Localized Structural Health Monitoring Using Energy-Efficient Wireless Sensor Networks

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***Abstract*—This paper presents a localized information processing approach for long-term, online structural health monitoring (SHM) using wireless sensor networks (WSNs). Based on the embedded AR-ARX method, each sensor independently calculates a statistical damage-sensitive coefficient using the measured acceleration data during each monitoring period. A nonlinear programming formulation is developed to identify damage presence, localize damage position, and quantify damage severity from the damage-sensitive coefficients in the whole sensing field. By limiting each sensor to exchange information among its neighboring sensors only, a localized near-optimal algorithm is proposed to reduce communication costs, thus alleviating the channel interference and prolonging the network lifetime. Simulation results on a steel frame structure prove the effectiveness of the proposed algorithm.**

***Index Terms*—Localized information processing, structure health monitoring (SHM), wireless sensor networks (WSNs).**

I. Introduction

Structural health monitoring (SHM) refers to the process of damage detection for civil, aerospace and mechanical engineering systems [1]. Here, the damage is defined as changes to the material or geometric properties of these systems due to either internal factors such as aging, or exterior forces such as natural disasters. During its normal operation, an SHM system periodically updates the health condition of a structure for low-power, long-term monitoring, whereas during extreme events such as earthquakes, SHM is used for real-time rapid structural condition screening [2]. Through acquisition and interpretation of critical structural response data, SHM aims for structural condition assessment at four levels: (1) identification of anomalies and damages in a structure; (2) localization of damage; (3) quantification of damage severity; (4) prediction of the remaining service life of the structure [3].

Traditionally, an SHM system collects the measured output from sensors installed in the structure and processes the data in a fusion center [4]. To reliably transmit the measurements, SHM systems often employ coaxial cables for communications between sensors and a fusion center. However, the installation of coaxial wires is both expensive and labor-intensive. For example, the cost of installing 350 sensing channels on the Tsing Ma suspension bridge in Hong Kong is estimated to have exceeded \$8 million [5].

Recent development in micro-fabrication and wireless communication technologies enables the replacement of coaxial wires by wireless sensors that are both affordable and easy to deploy [6]–[8]. The sensing and communication capabilities of wireless sensor networks (WSNs) lead to fundamentally new sensing architectures capable of monitoring large spatially distributed phenomena. Accompanied with the unprecedented data collection opportunities for SHM offered by large-scale WSNs, new challenges have also emerged due to two main constraints of network resources: communication bandwidth and battery power. In a large network, extensive communication between wireless sensors and the fusion center results in strong interference and hence packet loss. More importantly, communication is the main source of energy consumption for sensor nodes, thus decides the network lifetime.

To meet the rigid bandwidth requirements and extend the network lifetime, this paper develops a localized solution to long-term, online SHM. In a decentralized fashion, each sensor only exchanges information among its neighboring nodes to alleviate communication costs. Meanwhile, localized information processing is carried out at each sensor to make SHM decisions autonomously at reduced computational costs. Upon detecting damage, the refined damage information can be transmitted to a central console to help repair the structure. Since damage is generally scarce in a structure, information transmission from sensors to the central console is limited. On the contrary, frequent and extensive multihop data exchange between sensors and a fusion center is required in previous works [9], [10]. In addition to alleviating the channel interference and extending the network life time via limiting multihop data exchange, the localized SHM algorithm improves the network robustness with localized decision-making.

While the proposed localized communication and information processing framework can be coupled with a variety of damage detection methodologies, we take the AR-ARX method [2] as an example to demonstrate a distributed WSN for SHM at the following levels: identification of damage, localization of damage, and quantification of damage severity. In the AR-ARX method [2], the damage pattern of a structure is recognized through computing a statistical damage-sensitive coefficient from the measured acceleration data under ambient vibration during each monitoring period. Both theoretical and empirical analysis indicate that the damage-sensitive coefficient can be modeled by three contributing components: damage effect at the location of the sensor, damage effects of the neighboring

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area and an additive random noise. Based on this modeling, damages can be identified by comparing statistics between baseline and online measurements, which is conventionally done in a centralized manner [2]. In contrast, this paper formulates a localized damage pattern recognition problem. Main contributions of this work are the following.

