Deterministic Blind Identification of IIR Systems With Output-Switching Operations

Chengpu Yu, Lihua Xie, and Cishen Zhang

***Abstract*—In this paper, a deterministic blind identification approach is proposed for linear output-switching systems, which are modeled by multiple infinite impulse-response (IIR) dynamic functions. By adopting a new over-sampling strategy, the concerned single-input–single-output (SISO) output-switching system is equivalently transformed into a time-invariant multi-input–multi-output (MIMO) system. Further, by exploring the mutual relations among the multiple inputs, the time-invariant MIMO system model and subsequently the output-switching system model are identified uniquely up to a scalar constant using the proposed identification approach. Sufficient identifiability conditions are provided for output-switching systems and numerical simulations are carried out to validate the proposed approach.**

***Index Terms*—Blind identification, linear output-switching system, over-sampling operation, polyphase decomposition.**

I. Introduction

THIS paper investigates the blind identification of linear output-switching systems [1]. The output-switching system is widely used in many areas, such as wireless communications [2], [3], switching control [4]–[6], and circuit modeling [7]. One significant feature of the output-switching system is that only one communication channel is adopted to transmit several system outputs, thus increasing the communication efficiency and reducing the number of communication channels. Since a great number of system observations are lost during the switch process, it imposes great challenges to blindly identify the associated multiple transfer functions without accessing the system input.

The output-switching system is one kind of linear periodically time-varying (LPTV) system and is more general than the linear time-invariant (LTI) system. In the literature, there are several blind identification approaches for LPTV systems which can be generally classified into two categories: statistical methods [8]–[10] and deterministic methods [11], [12]. The statistical methods adopt statistical properties of the system input, while the deterministic methods do not. When the system input is an independent and identically distributed (i.i.d.) random signal and its mean and variance are known as *a priori* knowledge, a cyclic-moment based estimator was proposed in [13], [14] which can identify both the associated exponential bases and the channel functions. A multi-step linear predictor was designed for the blind identification of LPTV SIMO systems by assuming the system input to be white [8], [10]. A blind channel identification and interference rejection method was developed based on the assumption that the desired input and interfering signals are mutually independent white noises [9].

Deterministic blind identification methods do not rely on the statistical properties of the system input. There are a number of deterministic blind identification methods for SIMO FIR systems, such as cross-relation method [15], subspace method [16], least-square smoothing method [17] and so on. In contrast, not much work has been done on deterministic identification of LPTV (or output-switching) systems. A subspace-based deterministic method was developed for LPTV SIMO systems that are described by complex exponential basis expansion models [11]. In [12], SIMO systems modeled by more general basis functions are investigated; however, the multiple channels can only be identified up to an ambiguity matrix.

In this paper, a deterministic blind identification method for output-switching systems is developed. Using the over-sampling strategy proposed in [18], the concerned output-switching system is equivalently transformed into a time-invariant multi-input multi-output (MIMO) FIR system for which the transfer function matrix can be identified up to a constant ambiguity matrix under some mild conditions. Further, using the redundancy information introduced by the over-sampling operation, both the autoregressive parts of the involved transfer functions and the ambiguity matrix are identified by the proposed method. Although the over-sampling strategy proposed in our previous works [18], [19] is adopted here, the considered identification problem differs from those in [18], [19]: the output-switching system with multiple time-varying IIR transfer functions is considered in this paper, while only a single time-invariant IIR transfer function was considered in [18], [19]. More specifically, compared with [18], [19], the challenging points of the current work are as follows: 1) The sufficient conditions for the blind multi-channel identification are more complicated than that of the single-channel case. Especially, the condition on the over-sampling rate is more restrictive. 2) It is required to identify an extra ambiguity matrix for the multi-channel case. In addition, compared with the works in [11], [12], the concerned output-switching system

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is modeled by infinite impulse response (IIR) functions so that the corresponding identification problem is more challenging. Although the over-sampling technique has been widely applied to the blind identification of FIR systems [15], [20], over-sampling output-switching systems that are modeled by IIR functions are seldom investigated. The derived equivalent time-varying SIMO system model in this paper is much the same as that in [11]. However, the basis functions of the derived model are linearly dependent so that the identification problem cannot be solved by the method proposed in [11].

The rest of this paper is organized as follows. In Section II, the blind identification problem under consideration is formulated. The main result of this paper, namely a deterministic blind identification approach for output-switching systems is derived in Section III. Section IV shows several simulation examples followed by the conclusion in Section V.

Throughout the paper, the superscripts $'$, ^{T} and ^{H} represent the conjugate operator, matrix transpose and Hermitian transpose, respectively. The superscripts $^{-1}$ and † denote matrix inverse and Moore-Penrose inverse, respectively. I is an identity matrix of appropriate dimension and $j = \sqrt{-1}$ is an imaginary unit. $H(q)$ denotes the transfer function of $h(n)$ in time domain, and q^{-1} is a backward shift operator. $H(z)$ represents the transfer function $h(n)$ in z-domain. $\mathbb{Z}, \mathbb{N}$ and $\mathbb{C}$ stand for the integer number set, natural number set and the complex number set, respectively. $\uparrow L$ and $\downarrow L$ denote L-fold up-sampling and L-fold down-sampling operators as defined in [21], respectively, and $n \,\mathrm{Mod}\, M$ stands for the remainder of n divided by M.

II. Problem Formulation and Preliminaries

Consider a discrete-time M-channel output-switching system as follows [1]:

$$y(n) = H_{n\mathrm{Mod}M}(q)u(n) + w(n) \tag{1}$$

where $u(n)$ and $y(n)$ are system input and output, respectively, $w(n)$ is an additive noise, $\{H_i(q)\}_{i=0}^{M-1}$ denote M IIR transfer functions in time domain. Without loss of generality, we assume all IIR filters have the same denominator and denote

$$H_i(q) = \frac{F_i(q)}{A(q)} \quad i = 0, 1, \ldots, M-1$$

where $F_i(q) = f_{i0} + f_{i1}q^{-1} + \cdots + f_{iN}q^{-N}, A(q) = 1 + a_1 q^{-1} + \cdots + a_N q^{-N}$ and N is the system order which is assumed to be known. If the IIR filters have different denominators, they can be converted to a common denominator by some trivial manipulations.

The problem of interest is to blindly identify the multiple transfer functions $\{\frac{F_i(q)}{A(q)}\}_{i=0}^{M-1}$ from the system output $y(n)$ without accessing the system input.

Remark 1: The output-switching system is an important kind of multi-channel system in the communication area. Multiple channel outputs are transmitted through only one communication channel, thus lowering the communication cost and increasing the transmission efficiency. Compared with the SIMO system, the output-switching system in (1) transmits less observation samples to the receiver so that it consumes less communication bandwidth. Therefore, it is practically meaningful to investigate the identification problem of output-switching systems.

Remark 2: The output-switching system in (1) is one kind of linear periodically time-varying system. Compared with the existing works on blind identification of linear time-varying SIMO FIR systems [8]–[12], the considered output-switching system is modeled by multiple IIR transfer functions so that the corresponding identification problem is more general and challenging. Moreover, the concerned output-switching system in (1) has multiple channels but only one output, so the identification problem is quite challenging.

Throughout the paper, the following standard assumptions are adopted.

[A1.] The deterministic input $u(n)$ is not predictable, namely the current value of $u(n)$ cannot be determined by its past values. In addition, the input signal $u(n)$ has bounded amplitude.

[A2.] $\{\frac{F_i(q)}{A(q)}\}_{i=0}^{M-1}$ are stable, i.e., all poles have amplitudes smaller than one.

