_1)+y_1$$

$$\Rightarrow y=\frac{x-x_1}{x_2-x_1}y_2+\frac{x-x_2}{x_1-x_2}y_1$$

$$=y_1e_1(x)+y_2e_2(x)$$

~~$e_1(x_1)=0$~~.

$$\begin{cases} e_1(x_1)=1 \\ e_1(x_2)=0 \\ e_2(x_1)=0 \\ e_2(x_2)=1 \end{cases}$$

$e_1+e_2=1$.

构造 i 去. $y=\sum_i e_i(x)y_i + R_n(x)$

$$e_i(x_j)=d_{ij}.$$

$$e_i(x_j)=d_{ij}.$$

$$\begin{cases} e_1(x_1)=1. \\ e_1(x_2)=0 \\ \vdots \\ e_1(x_n)=0. \end{cases}$$

$$e_1(x)=\frac{(x-x_2)\cdots(x-x_n)}{(x_1-x_2)\cdots(x_1-x_n)}$$

$$e_i(x)=\frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)}$$

$$\underbrace{\sum_i e_i(x)}_{(n-1)\text{阶多项式}}=1.$$

$\underbrace{\sum_i e_i(x)}_{(n-1)\text{阶多项式}} - 1 = 0$.

其解有 $(x_1, \cdots, x_n)$ n个.

所以其值为1.

此式为恒等式.

Newton 插值.

$$f(x) = a_0 + a_1(x-x_1) + a_2(x-x_1)(x-x_2) + \cdots$$

$$f(x_1) = a_0.$$

$$f(x_2) = a_0 + a_1(x_2 - x_1)$$

$$f(x_3) = a_0 + a_1(x_3 - x_1) + a_2(x_3 - x_1)(x_3 - x_2).$$

$$\vdots$$

多项式：最后的结果是一样的. 误差也相同.

$$R_n \propto \prod_i (x - x_i)$$

Runge 现象.

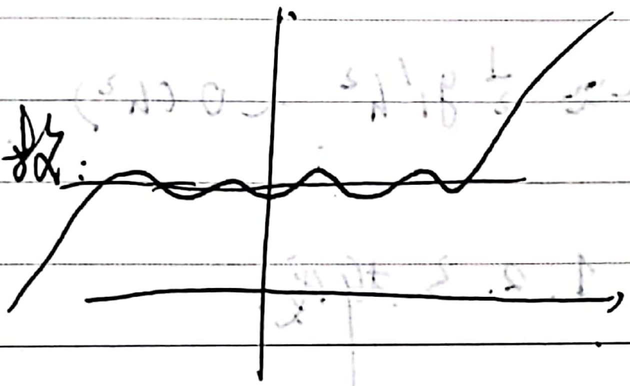
Runge-kutta. 算法

☆ Laplace - Runge - Lenz vector (LRL vector)

$$\vec{A} = \vec{p}\times\vec{L} - mk\hat{r}.$$

⇐ 一份材料.

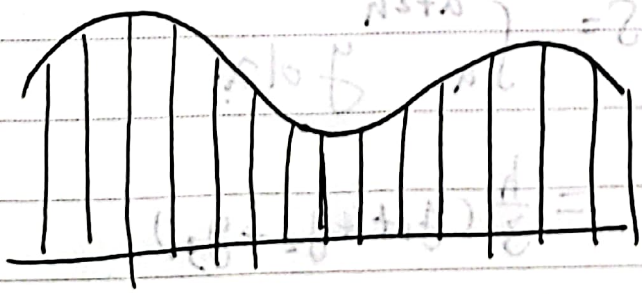
常值函数:



Mathematica. 两个命令.

1. Interpolation$[\{\{x_1, y_1\}, \{x_2, y_2\}, \cdots, \{x_n, y_n\}\}, x]$.

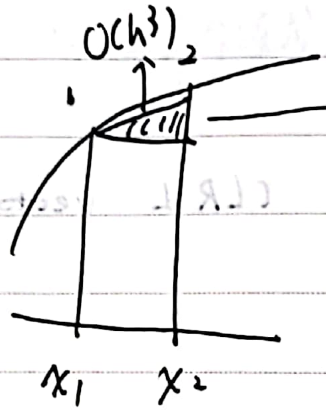
2. InterpolatingPolynominal$[\{\{x_1, y_1\}, \{x_2, y_2\}, \cdots, \{x_n, y_n\}\}, x]$.



插值近似.

low-dimension 函数.

积分. 低维 梯形近似.
高维: Monte-carlo.

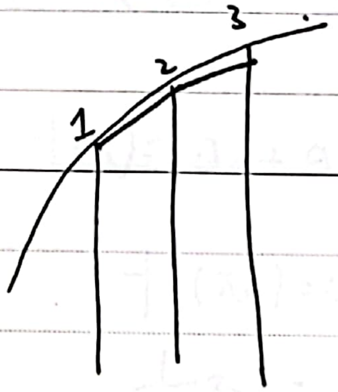


$\to S=(y_2-y_1)\times(x_2-x_1)\times\frac{1}{2}$.

$y_2=y_1+k(x_2-x_1)$.

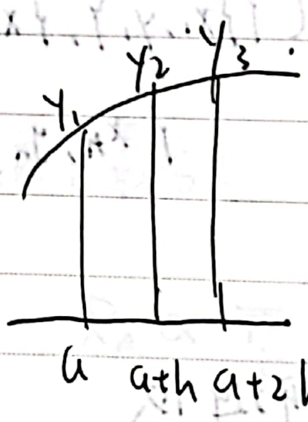
$k\approx y_1'$

$\approx\frac{1}{2}y_1'h^2\sim O(h^2)$.



1. 2. 3. 折线.

1. 2. 3. 曲线 ⇒ 近似.



$y=e_1(x_1)y_1+e_2(x_2)y_2+e_3(x_3)y_3$

$S=\int_a^{a+2h} y\,dx$

$=\frac{h}{3}(y_1+4y_2+y_3)$

辛普森算法. $O(h^4)$

Simpson公式.

$\frac{3}{8}h(y_1+3y_2+3y_3+y_4)$.

$\vdots$

Q：代码： BLAS.

$\sum_i a^*(i)\, b(i)$