1) We develop a localized data processing algorithm for SHM that considerably saves communication resources without degrading the damage detection accuracy. Information propagation across the decentralized large network is accomplished through a judiciously designed random gossip-like protocol, which ensures network-wide monitoring at scalable complexity and energy costs. This advantage is particularly critical to extending the lifetime of large-scale networks.
2) Taking an optimization approach, we propose a new 0-norm minimization formulation for estimation of the damage field. The damage severity coefficients of the entire sensing field are collected into a vector $\mathbf{c}$, where each element c_j is a non-negative scalar representing the damage severity at the position of sensor j. The spatial correlation of damage severity is modeled by a set of basis functions $f_i(j)$, which represents the influence of the damage occurred at sensor i on the output of sensor j. Since damage sources arise in a sparse manner across the large network field, we can treat the damage severity vector $\mathbf{c}$ as a sparse vector with only a small number of nonzero elements. Recognition of this important sparsity property allows us to develop novel SHM solutions that capitalize on recent advances in compressed sampling and sparse signal recovery [11].
3) We derive efficient algorithms to recover the damage severity vector $\mathbf{c}$ in a localized manner. The nonlinear 0-norm minimization problem is relaxed to a 1-norm linear programming problem, which can be efficiently solved to save computational costs. More importantly, the 1-norm linear programming formulation directly leads to an approximated localized minimization problem, which reduces the communication costs of an otherwise centralized implementation. Each sensor has its own objective function and constraints, and cooperates with others by low-cost local broadcasting. An iterative localized optimization framework is derived to attain fast convergence to a globally near-optimal solution.

The rest of this paper is organized as follows. In Section II, a literature survey is provided. The AR-ARX method is described in Section III. Section IV discusses the 0-norm, 1-norm and localized minimization formulations, and proposes an iterative localized optimization algorithm. Simulation results are provided in Section V to verify the effectiveness of the localized SHM algorithm. Section VI summarizes this paper along with future work.

II. Related Works

In this section, several damage identification algorithms are briefly discussed. Then we discuss the realization of online SHM based on different network infrastructure of WSNs.

A. Damage Identification Algorithms

Most of the current SHM algorithms are based on a simplified multiple-degree-of-freedom and time-invariant structure model to identify damage:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping and stiffness matrices; $\ddot{\mathbf{x}}(t), \dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are acceleration, velocity and displacement vectors; and $\mathbf{f}(t)$ is the force vector. Damage of the structure may result in variation in the mass, damping and stiffness matrices. Hence, through periodically identifying the system matrices of the operational (undamaged or damaged) state and comparing them with those of the initial (undamaged) state, the structural health condition can be monitored online.

Excitation of a structure is generally ambient vibration and it is difficult to measure the input force vector $\mathbf{f}(t)$ [12]. Recently the use of known active excitation has been explored for SHM, such as piezoelectric transducers (PZTs) [13]. However, without loss of generality, we focus on the case that the input $\mathbf{f}(t)$ is unknown.

One category of the structural damage identification methods is to observe frequency changes of measured data. However, damage itself is basically a local phenomenon while modal frequencies reflect the global property of the structure. Therefore, frequency shift has low sensitivity to damage and it is difficult to localize the position of damage [3].

The NExT-ERA method [14] uses the natural excitation technique (NExT) [12] and the eigensystem realization algorithm (ERA) [15] to identify modal parameters, and a least squares optimization to estimate the stiffness matrix. By comparing the stiffness matrices of the undamaged and damaged states, location and severity of damage are identified. However, this approach requires a reference output signal, which is used to compute the correlation function with each output signal, hence inapplicable to localized realization.

The extended DLV method [16] is based on determination of a special set of load vectors, named as damage locating vectors (DLVs). The DLVs have the property that when they are applied to the structure as static forces at the sensor locations, no stress is produced in the damaged elements. Denote the flexibility matrices before and after damage as $\mathbf{F}_u$ and $\mathbf{F}_d$, respectively, this property leads to $(\mathbf{F}_d - \mathbf{F}_u)\mathbf{L} = \mathbf{0}$, where the matrix $\mathbf{L}$ contains the DLVs. Each DLV is then applied to an undamaged analytical model of the structure. The stress in each structure element is calculated and a normalized cumulative stress is obtained. If an element has nearly zero normalized cumulative stress, then this element is a possible candidate of damage. To identify the flexibility matrices, which are the inverse of the stiffness matrices, the NExT and ERA methods can be employed when measurable input signals are available. When the external excitation is ambient, the mass perturbation method can be applied to calculate the flexibility matrices [17].