To address the identification problem, we first introduce an equivalent system model of (1). At time $lM + i$ for $0 \le i < M$, only the i-th channel is connected to the system output while others are disconnected. Note that the associated switch operation can be modeled by multiplying the channel output with a multiplication factor $\frac{1}{M}\sum_{k=0}^{M-1} W_M^{(n-i)k}$, where $W_M = e^{j2\pi/M}$. Indeed, it can be verified that

$$\frac{1}{M}\sum_{k=0}^{M-1} W_M^{(n-i)k} = \begin{cases} 1 & \text{if } i = n \,\mathrm{Mod}\, M \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the system model (1) can be reformulated as follows

$$\begin{aligned} y(n) &= \sum_{i=0}^{M-1}\left(\frac{1}{M}\sum_{k=0}^{M-1} W_M^{(n-i)k}\right) \\ &\quad \times \left(\frac{F_i(q)}{A(q)}u(n)\right) + w(n) \\ &= \sum_{k=0}^{M-1} W_M^{nk}\left[\frac{\left(\frac{1}{M}\sum_{i=0}^{M-1} W_M^{-ik}F_i(q)\right)}{A(q)}u(n)\right] \\ &\quad + w(n) \\ &= \sum_{k=0}^{M-1} W_M^{nk}\left[\frac{B_k(q)}{A(q)}u(n)\right] + w(n) \end{aligned} \tag{2}$$

where $B_k(q) = 1/M\sum_{i=0}^{M-1} W_M^{-ik}F_i(q)$ and its matrix form is as follows

$$\begin{bmatrix} B_0(q) \\ B_1(q) \\ \vdots \\ B_{M-1}(q) \end{bmatrix} = \frac{1}{M}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & W_M^{-1} & \cdots & W_M^{-(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W_M^{-(M-1)} & \cdots & W_M^{-(M-1)(M-1)} \end{bmatrix} \times \begin{bmatrix} F_0(q) \\ F_1(q) \\ \vdots \\ F_{M-1}(q) \end{bmatrix}. \tag{3}$$

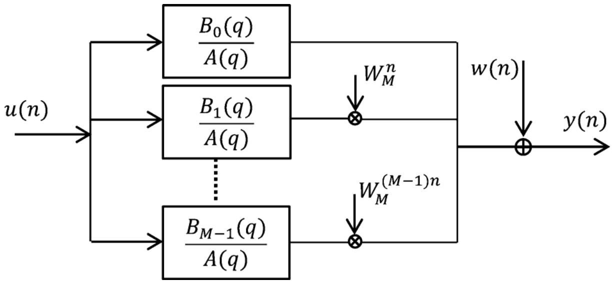

Fig. 1. An equivalent output-switching system with modulation at the output.

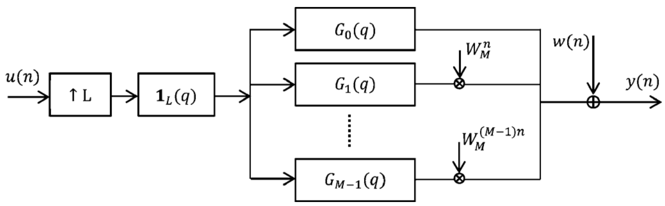

Fig. 2. An over-sampled output-switching system.

The equivalent output-switching system of (1) can be modeled by the superposition of several modulated channels as shown in Fig. 1. Since the system model in (1) and that in (2) are equivalent, we will focus on the blind identification of the system in (2) in the sequel.

III. Main Results

A. Over-Sampling the Output-Switching System

Using the over-sampling technique, the temporal channel diversity can be increased so that the channel identification can be realized without relying on the input statistics [9], [14]. For a discrete-time system with a fixed sampling period, it is impossible to over-sample the system output with a smaller sampling interval. Instead, we use the input holding strategy shown in Fig. 2 to replace the output over-sampling operation, where $G_i(q) = \frac{B_i(q)}{A(q)}$ and $\mathbf{1}_L(q) = 1 + q^{-1} + \cdots + q^{1-L}$. Let $L = L_1 \times L_2$, where $L_1, L_2 \in \mathbb{N}$. By adopting the novel over-sampling strategy proposed in [18] and using the polyphase decomposition techniques in ([21], Chapter 3), it can be established that

$$\begin{aligned} y(L_2 n + k) &= \sum_{i=0}^{M-1} W_M^{i(L_2 n+k)} \left(\left[\frac{q^k B_i(q) \mathbf{1}_{L_2}(q)}{A(q)} \right]_{\downarrow L_2} x(n) \right) \\ &\quad + w(L_2 n + k) \qquad k = 0, 1, \ldots, L_2 - 1, \end{aligned} \tag{4}$$

where $x(n)$ is generated from $u(n)$ with each symbol lasting for L_1 sampling periods. Then, an equivalent time-varying SIMO model is derived as shown in Fig. 3, where $G_{k,i}(q) = W_M^{ki}[\frac{q^k \mathbf{1}_{L_2}(q) B_i(q)}{A(q)}]_{\downarrow L_2}$, $w_k(n) = w(L_2 n + k)$ and $y_k(n) = y(L_2 n + k)$ for $0 \le i \le M-1, 0 \le k \le L_2 - 1$.

Lemma 1: Consider the down-sampled IIR transfer function $G_{k,i}(q) = W_M^{ki}[\frac{q^k \mathbf{1}_{L_2}(q) B_i(q)}{A(q)}]_{\downarrow L_2}$ for $i = 0, \ldots, M-1$. Let $\{\alpha_i\}_{i=1}^N$ be the zeros of the polynomial $A(q)$ and $\mathfrak{B}_i(q) = \mathbf{1}_{L_2}(q) B_i(q) \prod_{i=1}^N (1 + \alpha_i q^{-1} + \cdots + \alpha_i^{L_2-1} q^{1-L_2})$. Then, $G_{k,i}(q)$ can be equivalently expressed by

$$G_{k,i}(q) = W_M^{ki} \frac{Q_{k,i}(q)}{P(q)}, \tag{5}$$

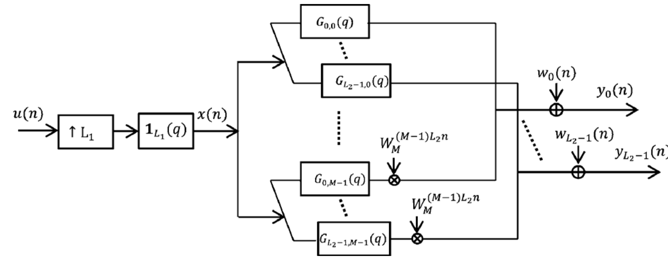

Fig. 3. An equivalent over-sampled output-switching system.

where

$$Q_{k,i}(q) = [q^k \mathfrak{B}_i(q)]_{\downarrow L_2}, P(q) = \prod_{i=1}^N \left(1 - \alpha_i^{L_2} q^{-1}\right).$$

The above lemma provides an alternative expression of a down-sampled IIR function and the details of its derivation can be found in [19]. It lays a foundation to the development of blind identification algorithms in the sequel. It is noted that the down-sampled IIR transfer functions have the same denominator polynomial. If $G_i(q)$ is a stable transfer function, $\{G_{k,i}(q)\}_{k=0}^{M-1}$ are all stable transfer functions.

B. Blind Identification of an MIMO FIR System

For notation simplicity, the noise effect will be neglected during the development of the deterministic identification method; however, it will be considered in numerical simulations. The matrix-vector multiplication form of the time-varying SIMO system shown in Fig. 3 is represented by

$$\bar{y}(n) = \sum_{i=0}^{M-1} W_M^{iL_2 n} \left[\bar{G}_i(q) x(n) \right], \tag{6}$$

where $\bar{y}(n) = \begin{bmatrix} y_0(n) \\ \vdots \\ y_{L_2-1}(n) \end{bmatrix}$ and $\bar{G}_i(q) = \begin{bmatrix} G_{0,i}(q) \\ \vdots \\ G_{L_2-1,i}(q) \end{bmatrix}$.

