

Supplemental File to “Communication-Efficient Decentralized Event Monitoring in Wireless Sensor Networks”

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1 DERIVATION OF THE PARTIAL CONSENSUS ALGORITHM

To solve (9) with the ADMM, we rewrite it as:

$$\begin{aligned} \min & \sum_{i=1}^L \tilde{f}_i(\mathbf{c}_{\mathcal{J}_i}^{(i)}), \\ \text{s.t.} & c_k^{(i)} = z_{ijk}, \forall v_i \in \mathcal{N}_k^E, \forall j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C, \forall k, \\ & c_k^{(j)} = z_{ijk}, \forall v_i \in \mathcal{N}_k^E, \forall j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C, \forall k. \end{aligned} \quad (1)$$

Here z_{ijk} is a slack variable attached to $c_k^{(i)}$ and $c_k^{(j)}$. The augmented Lagrangian function of (1) is:

$$\begin{aligned} L_a = & \sum_{i=1}^L \tilde{f}_i(\mathbf{c}_{\mathcal{J}_i}^{(i)}) \\ & + \sum_{k=1}^L \sum_{v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} \beta_{ijk}(c_k^{(i)} - z_{ijk}) \\ & + \frac{p}{2} \sum_{k=1}^L \sum_{v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} (c_k^{(i)} - z_{ijk})^2 \\ & + \sum_{k=1}^L \sum_{v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} \gamma_{ijk}(c_k^{(j)} - z_{ijk}) \\ & + \frac{p}{2} \sum_{k=1}^L \sum_{v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} (c_k^{(j)} - z_{ijk})^2. \end{aligned} \quad (2)$$

in which $\{\beta_{ijk}\}$ and $\{\gamma_{ijk}\}$ are Lagrange multipliers; p is a positive constant. At time t , the ADMM optimizes the augmented Lagrangian function as follows [1].

First, the optimization variables $\{\mathbf{c}_{\mathcal{J}_i}^{(i)}\}$ are optimized given $\{z_{ijk}(t)\}$, $\{\beta_{ijk}(t)\}$, and $\{\gamma_{ijk}(t)\}$. Note that the augmented Lagrangian function is separable to $\{\mathbf{c}_{\mathcal{J}_i}^{(i)}\}$. Hence the solution of $\tilde{\mathbf{c}}^{(i)}(t+1)$ is:

$$\begin{aligned} \mathbf{c}_{\mathcal{J}_i}^{(i)}(t+1) = & \arg \min \tilde{f}_i(\mathbf{c}_{\mathcal{J}_i}^{(i)}) \\ & + \sum_{k:v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} \beta_{ijk}(t)c_k^{(i)} \\ & + \frac{p}{2} \sum_{k:v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} (c_k^{(i)} - z_{ijk}(t))^2 \\ & + \sum_{k:v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} \gamma_{jik}(t)c_k^{(i)} \\ & + \frac{p}{2} \sum_{k:v_i \in \mathcal{N}_k^E} \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} (c_k^{(i)} - z_{jik}(t))^2, \end{aligned} \quad (3)$$

Second, the slack variables $\{z_{ijk}\}$ are optimized given $\{\mathbf{c}_{\mathcal{J}_i}^{(i)}(t+1)\}$, $\{\beta_{ijk}(t)\}$, and $\{\gamma_{ijk}(t)\}$. The augmented Lagrangian function is also separable and there is a neat closed-form solution for $z_{ijk}(t+1)$:

$$z_{ijk}(t+1) = \frac{1}{2} \left(c_k^{(i)}(t+1) + c_k^{(j)}(t+1) \right) + \frac{1}{2p} (\beta_{ijk}(t) + \gamma_{ijk}(t)). \quad (4)$$

Third, the Lagrange multipliers $\{\beta_{ijk}\}$ and $\{\gamma_{ijk}\}$ are updated with the method of multipliers:

$$\begin{aligned} \beta_{ijk}(t+1) &= \beta_{ijk}(t) + p \left(c_k^{(i)}(t+1) - z_{ijk}(t+1) \right), \\ \gamma_{ijk}(t+1) &= \gamma_{ijk}(t) + p \left(c_k^{(j)}(t+1) - z_{ijk}(t+1) \right). \end{aligned} \quad (5)$$

The updating rules (3), (4), and (5) can be further simplified. Substituting (4) to (5) yields:

$$\begin{aligned} \beta_{ijk}(t+1) &= \beta_{ijk}(t) - \frac{1}{2} (\beta_{ijk}(t) + \gamma_{ijk}(t)) \\ &+ \frac{p}{2} \left(c_k^{(i)}(t+1) - c_k^{(j)}(t+1) \right), \end{aligned} \quad (6)$$

$$\begin{aligned} \gamma_{ijk}(t+1) &= \gamma_{ijk}(t) - \frac{1}{2} (\beta_{ijk}(t) + \gamma_{ijk}(t)) \\ &+ \frac{p}{2} \left(c_k^{(j)}(t+1) - c_k^{(i)}(t+1) \right). \end{aligned} \quad (7)$$

Since we often set $\beta_{ijk}(0) = \gamma_{ijk}(0) = 0$, (6) implies that $\beta_{ijk}(t) = -\beta_{jik}(t) = -\gamma_{ijk}(t) = \gamma_{jik}(t)$. Then (4) is:

$$z_{ijk}(t+1) = \frac{1}{2} \left(c_k^{(i)}(t+1) + c_k^{(j)}(t+1) \right), \quad (8)$$

and (6) becomes:

$$\begin{aligned} \beta_{ijk}(t+1) &= \beta_{ijk}(t) + \frac{p}{2} \left(c_k^{(i)}(t+1) - c_k^{(j)}(t+1) \right), \\ \gamma_{ijk}(t+1) &= \gamma_{ijk}(t) + \frac{p}{2} \left(c_k^{(j)}(t+1) - c_k^{(i)}(t+1) \right). \end{aligned} \quad (9)$$

Subtracting the second equation from the first one in (9) and defining a new Lagrange multiplier $\alpha_{ik} = \sum_{v_j \in \mathcal{N}_k^E \cap \mathcal{N}_i^C} (\beta_{ijk} + \gamma_{jik})$, the updating rule for α_{ik} is (11).

Substituting (8), (11), and the definition of α_{ik} to (3), the updating rule for $\mathbf{c}_{\mathcal{J}_i}^{(i)}$ is (10).

REFERENCES

- [1] D. Bertsekas and J. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*, Second Edition, Athena Scientific, 1997