Beam Orientation Optimization for Intensity Modulated Radiation Therapy using L_{21} Minimization

Xun Jia
xunjia@ucsd.edu
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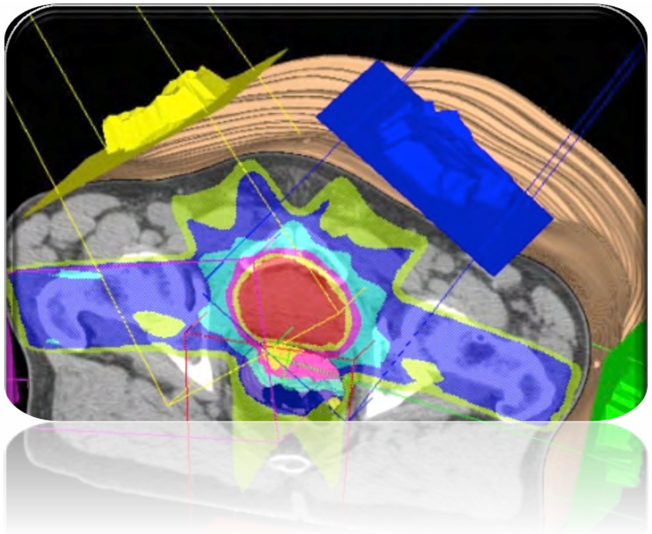
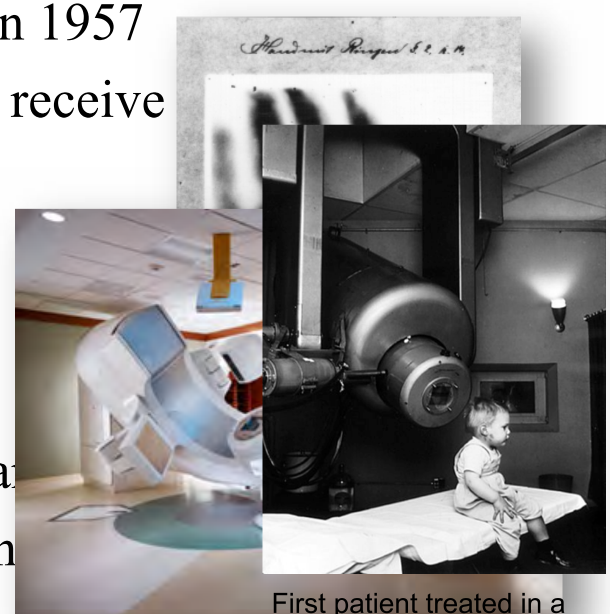
Outline

- Introduction to Radiation Therapy
- Motivation of Beam Orientation Optimization (BOO)
- BOO
  - Model and Rationale
  - Algorithm
  - Validation
- Conclusions



Radiotherapy

- Medical applications of radiation for cancer treatment
  - Discovery of x-ray in 1895
  - First cancer treatment in US in 1896
  - First treatment in MV linear accelerator in 1957
  - Nowadays, ~2/3 of cancer patients receive therapy as part of their cancer treatment
- Mechanism
  - Damaging the DNA of cancerous cells
- Objectives
  - Deliver a prescribed amount of dose to ca
  - Spare radiation dose to surrounding organ

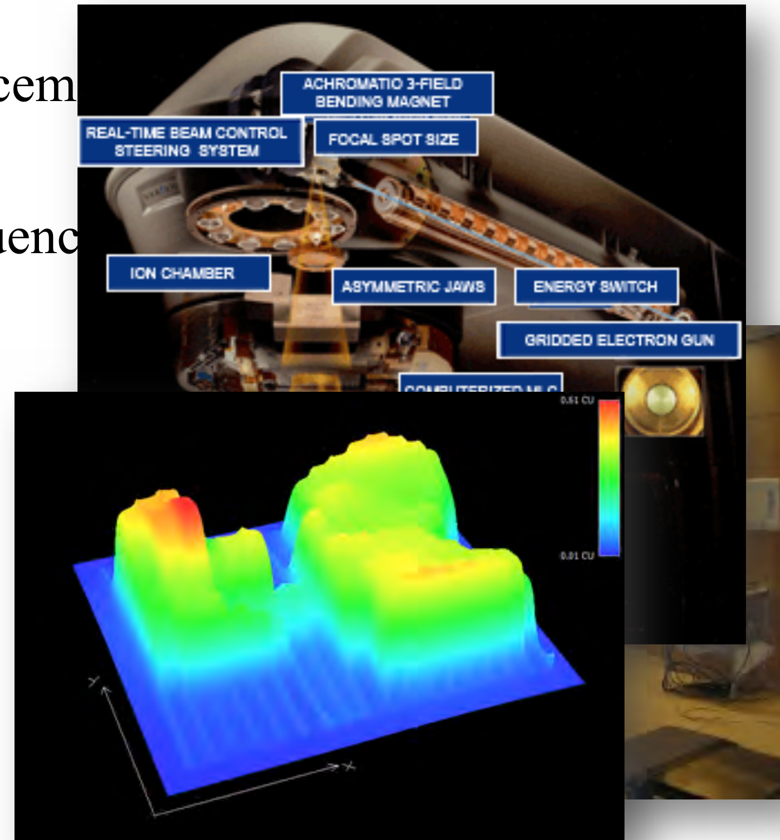


First patient treated in a MV linear accelerator

A TrueBeam linear accelerator

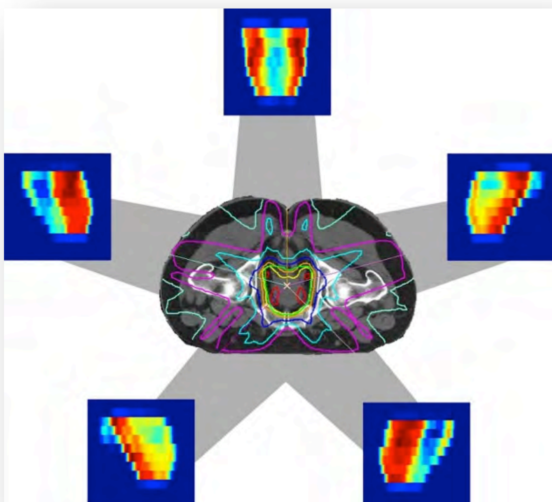
Linear Accelerator

- A linear accelerator (Linac) produces high energy radiation beams for the treatment
- Flexible geometry allows the free placem
- Multi-leaf Collimator (MLC)
  - To shape the beam and form a fluenc