Let $s(n) = \frac{x(n)}{P(q)}$ be the common source signal and $\bar{F}_i(q) = [\, W_M^{0i} Q_{0,i}(q) \quad \cdots \quad W_M^{(L_2-1)i} Q_{L_2-1,i}(q)\,]^T$. Denote $\bar{T}_i(q) = \bar{F}_i(W_M^{-iL_2} q)$ and $\mathbf{T}(q) = [\, \bar{T}_0(q) \quad \cdots \quad \bar{T}_{M-1}(q)\,]$. By several trivial manipulations, (6) can be alternatively represented as

$$\bar{y}(n) = \mathbf{T}(q) \bar{s}(n), \tag{7}$$

where $\bar{s}(n) = [\, W_M^{0L_2 n} s(n) \quad \cdots \quad W_M^{(M-1)L_2 n} s(n)\,]^T$.

Equation (7) is an MIMO model with multiple unknown deterministic inputs. The advantage of such a representation is that the channel matrix $\mathbf{T}(q)$ is time-invariant so that the blind identification framework for MIMO systems [22]–[25] can be applied here. In order to identify the channel matrix $\mathbf{T}(q)$ in (7) up to a constant ambiguity matrix, the following conditions should also be satisfied [22]: (a) all columns of $\mathbf{T}(q)$ have the same degree (the degree of a polynomial vector is the largest polynomial order of its entries); (b) the channel matrix $\mathbf{T}(q)$ is irreducible and column reduced. Next, we will discuss how these conditions can be satisfied.

From the derivation of Lemma 1 in [19], it can be found that the condition (a) mentioned above can be easily satisfied when all $\{B_i(q)\}_{i=0}^{M-1}$ have the same order N. However, if the order of $B_i(q)$ is less than the N, the following lemma can help to determine whether the column vector $\bar{T}_i(q)$ has degree N or not.

Lemma 2: According to the derivation of $Q_{k,i}(q)$ in Lemma 1, the column vector $\bar{T}_i(q)$ has degree N if the order of $B_i(q)$ is larger than $N - L_2$.

Proof: Denote by l_B the order of $B_i(q)$. When a_N is non-zero, the length of the polynomial $\mathbf{1}_{L_2}(q)B_i(q)\prod_{i=1}^{N}(1+\alpha_i q^{-1}+\cdots+\alpha_i^{L_2-1}q^{1-L_2})$ is $(N+1)L_2 + l_B - N$. If $l_B > N - L_2$ or $\frac{N-l_B}{L_2} < 1$, it can be inferred that at least one element of $\{Q_{k,i}(q)\}_{k=0}^{M-1}$ has length $N+1$ or order N. Since the degree of a polynomial vector is defined as the maximum order of its elements, the result of this lemma is then established. ■

For an SIMO IIR system, the involved non-zero polynomial vector is surely column reduced, and its irreducibility can be guaranteed under several conditions [19]. Compared with the SIMO system, the analysis of the MIMO IIR system is much more complicated. The polynomial matrix $\mathbf{T}(q)$ is determined not only by $\{B_i(q)\}_{i=0}^{M-1}$ and $A(q)$, but also the over-sampling rate L_2. Using the polyphase decomposition formula in z-domain [21], $G_{k,i}(z)$ can be expressed in terms of $A(z), B_i(z)$ and L_2 as follows:

$$G_{k,i}(z) = \frac{W_M^{ki}}{L_2}\sum_{m=0}^{L_2-1}\left[\frac{\left(\zeta W_{L_2}^m\right)^k \mathbf{1}_{L_2}\left(\zeta W_{L_2}^m\right) B_i\left(\zeta W_{L_2}^m\right)}{A\left(\zeta W_{L_2}^m\right)}\right], \quad (8)$$

where $\zeta = z^{1/L_2}$. Further, based on the result in Lemma 1, it can be established that

$$\begin{aligned}\bar{F}_{k,i}(z) &= G_{k,i}(z)P(z)\\ &= \frac{W_M^{ki}}{L_2}\sum_{m=0}^{L_2-1}\Big[\left(\zeta W_{L_2}^m\right)^k \mathbf{1}_{L_2}\left(\zeta W_{L_2}^m\right)\\ &\quad\times B_i\left(\zeta W_{L_2}^m\right)\Pi_{l=0,l\neq m}^{L_2-1}A\left(\zeta W_{L_2}^l\right)\Big].\end{aligned}$$

Let $\bar{\mu}_k(\zeta)$ and $\bar{\eta}_i(\zeta)$ denote the vectors as follows:

$$\begin{aligned}\bar{\mu}_k(\zeta) &= \left[\left(\zeta W_{L_2}^0\right)^k \quad \left(\zeta W_{L_2}^1\right)^k \quad \cdots \quad \left(\zeta W_{L_2}^{(L_2-1)}\right)^k\right]^T,\\ \bar{\eta}_i(\zeta) &= \begin{bmatrix}\mathbf{1}_{L_2}\left(\zeta W_{L_2}^0\right)B_i\left(\zeta W_{L_2}^0\right)\Pi_{l=0,l\neq 0}^{L_2-1}A\left(\zeta W_{L_2}^l\right)\\ \mathbf{1}_{L_2}\left(\zeta W_{L_2}^1\right)B_i\left(\zeta W_{L_2}^1\right)\Pi_{l=0,l\neq 1}^{L_2-1}A\left(\zeta W_{L_2}^l\right)\\ \vdots\\ \mathbf{1}_{L_2}\left(\zeta W_{L_2}^{L_2-1}\right)B_i\left(\zeta W_{L_2}^{L_2-1}\right)\Pi_{l=0,l\neq L_2-1}^{L_2-1}A\left(\zeta W_{L_2}^l\right)\end{bmatrix}.\end{aligned}$$

Then, the matrix-vector multiplication form of $\bar{F}_{k,i}(z)$ is as follows:

$$\bar{F}_{k,i}(z) = \frac{W_M^{ki}}{L_2}\bar{\mu}_k^T(\zeta)\bar{\eta}_i(\zeta).$$

The entry $T_{k,i}(z)$ of the polynomial matrix $\mathbf{T}(z)$ can be expressed by

$$\begin{aligned}T_{k,i}(z) &= \bar{F}_{k,i}\left(W_M^{-iL_2}z\right)\\ &= \frac{W_M^{ki}}{L_2}\bar{\mu}_k^T\left(W_M^{-i}\zeta\right)\bar{\eta}_i\left(W_M^{-i}\zeta\right).\end{aligned}$$

Then, the polynomial matrix $\mathbf{T}(z)$ can be compactly formulated by

$$\begin{aligned}\mathbf{T}(z) = &\begin{bmatrix}\frac{W_M^{00}}{L_2} & \cdots & \frac{W_M^{0(M-1)}}{L_2}\\ \vdots & \ddots & \vdots\\ \frac{W_M^{(L_2-1)0}}{L_2} & \cdots & \frac{W_M^{(L_2-1)(M-1)}}{L_2}\end{bmatrix}\\ &\odot\left(\begin{bmatrix}\bar{\mu}_0^T\left(W_M^{-0}\zeta\right)\\ \vdots\\ \bar{\mu}_{L_2-1}^T\left(W_M^{-(M-1)}\zeta\right)\end{bmatrix}\right.\\ &\left.\times\left[\bar{\eta}_0\left(W_M^{-0}\zeta\right) \quad \cdots \quad \bar{\eta}_{M-1}\left(W_M^{-(M-1)}\zeta\right)\right]\right) \quad (9)\end{aligned}$$

where $\odot$ denotes the Hadamard product.

