1. In implementing time-dependent Fourier representations, we employ sampling in both the time and frequency dimensions. In this problem we investigate the effects of both types of sampling.

Consider a sequence x[n] with conventional Fourier transform:

$$X(e^{jw}) = \sum_{m=-\infty}^{\infty} x[m]e^{-jwm}$$

(a) If periodic function $X(e^{jw})$ sampled at frequencies $w_k = 2\pi k / N, k = 0,1,...,N-1$, we obtain

$$\tilde{X}[k] = \sum_{m=-\infty}^{\infty} x[m]e^{-j\frac{2\pi}{N}m}$$

These samples can be thought of as the discrete Fourier transform of the sequence $\tilde{x}[n]$ given by

$$\tilde{x}[n] = \sum_{k=0}^{N-1} \frac{1}{N} \tilde{X}[k]e^{j\frac{2\pi}{N}km}$$

Show that

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n+rN]$$

(b) What are the conditions on x[n] so that no aliasing distortion occurs in the time domain when $X(e^{jw})$ is sampled?

(c) Now consider "sampling" the sequence x[n]; i.e., let us form the new sequence

$$y[n] = x[nM]$$

consisting of every M^{th} sample of x[n]. Show that the Fourier transform of y[n] is:

$$Y(e^{jw}) = \frac{1}{M} \sum_{k=0}^{M-1} X(e^{j(w-2\pi k)/M})$$

In proving this result you may wish to begin by considering the sequence:

$$v[n] = x[n]p[n]$$

where

$$p[n] = \sum_{r=-\infty}^{\infty} \delta[n+rM]$$

Then note that $y[n] = v[nM] = x[nM]$.

(d) What are the conditions on $X(e^{jw})$ so that no aliasing distortion in the frequency domain occurs when x[n] is sampled?

2. A linear time-invariant system has the transfer function,

$$H(z) = 8\left[\frac{1-4z^{-1}}{1-\frac{1}{6}z^{-1}}\right]$$

(a) Determine the complex cepstral coefficients, $\hat{h}(n)$, for all n.

(b) Plot $\hat{h}(n)$ versus n for the range $-10 \le n \le 10$.

(c) Determine the(real) cepstrum coefficients, $c[n]$, for all n.

3. A casual LTI system has system function:

$$H(z) = \frac{1-4z^{-1}}{1-0.25z^{-1}-0.75z^{-2}-0.875z^{-3}}$$

(a) Use the Levinson-Durbin recursion to determine whether or not the system is stable.

(b) Is the system minimum phase?