IMRT

- Intensity Modulated Radiation Therapy
  - A few beam angels are selected
  - A (non-flat) fluence map is delivered at each beam angle
  - Conformed dose distribution to target
  - Sparing dose to critical organs by beam angle selection and fluence map modulation



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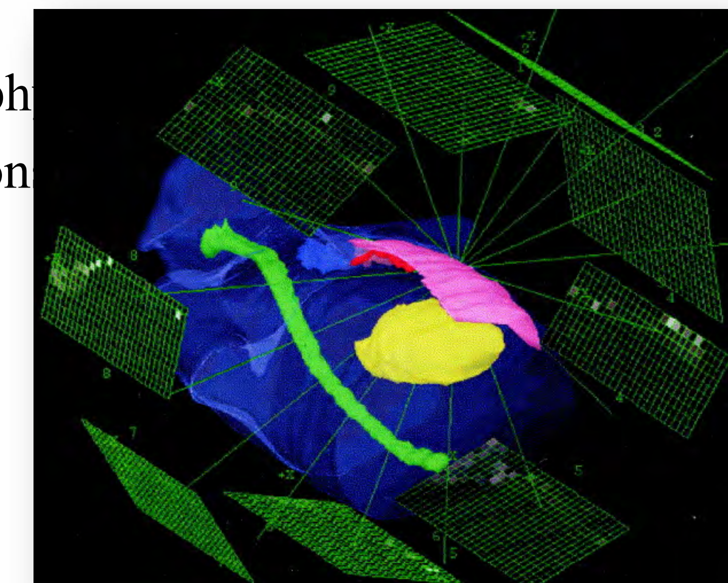
Mathematical View

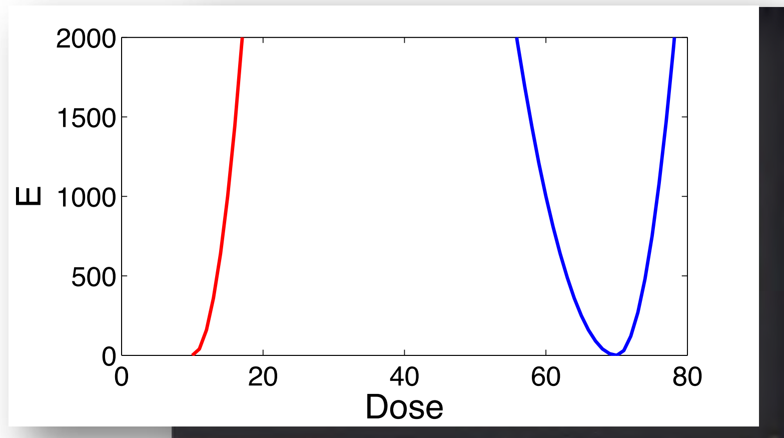
- Discretize fluence maps into beamlets $\{x_{j,\theta}\}$
- Discretize 3D patient body into voxels $i \in T \bigcup C$
  - T --- target, C --- critical organs
- Dose deposition matrix D
  - $D_{i,j,\theta}$: the dose to the voxel i from the beamlet j at angle θ at unit intensity
  - Patient-specific, determined by ph
- Dose calculation: sum the contribution

$$z_i = \sum_{j \in s_\theta, \theta \in \Theta} D_{i,j,\theta} x_{j,\theta}$$



Mathematical View

- Optimization problem:
  - Determine the values of a set of treatment parameters, such that the dose distribution z agrees with the prescription dose p



- Objective function
  - Designed for

various considerations
  - A typical (and simple) one

$$E = \sum_{i \in T \bigcup C} E_i[z_i]$$

$$E_i[z_i] = \alpha_i \max(0, z_i - p_i)^2 \quad i \in C$$

$$E_i[z_i] = \alpha_i \max(0, z_i - p_i)^2 + \beta_i \max(0, p_i - z_i)^2 \quad i \in T$$

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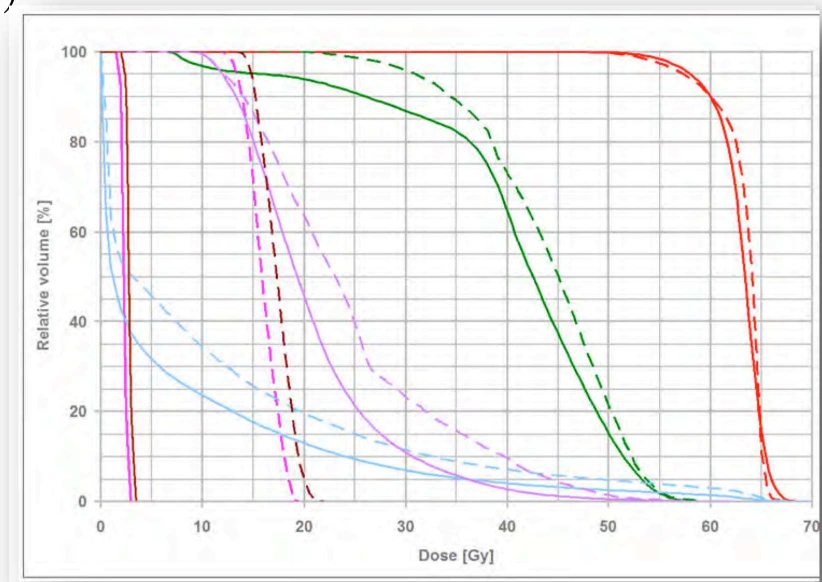
DVH