Remark 3: The expression of the polynomial matrix $\mathbf{T}(z)$ in terms of $\{A(z), B_0(z), \ldots, B_{M-1}(z), L_2\}$ is shown in (9). If the right-hand side of (9) is of full column rank for all values of ζ (or z), then $\mathbf{T}(z)$ is irreducible. If the leading coefficient matrix of the right-hand side of (9) is of full column rank, then $\mathbf{T}(z)$ is column reduced. The polynomial matrix $\mathbf{T}(z)$ is irreducible and column reduced if and only if [26] the block Toeplitz matrix $\mathcal{T}_K$ has full column rank when $K > MN$ and $L_2 > M$, where $\mathcal{T}_K$ is defined as

$$\mathcal{T}_K = \begin{bmatrix}T_0 & \cdots & T_N & & \\ & \ddots & & \ddots & \\ & & T_0 & \cdots & T_N\end{bmatrix}_{L_2K\times M(K+N)}, \quad (10)$$

and $\{T_i\}_{i=0}^N$ are the coefficient matrices of $\mathbf{T}(z)$.

Remark 4: Although it is difficult to derive closed-form sufficient conditions for the irreducible and column reduced matrix $\mathbf{T}(z)$ in terms of $\{A(z), \{B_i(z)\}_{i=0}^{M-1}, L_2\}$, several necessary conditions can be derived from the expression of $\mathbf{T}(z)$ in (9) as follows:

a) $B_i(z)$ has no factor $1 - z^{-1}$ or other factors in the form $1 - c_0 z^{-L_2}$ for any $c_0 \in \mathbb{C}$;
b) $A(z)$ has no zero pairs $\{c_0, c_0 e^{j\frac{2\pi k}{L_2}}\}$ for $c_0 \in \mathbb{C}$ and $k \in \{1, 2, \ldots, L_2 - 1\}$;
c) $A(z)$ has no factors in the form $1 - c_0 z^{-M}$ for any $c_0 \in \mathbb{C}$;
d) $A(z)$ has no zeros $e^{j\frac{2\pi k}{L_2}}$ for $k = 1, 2, \ldots, L_2 - 1$, which is guaranteed under Assumption A2;
e) $\frac{B_i(z)}{A(z)}$ is irreducible for $i = 0, 1, \ldots, M-1$.

To obtain an irreducible and column reduced matrix $\mathbf{T}(z)$, it is not necessary for all $\{B_i(z)\}_{i=0}^{M-1}$ or $\{F_i(z)\}_{i=0}^{M-1}$ to be coprime. Also, it does not require all the functions $\{H_i(z)\}_{i=0}^{M-1}$ in (1) to be irreducible. For instance, the generated polynomial matrix is irreducible and column reduced when $H_1(z) = \frac{(1+0.6z^{-1})(1+0.4z^{-1})}{(1+0.6z^{-1})(1-0.7z^{-1})}, H_2(z) = \frac{z^{-1}(1+0.4z^{-1})}{(1+0.6z^{-1})(1-0.7z^{-1})}$ and $L_2 = 2$. However, all $\{H_i(z)\}_{i=0}^{M-1}$ should not have identical zeros and poles. Otherwise, it can be inferred from (3) that $\{\frac{B_i(z)}{A(z)}\}_{i=0}^{M-1}$ are reducible, so is the polynomial matrix $\mathbf{T}(z)$.

By concatenating K vectors of $\bar{y}(n)$ in (7) vertically, we can get that

$$\bar{\mathbf{y}}(n) = \mathcal{T}_K\bar{\mathbf{s}}(n), \quad (11)$$

where

$$\begin{aligned}\bar{\mathbf{y}}(n) &= [\,\bar{y}^T(n) \quad \bar{y}^T(n-1) \quad \cdots \quad \bar{y}^T(n-K+1)\,]^T,\\ \bar{\mathbf{s}}(n) &= [\,\bar{s}^T(n) \quad \bar{s}^T(n-1) \quad \cdots \quad \bar{s}^T(n-K-N+1)\,]^T.\end{aligned} \tag{12}$$

Further, by concatenating all available vectors of $\bar{\mathbf{y}}(n)$ horizontally, it follows that

$$Y(n) = \mathcal{T}_K S(n), \tag{13}$$

where $Y(n) = [\bar{\mathbf{y}}(n)\cdots\bar{\mathbf{y}}(K+N)]$ and $S(n) = [\bar{\mathbf{s}}(n)\cdots\bar{\mathbf{s}}(K+N)]$.

To ensure the sufficient richness of the signal $s(n)$ to excite the system, conditions on the source signal $u(n)$ are to be investigated. By Assumption A1 that $u(n)$ is deterministic, so we introduce the linear complexity to describe its richness [15]. The linear complexity of the sequence $\{u(n) : n = 0,\ldots,N_u\}$ is defined by the smallest value of c for which there exist $\{\beta_i\}_{i=1}^{c}$ such that $u(n) = \sum_{i=1}^{c}\beta_i u(n-i)$ for $n = c,\ldots,N_u$. Denote by $\mathcal{C}(u(n))$ the linear complexity of $\{u(n)\}_{n=0}^{N_u}$. In the above case, it has that $\mathcal{C}(u(n)) = c$.

Lemma 3: [18] Let $\mathcal{C}(u(n))$ denote the linear complexity of $\{u(n)\}_{n=0}^{N_u}$. According to Fig. 3, $x(n)$ is an over-sampled signal of $u(n)$ with an over-sampling rate L_1. Then, the linear complexity of $s(n) = \frac{x(n)}{P(q)}$ satisfies that $\mathcal{C}(s(n)) \geq L_1\mathcal{C}(u(n)) - L_1 - N + 1$.

The following theorem gives a sufficient and necessary condition for the full row rank of the matrix $S(n)$ in terms of L_1 and L_2.

Theorem 1: Assume that $\mathcal{C}(u(n)) \geq \lceil\frac{K+2N-1}{L_1}\rceil + 1$, where $\lceil\cdot\rceil$ is the ceil function. When $K \geq L_1$ and the number of observation samples is much larger than $(M+1)(K+N)-1$, the matrix $S(n)$ has full row rank if and only if L_1L_2 and M are mutually coprime.

The proof of the above theorem can be found in Appendix A.

When the matrix $\mathcal{T}_K$ has full column rank and the matrix $S(n)$ has full row rank, $Y(n)Y^H(n)$ has the same column space with $\mathcal{T}_K$. By taking eigenvalue decomposition for the Hermitian matrix $Y(n)Y^H(n)$, we can get that

$$Y(n)Y^H(n) = [\,U_s \quad U_o\,]\begin{bmatrix}\Sigma & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}U_s^H\\ U_o^H\end{bmatrix}. \tag{14}$$

Since the block Toeplitz matrix $\mathcal{T}_K$ has the same column space with that of $Y(n)$, it can be established that

$$U_o^H\mathcal{T}_K = 0. \tag{15}$$

By rearranging the above equation, we can get that

$$\mathcal{U}_o\begin{bmatrix}T_0\\ \vdots\\ T_N\end{bmatrix} = \mathcal{U}_o\bar{T} = 0, \tag{16}$$

where $\bar{T} = [T_0^T\cdots T_N^T]^T$ and $\mathcal{U}_o$ is a block Toeplitz matrix constructed from the matrix U_o. It can be observed that $\bar{T}\Gamma$, for any nonsingular $M\times M$ matrix Γ, is also a solution to (16). For the blind MIMO system identification problem, using only the second-order statistics of the system output observations, it is impossible to determine the ambiguity matrix Γ [22]. However, the multiple inputs in (7) are generated by modulating the same sequence by different complex exponentials. Thus, mutual relations among multiple inputs will be exploited to determine the ambiguity matrix Γ.

C. Identification of the Ambiguity Matrix and the Denominator Polynomial

In order to identify the denominator polynomial $P(q)$ and the ambiguity matrix Γ, we first compute the multiple inputs $\bar{s}(n)$ in (7). Denote by $\hat{\mathbf{T}}(q)$ the estimate of the channel matrix $\mathbf{T}(q)$ from adequate output observations, it follows that

$$\mathbf{T}(q) = \hat{\mathbf{T}}(q)\Gamma.$$

Without noise effects, we can obtain that

$$\hat{\bar{s}}(n) = \hat{\mathbf{T}}^{\dagger}(q)\bar{y}(n) = \Gamma\bar{s}(n). \tag{17}$$

When the polynomial matrix $\hat{\mathbf{T}}(q)$ is a tall and irreducible matrix, its Moore-Penrose inverse, which is also a polynomial matrix, can always be calculated [26]. Further, it can be established that

$$\Gamma^{-1}\hat{\bar{s}}(n) = \Gamma^{-1}\begin{bmatrix}\hat{\bar{s}}_0(n)\\ \vdots\\ \hat{\bar{s}}_{M-1}(n)\end{bmatrix} = \begin{bmatrix}W_M^{0L_2n}\\ \vdots\\ W_M^{(M-1)L_2n}\end{bmatrix}s(n). \tag{18}$$

It is obvious that one necessary condition to determine the ambiguity matrix Γ is that all elements in $\{W_M^{iL_2}\}_{i=0}^{M-1}$ should be distinct. The following lemma provides a criterion to choose the over-sampling rate L_2 so as to meet the above requirement.

Lemma 4: $\{W_M^{iL_2}\}_{i=0}^{M-1}$ have different values if and only if M and L_2 are mutually prime.

Proof: The result is a special case of the Chinese Remainder Theorem: If M and L_2 are mutually prime numbers, for each $k, 0 \leq k < ML_2$, there exists a unique integer pair (α,β) satisfying $\alpha = k \,\mathrm{Mod}\, L_2$ and $\beta = k \,\mathrm{Mod}\, M$. That means the map from k to (α,β) is one-to-one.

Let $k = iL_2, 0 \leq i < M$. Then, according to the Chinese Remainder Theorem, we can get that $\{iL_2 \,\mathrm{Mod}\, M\}_{i=0}^{M-1}$ have different values in the range $\{0,\ldots,M-1\}$. Otherwise, if M and L_2 are not mutually coprime, we can always find that several elements in $\{W_M^{iL_2}\}_{i=0}^{M-1}$ are identical. ■

It is shown in ([11], Theorem 4) that the ambiguity matrix Γ can be determined up to a scalar factor if the complex exponential bases $\{W_M^{iL_2}\}_{i=0}^{M-1}$ are distinct. However, we find that all these exponential bases are linearly dependent such that the ambiguity matrix T cannot be determined up to a scalar factor. Thus, the existing deterministic blind identification methods [11], [14] cannot solve this identification problem.

According to the over-sampled output-switching system in Fig. 3, the input signal $x(n)$ of the time-varying SIMO system (6) is still a piece-wise constant signal with each symbol lasting for L_1 sampling periods. Next, based on the estimated signal $\hat{\bar{s}}(n)$ and the piece-wise constant property of $x(n)$, an identification method for the denominator polynomial $P(q)$ and the ambiguity matrix Γ will be developed.

Theorem 2: Assume that all conditions in Theorem 1 and Assumption A2 hold. The ambiguity matrix Γ can then be determined up to a scalar constant and the denominator polynomial $P(q)$ can be uniquely determined from (18) if the following two conditions are satisfied:

a) $L_1 L_2$ and M are mutually coprime;
b) $P(q)$ (or $A(q)$) has no factors in the form $1 - c_0 q^{-L_1}$ for any $c_0 \in \mathbb{C}$.

The proof of the above theorem can be found in Appendix B.

Denote by $\{\gamma_{i,k}\}_{i=0,k=0}^{M-1,M-1}$ the entries of Γ^{-1}. Since $s(n) = \frac{x(n)}{P(q)}$, (18) can be equivalently written as

$$x(n) = \gamma_{i,0} P(q) \left(W_M^{-iL_2 n} \hat{\bar{s}}_0(n) \right) + \cdots + \gamma_{i,M-1} P(q) \left(W_M^{-iL_2 n} \hat{\bar{s}}_{M-1}(n) \right) \quad i = 0, 1, \ldots, M-1. \quad (19)$$

Let $n = lL_1$ with $l \in \mathbb{N}$. By making use of the piecewise constant property of $x(n)$, (20), shown at the bottom of the page, can be derived, where $i \in \{0, \ldots, M-1\}, k \in \{0, \ldots, L_1 - 2\}$.

In Equation (20), the unknown variables $\{\gamma_{i,k}\}_{k=0}^{M-1}$ and $\{p_k\}_{k=1}^{N}$ are coupled together, so it is a bilinear identification problem. To deal with such a problem, we alternatively compute the nontrivial solutions of $\{\gamma_{i,k}\}_{k=0}^{M-1}$ and $\{p_k\}_{k=1}^{N}$ from (20). It is noteworthy that the initial point is crucial for the above alternating optimization method and should be carefully selected. Applying the overparameterization technique [27] and denoting

$$\bar{\alpha} = \begin{bmatrix} \gamma_{0,0} \begin{pmatrix} 1 \\ \vdots \\ p_N \end{pmatrix}^T & \cdots & \gamma_{M-1,M-1} \begin{pmatrix} 1 \\ \vdots \\ p_N \end{pmatrix}^T \end{bmatrix}^T$$

as an augmented coefficient vector, $\bar{\alpha}$ can be computed as a nontrivial solution of (20). Note that there exists a scalar ambiguity for the estimation of $\bar{\alpha}$. Denote by $\{\bar{\alpha}_i\}_{i=1}^{M^2(N+1)}$ the components of the vector $\bar{\alpha}$. Then, the initial value of the coefficient vector of $P(q)$ is computed as follows:

$$\begin{bmatrix} 1 \\ \hat{p}_1 \\ \vdots \\ \hat{p}_N \end{bmatrix} = \frac{1}{M^2} \sum_{i=0}^{M^2-1} \frac{1}{\bar{\alpha}_{i(N+1)+1}} \begin{bmatrix} \bar{\alpha}_{i(N+1)+1} \\ \bar{\alpha}_{i(N+1)+2} \\ \vdots \\ \bar{\alpha}_{i(N+1)+N+1} \end{bmatrix}. \quad (21)$$

D. Blind Identification of the Output-Switching System

Suppose that the estimates of the ambiguity matrix Γ and the denominator polynomial $P(q)$ have been achieved. Then, the channel matrix $\mathbf{T}(q)$ can be uniquely determined up to a scalar factor. Since $\bar{T}_i(q) = \bar{F}_i(W_M^{-iL_2} q)$, the estimate of $\bar{F}_i(q) = [W_M^{0i} Q_{0,i}(q) \ \cdots \ W_M^{(L_2-1)i} Q_{L_2-1,i}(q)]$ or $\{Q_{k,i}(q)\}_{k=0}^{L_2-1}$ can be obtained as well. From Lemma 1, we can get that

$$\frac{Q_{k,i}(q)}{P(q)} = \left[\frac{q^k \mathbf{1}_{L_2}(q) B_i(q)}{A(q)} \right]_{\downarrow L_2} \qquad k = 0, \ldots, L_2 - 1. \quad (22)$$

By applying the polyphase decomposition identity in z-domain ([21], Chapter 3), it can be established that

$$\frac{\mathbf{1}_{L_2}(z) B_i(z)}{A(z)} = \sum_{k=0}^{L_2-1} \frac{z^{-k} Q_{k,i}\left(z^{L_2}\right)}{P\left(z^{L_2}\right)} = \frac{\sum_{k=0}^{L_2-1} z^{-k} Q_{k,i}\left(z^{L_2}\right)}{P\left(z^{L_2}\right)}. \quad (23)$$

It is shown in Lemma 1 that the expression of $P(z)$ can be derived from $A(z)$. However, $A(z)$ cannot be obtained directly from $P(z)$.