- Dose Volume Histogram:
  - To summarize 3D dose distributions in a graphical 2D format
  - A organ-specific curve *V(z)* --- at least V% of the organ receives a dose level of z

$$V(z) = 1 - \int_0^z \mathrm{d}z' \; p(z')$$

- Ideal DVH curves
- In reality…



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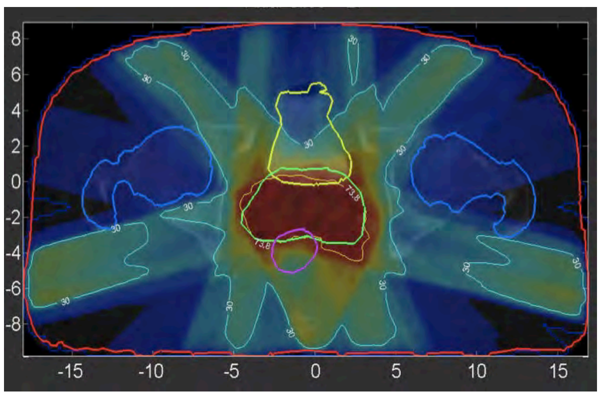
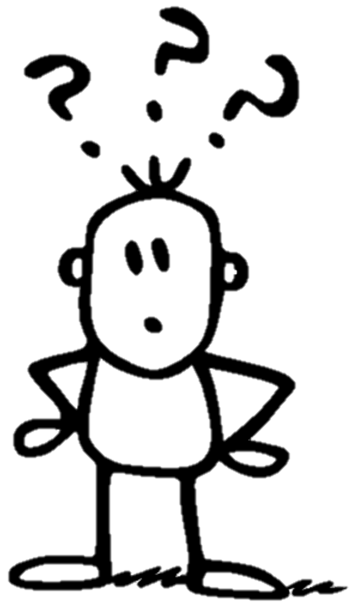
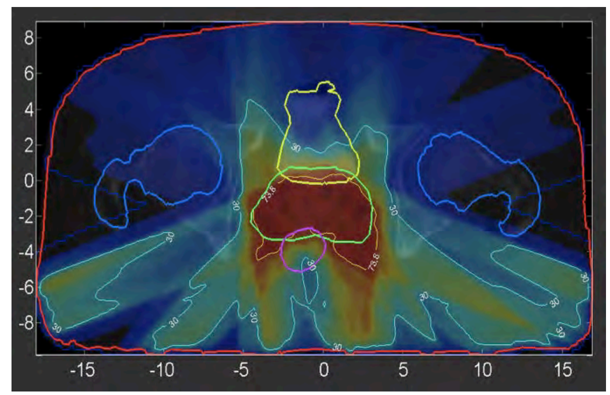
Beam Orientation Optimization

- Motivation for BOO
  - IMRT optimization
    - Find fluence maps at a certain angles for a good treatment plan

  - At what angles?







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BOO

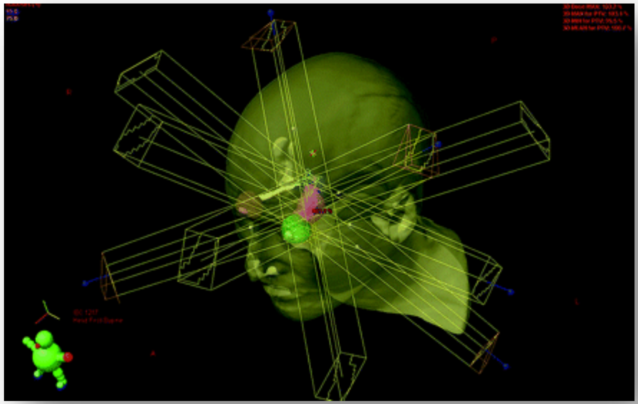
- Notations
  - Fluence map $x_{j,\theta}$
  - Dose deposition matrix $D_{i,j,\theta}$
  - Dose distribution

$$z_i = \sum_{j \in s_\theta, \theta \in \Theta} D_{i,j,\theta} x_{j,\theta}$$

- Find a small set of angles for a good plan

$$\Theta = \text{argmin}_\Theta \left[\min_{x_{j,\theta}} E[z] \quad \text{s.t.} \quad \theta \in \Theta, \ x_{j,\theta} \geq 0 \right]$$

- Available approaches
  - Trial-and-error
  - Enumeration
  - Geometry consideration
  - Ranking method
  - …



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Model

- The idea of sparsity
  - Find a solution that has only a few non-zero elements, such that…
  - For BOO, select only a few beam angles among all candidates
  - Sparsity only at the beam angle level

- Dosimetric objective

$$E_{Dose} = \sum_i \alpha_i [\max(0, p_i - z_i(x))]^2 + \beta_i [\max(0, z_i(x) - p_i)]^2$$