As it is shown in Remark 4, to guarantee the blind identifiability, the polynomial vector $[B_0(z) \ \cdots \ B_{M-1}(z) \ A(z)]^T$ should be irreducible. Based on (23) and in view of Lemma 1, we can obtain that

$$\begin{bmatrix} \sum_{k=0}^{L_2-1} z^{-k} Q_{k,0}\left(z^{L_2}\right) \\ \vdots \\ \sum_{k=0}^{L_2-1} z^{-k} Q_{k,M-1}\left(z^{L_2}\right) \\ \mathbf{1}_{L_2}(z) P\left(z^{L_2}\right) \end{bmatrix} = \begin{bmatrix} B_0(z) \\ \vdots \\ B_{M-1}(z) \\ A(z) \end{bmatrix} C(z) \quad (24)$$

where $C(z)$ is a common factor of polynomial vector on the left side of the above equation. Equation (24) can be considered as an SIMO system model where $C(z)$ is a common source signal; thus, it can be solved by the subspace-based blind identification method developed in [16].

Theorem 3: Assume that all conditions in Theorems 1–2 and Assumptions A1–A2 hold. Assume also that the Toeplitz matrix $\mathcal{T}_K$ is of full column rank when $K > MN$ and $L_2 > M$. In the absence of noise, all the transfer functions $\{\frac{B_i(q)}{A(q)}\}_{i=0}^{M-1}$ or $\{\frac{F_i(q)}{A(q)}\}_{i=0}^{M-1}$ can be identified up to a scalar constant.

The above theorem can be easily derived from the results in Theorems 1–2.

In summary, the proposed blind identification method includes the following steps:

Step 1. Construct the matrix $Y(n)$ in (13) and compute $\bar{T}$ using the subspace-based method as shown in (14)–(16).
Step 2. Compute the multiple source signals $\bar{s}(n)$ from (17).
Step 3. Determine the ambiguity matrix Γ and coefficients of $P(q)$ from (20) using the method described in Subsection III.C.

$$[\gamma_{i,0} P(q) \ \cdots \ \gamma_{i,M-1} P(q)] \begin{bmatrix} W_M^{-iL_2(n-k)} \hat{\bar{s}}_0(n-k) - W_M^{-iL_2(n-k-1)} \hat{\bar{s}}_0(n-k-1) \\ \vdots \\ W_M^{-iL_2(n-k)} \hat{\bar{s}}_{M-1}(n-k) - W_M^{-iL_2(n-k-1)} \hat{\bar{s}}_{M-1}(n-k-1) \end{bmatrix} = 0 \quad (20)$$

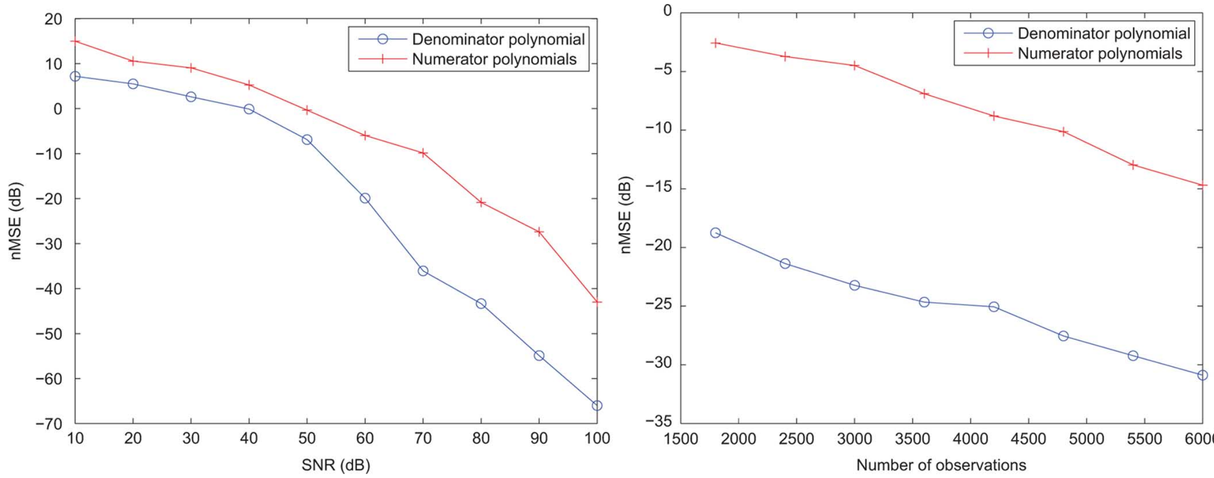

Fig. 4. Identification performance of the proposed algorithm.

Step 4. Compute the estimate of $\{B_i(q)\}_{i=0}^{M-1}$ and $A(q)$ from (24) using the subspace-based method [16].

Step 5. Estimate the coefficients of $\{H_i(q)\}_{i=0}^{M-1}$ based on (2)–(3).

IV. Numerical Simulations

In this section, numerical simulations are carried out to validate the proposed blind identification algorithm for output-switching systems. The input signal $u(n)$ is generated as a truncated Gaussian white noise so that the linear complexity condition can be guaranteed with probability one. The noise $w(n)$ is also generated as a truncated Gaussian white noise. The identification performances are evaluated with respect to different noise levels and different dimensions of observation samples. The noise level is described by the signal-noise ratio (SNR), which is defined by

$$\mathrm{SNR} = \frac{1}{T}\sum_{i=1}^{T}\frac{\sum_n \left(H_{n\mathrm{Mod}M}(q)u^i(n)\right)^2}{\sum_n \left(w^i(n)\right)^2}, \quad (25)$$

where T is the number of Monte-Carlo trials which is set to 200 in the following simulations, $u^i(n)$ and $w^i(n)$ are generated input signal and noise sequences, respectively, in the i-th trial. The identification performance is evaluated by the normalized mean-square error (nMSE). For the identification of the numerator polynomials of $H_i(q) = \frac{F_i(q)}{A(q)}$ for $i = 0, \ldots, M-1$, the corresponding nMSE is defined by

$$\mathrm{nMSE} = \frac{1}{T}\sum_{i=1}^{T}\frac{\min_{\eta\in\mathbb{C}}\sum_{k=0}^{M-1}\|\eta\hat{\mathbf{f}}_k^i - \mathbf{f}_k\|^2}{\sum_{k=0}^{M-1}\|\mathbf{f}_k\|^2}, \quad (26)$$

where $\hat{\mathbf{f}}_k^i$ denotes the estimate of the coefficient vector of $F_k(q)$ in the i-th trial and the minimization with respect to η is to eliminate the scalar ambiguity. Similarly, the corresponding nMSE for the denominator polynomial $A(q)$ is defined by

$$\mathrm{nMSE} = \frac{1}{T}\sum_{i=1}^{T}\frac{\|\hat{\mathbf{a}}^i - \mathbf{a}\|^2}{\|\mathbf{a}\|^2}, \quad (27)$$

where $\mathbf{a}$ is the true coefficient vector of $A(q)$ and $\hat{\mathbf{a}}^i$ denotes the i-th estimate of $\mathbf{a}$. It is noted that there exist no scalar ambiguity for the identification of $A(q)$ since it is a monic polynomial. Identification performances of the numerator polynomials and the denominator polynomial are evaluated separately.

Next, three examples will be given to demonstrate the performances of the proposed algorithm: (1) the transfer functions $\{H_k(q)\}_{k=0}^{M-1}$ have the same denominator and have a common factor in their numerators; (2) the transfer functions $\{H_k(q)\}_{k=0}^{M-1}$ have different denominators; (3) there exists discrepancy between the real transfer functions and the desired transfer functions.

Case 1: The involved transfer functions of an output-switching system with $M = 3$ are defined as follows

$$H_0(q) = \frac{(1+0.4q^{-1})(1+1.6q^{-1})}{(1+0.8q^{-1})(1-0.7q^{-1})},$$
$$H_1(q) = \frac{(1+0.4q^{-1})(1-0.7q^{-1})}{(1+0.8q^{-1})(1-0.7q^{-1})},$$
$$H_2(q) = \frac{(1+0.4q^{-1})(1-1.2q^{-1})}{(1+0.8q^{-1})(1-0.7q^{-1})}.$$

It can be observed that all transfer functions share a common factor $(1+0.4q^{-1})$, and $H_1(q)$ and $H_2(q)$ are reducible. By setting $L_1 = 2, L_2 = 4, K = 7$, it can be verified that sufficient conditions in Theorem 3 are satisfied.

Fig. 4 shows the identification results obtained by the proposed identification method. The left part of Fig. 4 is obtained when the number of observation samples is set to 3600 and the right part is obtained when $\mathrm{SNR} = 70$. It can be observed from the left part that accurate identification results can be obtained when the SNR is high enough. The results on the right part imply that the estimates of multiple transfer functions approach their true values when the number of observation samples tends to infinity.

From Fig. 4, we can find that the identification performances at low SNRs, especially when $\mathrm{SNR} \leq 40$, are not satisfactory. The reason is that the proposed identification method is a two-stage method so that the estimation error in the first stage may propagate or be amplified in the second stage. In addition, the coefficient matrix of the numerator polynomial matrix $\mathbf{T}(z)$ is estimated by $\bar{T}\Gamma$, where $\bar{T}$ is obtained by the subspace-based method and Γ are estimated by solving (19). The estimation errors in different parts are multiplied, so the identification per-

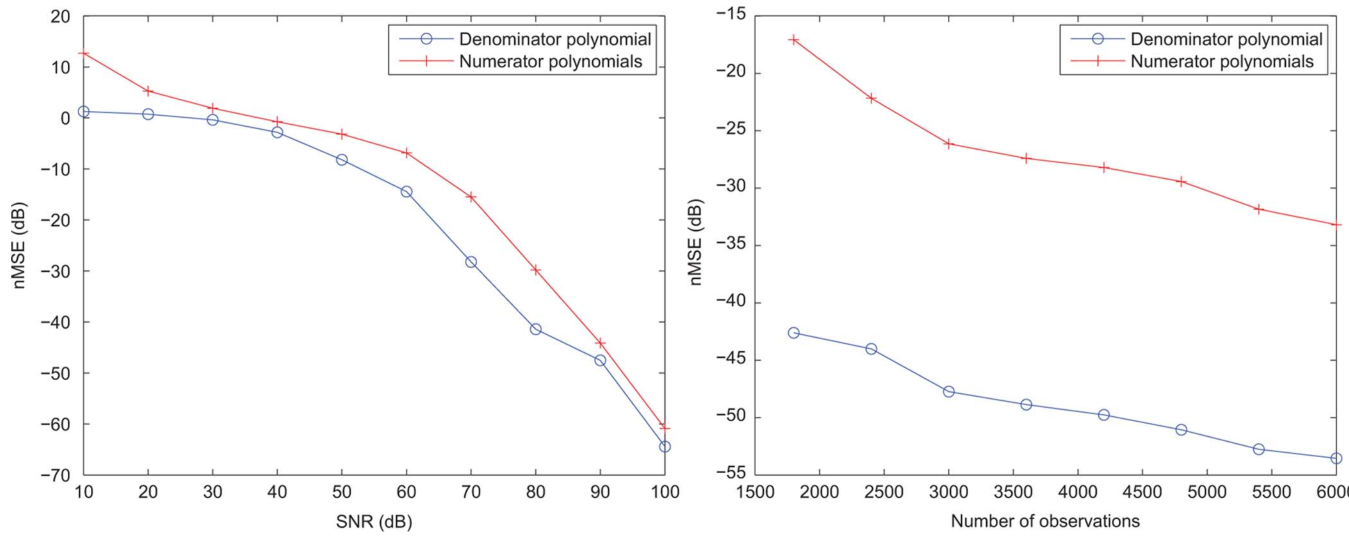

Fig. 5. Identification performance of the proposed algorithm.

formances of the numerator polynomials are worse than that of the denominator polynomial.

Case 2: The involved transfer functions of an output-switching system with $M = 2$ are defined as follows

$$H_0(q) = \frac{1 + 0.5q^{-1}}{1 - 0.7q^{-1}}, \quad H_1(q) = \frac{1 + 0.4q^{-1}}{1 + 0.8q^{-1}}.$$

It can be observed that these two transfer functions have different denominators; however, they can be converted into the transfer functions with the same denominator. By setting $L_1 = 3, L_2 = 3, K = 5$, it can be verified that sufficient conditions in Theorem 3 are satisfied. From Fig. 5, it can be observed that the identification performances in this case are quite similar to that in **Case 1**: both the numerator polynomials and the denominator polynomial can be accurately estimated when the SNR is high enough and the available observation samples are adequate.

Case 3: The transfer functions and other simulation settings are the same as in **Case 2**. Here, $\{H_i(q)\}_{i=1}^2$ are called desired transfer functions, which have infinitely long coefficient sequences. And the FIR filters $\{D_i(q)\}_{i=1}^2$ are called real transfer functions, which are constructed by the truncated coefficient sequences of $\{H_i(q)\}_{i=1}^2$. In the simulation, the system outputs are generated according to the real transfer functions, while the desired transfer functions are to be estimated based on the available system outputs. The aim of this case is to show the identification performance with respect to the discrepancy between the real transfer function and the desired transfer functions.

Since the coefficients of $\{H_i(q)\}_{i=1}^2$ decrease exponentially, we obtain the truncated coefficient sequences by keeping the first few coefficients. The discrepancy is described by the number of preserved coefficients in the simulation. It is obvious that the discrepancy is smaller when the number of truncated coefficients becomes larger. Fig. 6 shows the identification performance with respect to the discrepancy between the transfer functions. It can be found that the nMSE curves decay along with the decrease of the discrepancy. As a matter of fact, the discrepancy between transfer functions can be cast as

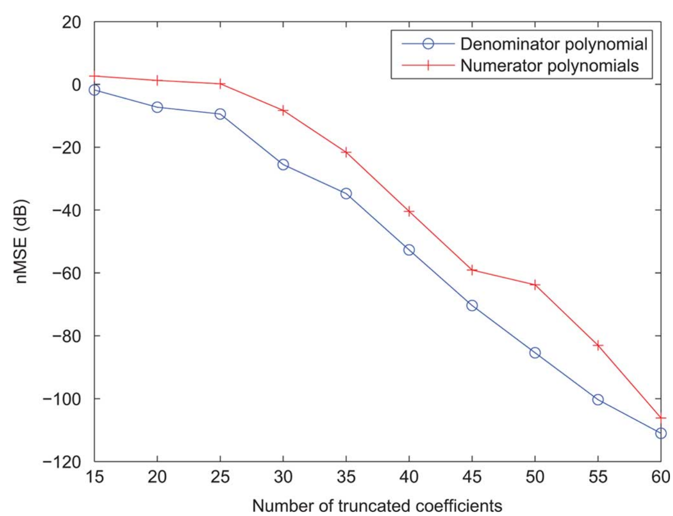

Fig. 6. Identification performance of the proposed algorithm with respect to the discrepancy between transfer functions.

measurement noises; thus, the identification performance with respect to the discrepancy is similar to that with respect to the signal noise ratio.

V. Conclusion

In this paper, a blind identification algorithm has been proposed for output-switching systems using a new over-sampling strategy. Under the provided identification framework, sufficient identifiability conditions have been given and the selection of over-sampling rates has been investigated, where it is shown that the over-sampling rates have to be mutually coprime to the number of channels involved in the output-switching system. Due to the linearly dependent complex exponential basis, the equivalent output-switching system with modulation at the output cannot be properly identified by traditional subspace-based methods. However, by taking into account of the redundancy information which is introduced by the over-sampling operation, it has been resolved by the proposed method. Numerical simulations have been carried out to show the performances of the proposed algorithm.