- BOO objective

$$E_{Angle} = \sum_\theta \mu_\theta [\sum_j (x_{j,\theta})^2]^{1/2}$$

- Optimization model

$$x = \text{argmin}_x \mu E_{Dose} + E_{Angle}$$

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L_{21} Norm

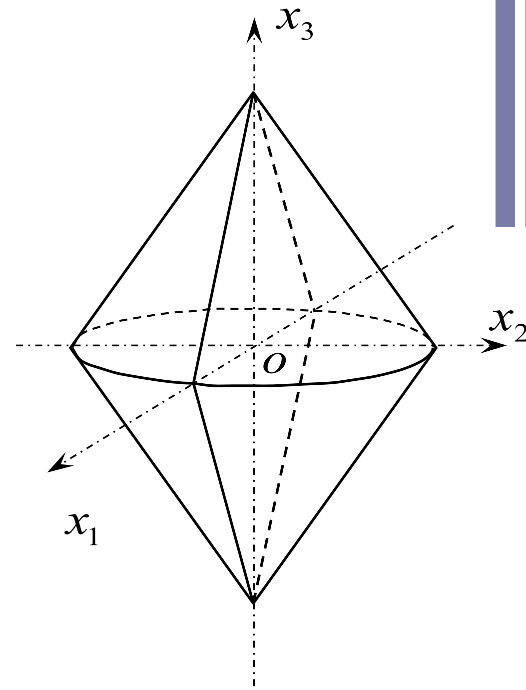
- For a beamlet vector

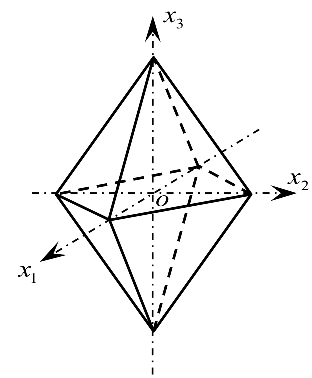
$$x = (x_{1,1}, x_{2,1}, x_{3,1}, \ldots x_{1,2}, x_{2,2}, \ldots)$$

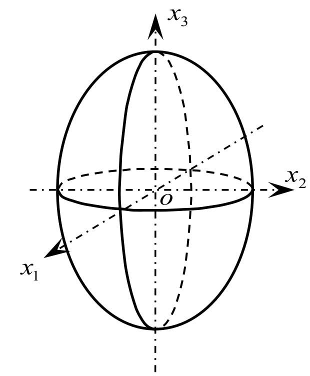
- Define L_{21} norm

$$|x|_{2,1} = \sum_{\theta}[\sum_{j}(x_{j,\theta})^2]^{1/2}$$

- Minimization of an L_{21} norm leads to sparsity only at beam angle level, while treating all beamlets in an angle equally







Algorithm

- Optimalily condition

$$0 \in \mu \frac{\partial E_{Dose}}{\partial x} + \frac{\partial E_{Angle}}{\partial x}$$

- Split

$$0 \in x - g - \lambda\mu \frac{\partial E_{Dose}}{\partial x}$$

$$0 \in x - g + \lambda \frac{\partial E_{Angle}}{\partial x}$$

- Algorithm

$$g = x - \lambda\mu \frac{\partial E_{Dose}}{\partial x}$$

$$x = \arg\min \frac{1}{2}|x - g|_2^2 + \lambda E_{Angle}$$

$$x^\theta = g^\theta \max\left(1 - \frac{\lambda\mu}{|g^\theta|_2}, 0\right)$$

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Algorithm

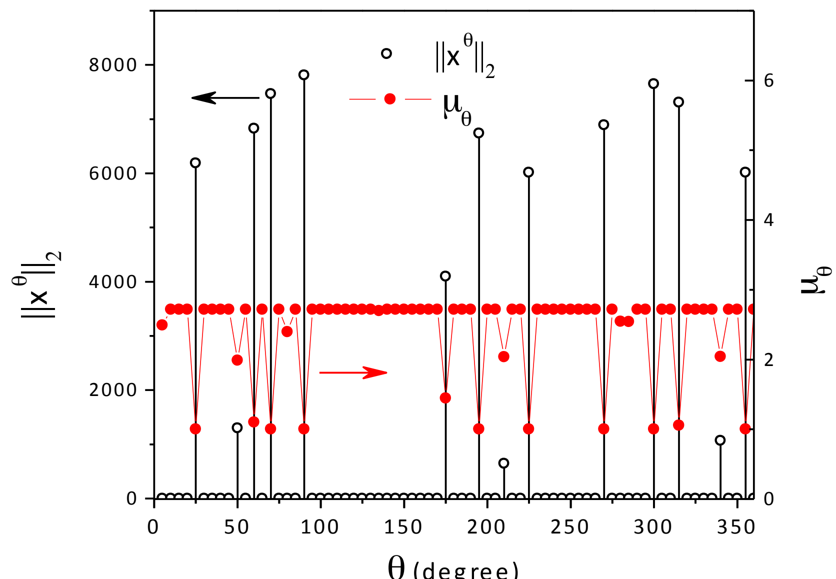
- Varying μ_θ $\quad E_{Angle} = \sum_\theta \mu_\theta [\sum_j (x_{j,\theta})^2]^{1/2} = \sum_\theta \mu_\theta |x^\theta|_2$

  large $|x^\theta|_2 \rightarrow$ more likely a good angle $\rightarrow$ small μ_θ

  Only compare to its neighbors

- A heuristic method to speed up the convergence

  1. locate two nearby beams θ_+ and θ_- with non vanishing $|x^\theta|_2$
  2. find $A = \max[|x^\theta|_2, |x^{\theta_+}|_2, |x^{\theta_-}|_2]$
  3. compute $\mu_\theta = \exp[-(\frac{|x^\theta|_2}{A} - 1)]$



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Algorithm

- Summary of algorithm
  - Sparsify fluence map:

$$g = x - \lambda\mu \frac{\partial E_{Dose}}{\partial x}$$

$$x^\theta = g^\theta \max(1 - \frac{\lambda\mu_\theta}{|g^\theta|_2}, 0)$$

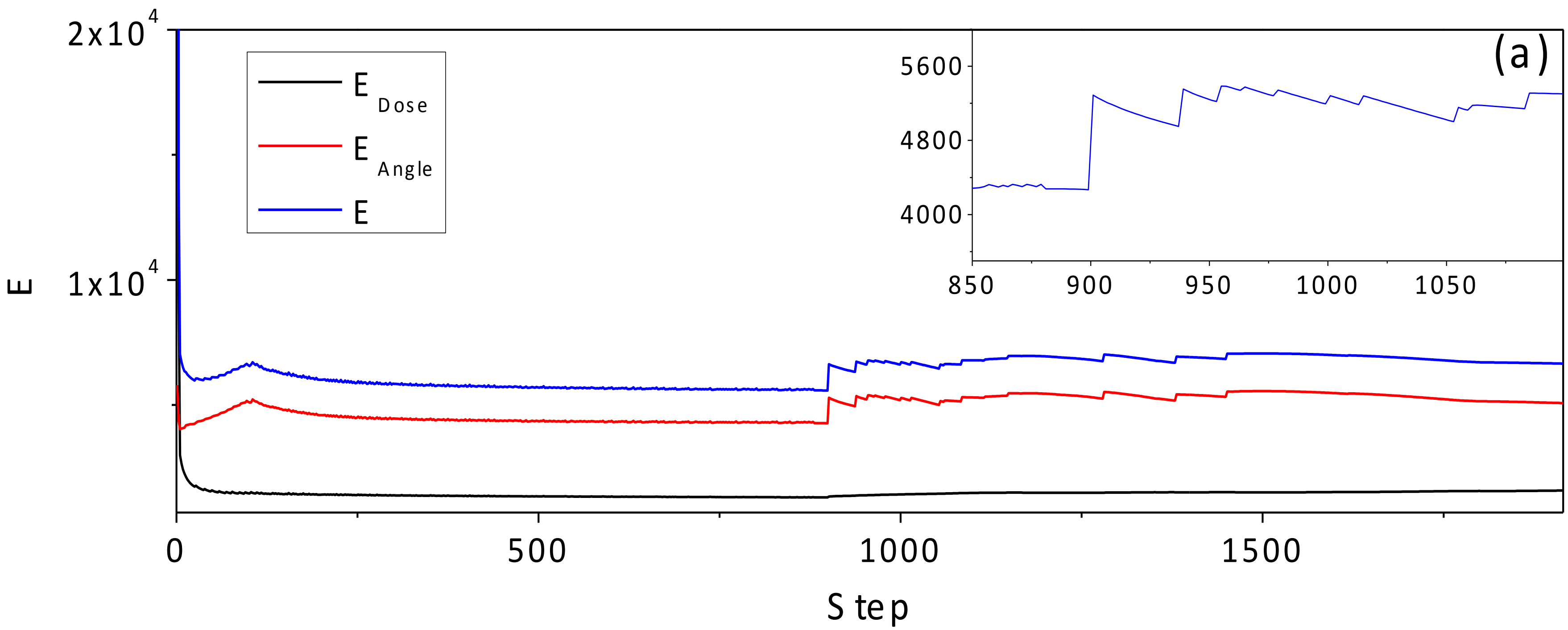
  - Adjust weighting factor

$$\mu_\theta = \exp[-(\frac{|x^\theta|_2}{A} - 1)]$$

  - Count the number of beam angles; if more than desired, go back to the first step

Iteration Process

- Objective function value



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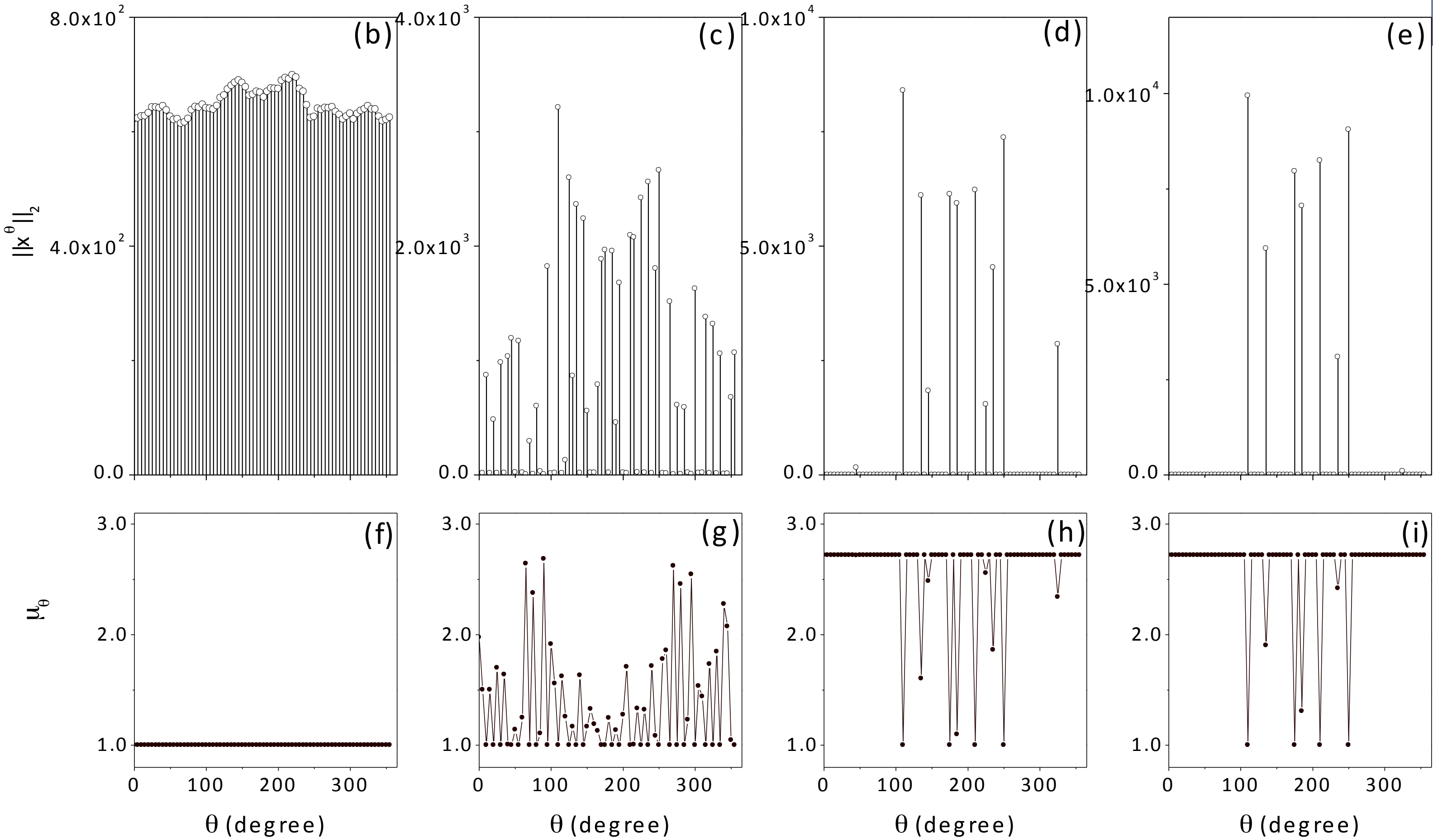
Iteration Process



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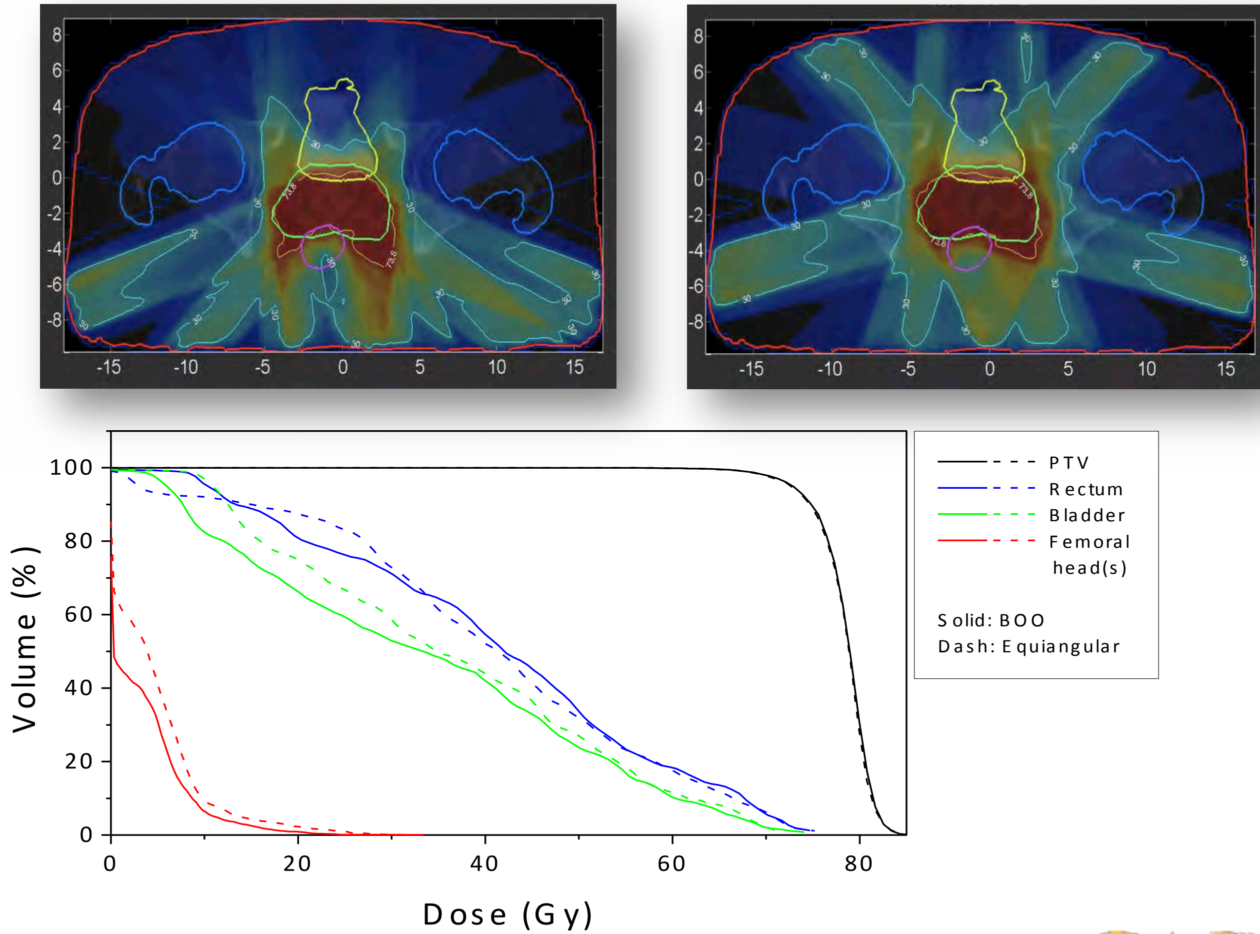
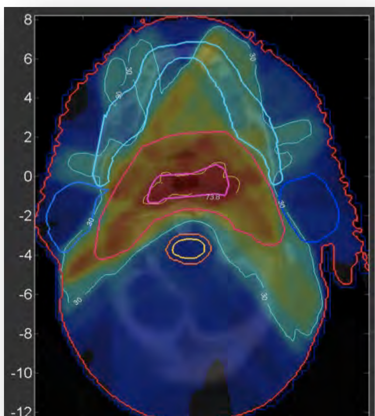
Results