It is well known that the coprime condition is necessary for the blind identification of an SIMO system. However, as it is shown in this paper, by holding the system input while adding a switching operator (or modulations) at the output, the coprime condition on multiple transfer functions may not be necessary. Thus, the proposed algorithm has many promising applications. In our future works, we will investigate how to estimate the orders of multiple IIR transfer functions, and develop robust identification methods with respect to the channel order overestimation.

Appendix A
Proof of Theorem 1

When $n \geq (M+1)(K+N)-1$, the matrix $S(n)$ is a fat matrix. The z-transform of the input vector $\bar{s}(n)$ in (7) is written by

$$\bar{S}(z) = \left[S(z) \quad S\left(W_M^{-L_2} z\right) \quad \cdots \quad S\left(W_M^{-(M-1)L_2} z\right) \right]^T$$

where $S(z) = \frac{X(z)}{P(z)} = \frac{\mathbf{1}_{L_1}(z) U(z^{L_1})}{P(z)}$. It can be established that

$$\begin{bmatrix} P(z) & & \\ & \ddots & \\ & & P\left(W_M^{-(M-1)L_2} z\right) \end{bmatrix} \bar{S}(z) = \begin{bmatrix} \mathbf{1}_{L_1}(z) U\left(z^{L_1}\right) \\ \vdots \\ \mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) U\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right) \end{bmatrix}. \quad (28)$$

If $L_2 L_1$ and M have a common divisor, according to the result in Lemma 4, several elements in the set $\{U(W_M^{-iL_2L_1} z^{L_1})\}_{i=0}^{M-1}$ are identical. It implies that there exists a non-zero polynomial vector $[\, C_0(z) \quad \cdots \quad C_{M-1}(z)]$ of degree $L_1 - 1$ [26] such that

$$\begin{aligned} &[\, C_0(z) \quad \cdots \quad C_{M-1}(z)] \\ &\times \begin{bmatrix} \mathbf{1}_{L_1}(z) U\left(z^{L_1}\right) \\ \vdots \\ \mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) U\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right) \end{bmatrix} \\ &= \left[C_0(z) P(z) \quad \cdots \quad C_{M-1}(z) P\left(W_M^{-(M-1)L_2} z\right) \right] \\ &\times \bar{S}(z) = 0. \end{aligned} \quad (29)$$

The z-transform of the vector $\bar{\mathbf{s}}(n)$ in (11) is written by $\bar{\mathbf{S}}(z) = [\bar{S}^T(z) \quad \cdots \quad z^{1-K-N} \bar{S}^T(z)]^T$. When $M(K+N) \geq M(L_1+N)$ or $K \geq L_1$, it can be found that the elements in the vector $\bar{\mathbf{S}}(z)$ are linearly dependent. Thus, the matrix $S(n)$ is row-rank deficient and the necessity has been proven.

When $L_2 L_1$ and M are mutually coprime, all elements in $\{U(W_M^{-iL_2L_1} z^{L_1})\}_{i=0}^{M-1}$ are distinct. By Theorem 1 in [18], the signal $u(n)$ has sufficient richness when $\mathcal{C}(u(n)) \geq \lceil \frac{K+2N-1}{L_1} \rceil + 1$. Then, it can be verified that there exists no nontrivial polynomial vector of order less than K to make the equality (29) hold. Thus, the matrix $S(n)$ has full row rank and the sufficiency has been proven.

Appendix B
Proof of Theorem 2

By taking into account of $S(z) = \frac{X(z)}{P(z)} = \frac{\mathbf{1}_{L_1}(z) U(z^{L_1})}{P(z)}$, (18) can be equivalently written in z-domain as follows:

$$\Gamma^{-1} \hat{\bar{S}}(z) = \begin{bmatrix} S(z) \\ \vdots \\ S\left(W_M^{-(M-1)L_2} z\right) \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{1}_{L_1}(z) U\left(z^{L_1}\right)}{P(z)} \\ \vdots \\ \frac{\mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) U\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right)}{P\left(W_M^{-(M-1)L_2} z\right)} \end{bmatrix}. \quad (30)$$

Suppose that $\{\tilde{P}(z), \tilde{\Gamma}, \tilde{U}(z^{L_1})\}$ is another solution satisfying the above equation and the conditions (a–b) in the theorem. Denote $\hat{\Gamma} = \Gamma^{-1}\tilde{\Gamma}$. We can then get that

$$\hat{\Gamma} \begin{bmatrix} \frac{\mathbf{1}_{L_1}(z) \tilde{U}\left(z^{L_1}\right)}{\tilde{P}(z)} \\ \vdots \\ \frac{\mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) \tilde{U}\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right)}{\tilde{P}\left(W_M^{-(M-1)L_2} z\right)} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{1}_{L_1}(z) U\left(z^{L_1}\right)}{P(z)} \\ \vdots \\ \frac{\mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) U\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right)}{P\left(W_M^{-(M-1)L_2} z\right)} \end{bmatrix}. \quad (31)$$

Denote by $\{\hat{\gamma}_{i,j}\}_{i=0,j=0}^{M-1,M-1}$ the entries of the matrix $\hat{\Gamma}$. It follows that

$$\begin{aligned} &\hat{\gamma}_{i,0} \frac{\mathbf{1}_{L_1}(z) \tilde{U}\left(z^{L_1}\right)}{\tilde{P}(z)} + \cdots \\ &+ \hat{\gamma}_{i,M-1} \frac{\mathbf{1}_{L_1}\left(W_M^{-(M-1)L_2} z\right) \tilde{U}\left(W_M^{-(M-1)L_2 L_1} z^{L_1}\right)}{\tilde{P}\left(W_M^{-(M-1)L_2} z\right)} \\ &= \frac{\mathbf{1}_{L_1}\left(W_M^{-iL_2} z\right) U\left(W_M^{-iL_2 L_1} z^{L_1}\right)}{P\left(W_M^{-iL_2} z\right)}. \end{aligned} \quad (32)$$

If the condition (b) is satisfied, the polynomial $\tilde{P}(z)$ (or $P(z)$) cannot be absorbed by $\tilde{U}(z^{L_1})$ (or $U(z^{L_1})$). Since L_2 and M are mutually coprime, all polynomials $\{\mathbf{1}_{L_1}(W_M^{-iL_2} z)\}_{i=0}^{M-1}$ are distinct. If z_0 is a zero of $U(z)$, then $\{z_0 W_M^{iL_2} W_{L_1}^k\}_{k=0}^{L_1-1}$ are all zeros of $U(W_M^{-iL_2L_1} z^{L_1})$ or the left side of (32).

Since M and $L_1 L_2$ are mutually coprime, all elements in $\{\tilde{U}(W_M^{-kL_2L_1} z^{L_1})\}_{k=0}^{M-1}$ are different. In addition, $U(z)$ has a sufficiently large order, it can be inferred that $U(W_M^{-iL_2L_1} z^{L_1})$ and one element in $\{\tilde{U}(W_M^{-kL_2L_1} z^{L_1})\}_{k=0}^{M-1}$ have the same zeros, namely only one element in $\{\hat{\gamma}_{i,k}\}_{k=0}^{M-1}$ is non-zero valued. In addition, $\mathbf{1}_{L_1}(W_M^{-iL_2} z)$ is a common factor of both sides of (32). Thus, we can obtain the following results:

a) $\hat{\gamma}_{i,j} = 0$ when $i \neq j$, i.e., the matrix $\hat{\Gamma}$ is diagonal;

b) $U(W_M^{-iL_2L_1}z^{L_1})$ and $\tilde{U}(W_M^{-iL_2L_1}z^{L_1})$ are identical with a scalar ambiguity $\hat{\gamma}_{i,i}$;

c) $\tilde{P}(z)$ and $P(z)$ are the same.

Further, it follows from (31) that all diagonal entries of the matrix $\hat{\Gamma}$ have the same value. Thus, the theorem is proven.

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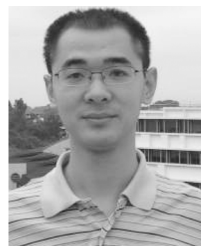

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