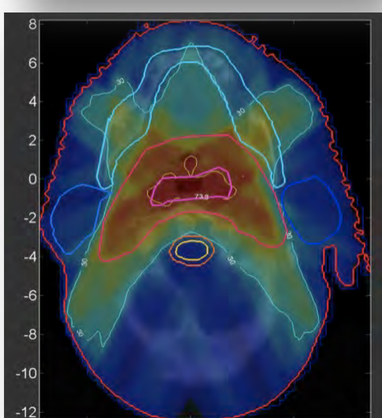
- A prostate case

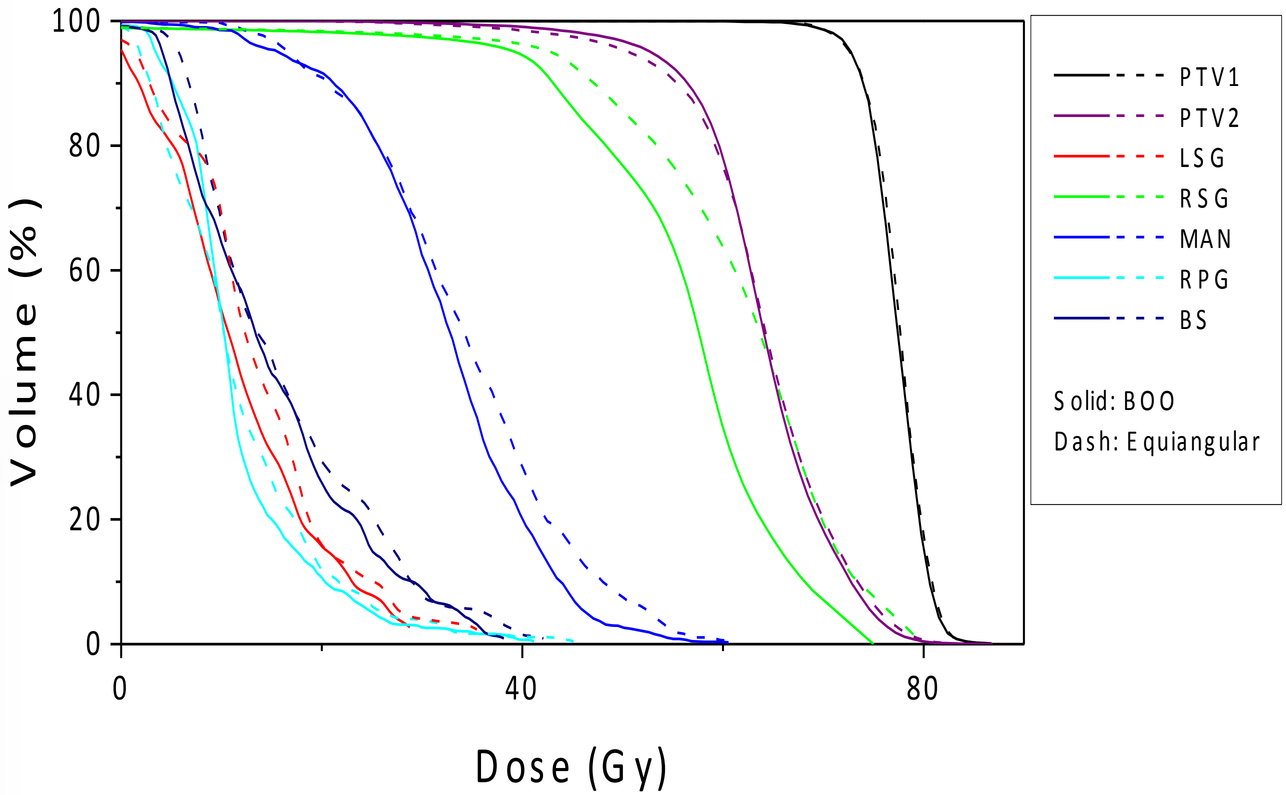


Results

- A head-and-neck case







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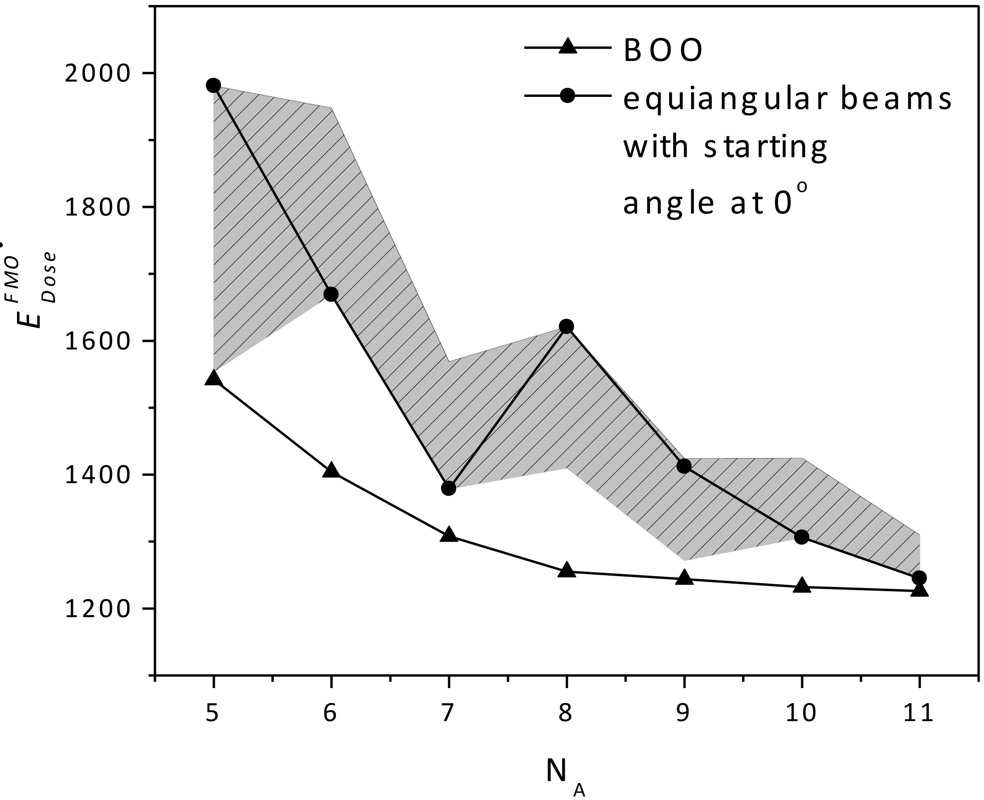
Varying Beam Angles

- Perform FMO based on given angles and compare the resulting objective function value

- Equiangular plans with various starting angles

- BOO plans are always better than non-BOO plans

- Gain of using more angels become diminishing



Objective Function Values

- Summary of FMO objective function values in all cases

Case \ N_A	5	6	7	8	9
P1	**3218**/4655 [4655-6004]	**3026**/6343 [4705-6699]	**2966**/4052 [3761-4473]	**2725**/4966 [4054-4966]	**2610**/3766 [3766-3511]
P2	**2098**/2268 [2133-2524]	**2023**/2070 [2070-2879]	**1824**/1934 [1861-1949]	**1812**/2009 [1916-2012]	**1703**/1864 [1714-1893]
P3	**1541**/1981 [1554-1981]	**1404**/1669 [1669-1948]	**1308**/1379 [1379-1569]	**1255**/1621 [1410-1621]	**1244**/1412 [1272-1424]
P4	**1946**/2446 [1930-2446]	**1919**/2002 [2002-2250]	**1815**/1874 [1845-2035]	**1799**/2017 [1972-2042]	**1627**/1816 [1691-1939]
P5	**2289**/2689 [2391-2834]	**2111**/2576 [2433-2815]	**1963**/2140 [2132-2250]	**1956**/2353 [2188-2373]	**1938**/2130 [2003-2188]
H1	**191**/185 [185-240]	**163**/238 [237-265]	**157**/181 [163-181]	**155**/189 [172-201]	**144**/152 [148-168]

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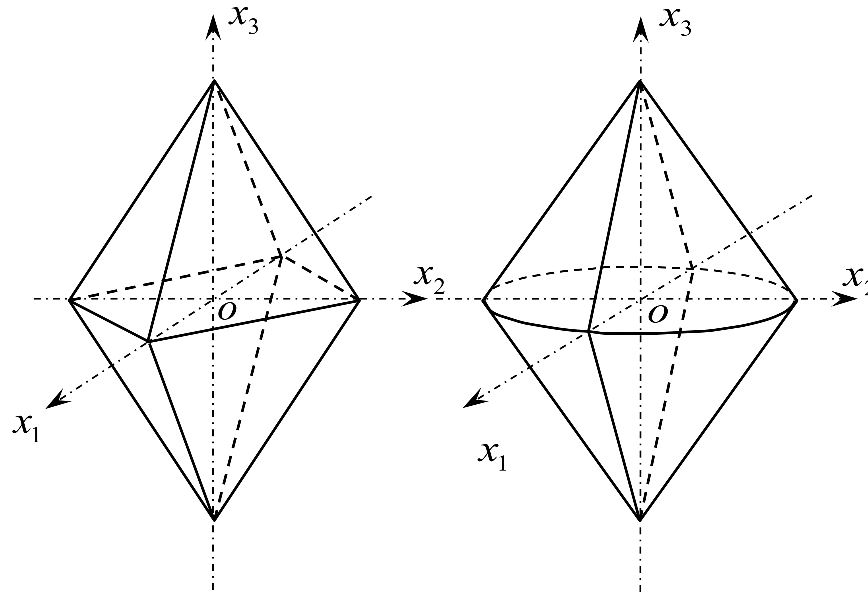
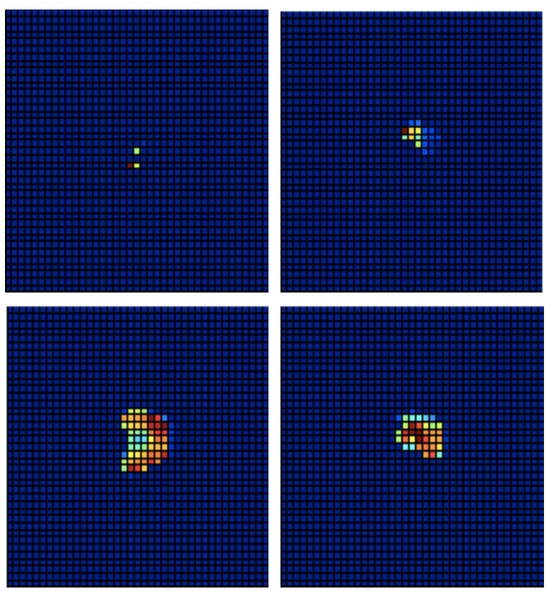
Discussions

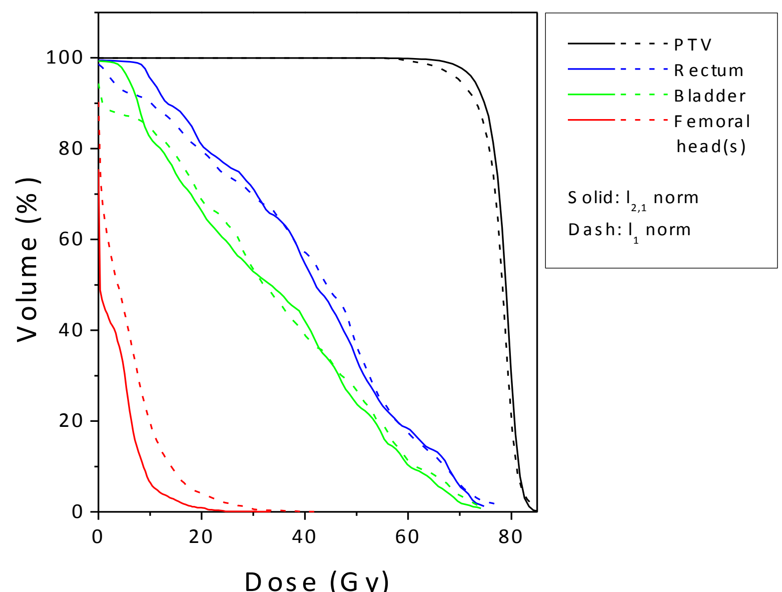
- L_{21} VS L_{11} norms

$$E[x] = \sum_{\theta} \sum_{i} |x_i^{\theta}|$$

$$E[x] = \sum_{\theta} [\sum_{i} (x_i^{\theta})^2]^{1/2}$$







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Conclusion

- Beam Orientation Optimization
  - It can be approximately solved by an L_{21} minimization approach and the problem is convex
  - We developed an efficient algorithm to solve the optimization problem
  - We have validated this approach in patient cases
  - L_{21} is a good approximation to the BOO problem

Acknowledgement

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