Different from identifying modal parameters in the NExT-ERA method and the extended DLV method, the AR-ARX method classifies the damaged pattern via comparing the statistics between baseline measurements and current measurements [2]. The idea is that if there is damage in the

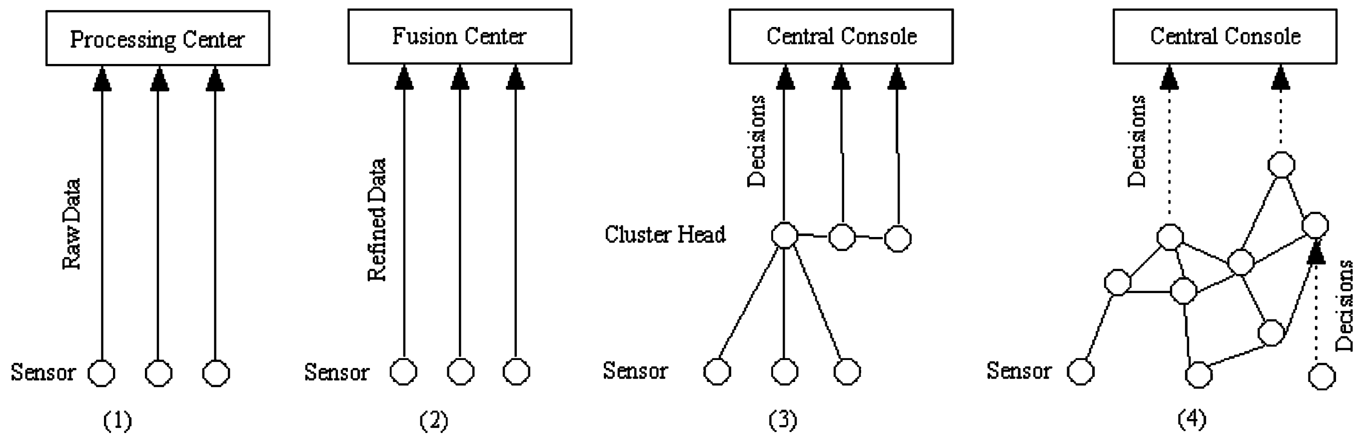

Fig. 1. Four categories of network infrastructures: centralized, distributed, hierarchical, and localized. (a) Centralized infrastructure. (b) Distributed infrastructure. (c) Hierarchical infrastructure. (d) Localized infrastructure.

structure, the auto-regressive (AR) and autoregressive with exogenous input (ARX) models previously identified using the undamaged time history data are unable to reproduce the newly obtained time series measured from the damaged structure. Furthermore, measurement points near to the actual damage locations will have relatively large predictive residual errors.

Various other damage detection algorithms have been proposed in SHM, for example, wavelet analysis [18], adaptive extended Kalman filter [19] and Hilbert–Huang transformation [20]. For detailed review of vibration-based damage identification methods, readers are referred to [3] and [7].

B. Online SHM Based on WSNs

Online SHM based on WSNs has emerged in recent years as a promising technique to monitor the health condition of structures. A wireless sensor is equipped with sensing, communication and computation units. The communication unit enables the sensor to transmit information without using expensive coaxial wires. The computation unit is able to process raw data and make decisions. Extensive surveys of SHM based on WSNs can be found in [4], [5], [7].

There are four categories of network infrastructures according to the way of data processing and information transmitting, as shown in Fig. 1.

1) *Centralized infrastructure*: Sensors send back raw measurement data to a processing center. The processing center identifies damage from the raw data.
2) *Distributed infrastructure*: Sensors send back refined data, which is extracted from the raw data, to a fusion center. The fusion center fuses the refined data and makes decisions on structural health condition.
3) *Hierarchical infrastructure*: Sensors are divided into several clusters. Within a cluster, sensors exchange information with a cluster head. Cluster heads may exchange information, make decisions and feedback decisions to a central console.
4) *Localized infrastructure*: Each sensor exchanges information only with its neighboring sensors, and makes decision autonomously. If one sensor identifies a damage, it informs to a central console via multihop communications.

A WSN has limited bandwidth and battery energy. The centralized infrastructure is subject to high packet loss and energy consumption, since a large amount of measurements are generated in each sampling period. The distributed infrastructure, though still using centralized data fusion, is able to save the bandwidth and energy resources through preprocessing the raw data and transmitting only refined data. However, in a large-scale WSN, one-hop communication between sensors and the fusion center is generally impossible, while multihop communication aggravates the burden on bandwidth and energy consumption. In the hierarchical infrastructure, each sensor is limited to exchange information with a cluster head, and the cluster head is in charge of high-level communications. However, it is difficult to predefine clusters in a large-scale WSN. Furthermore, failure of a cluster head leads to the malfunction of a whole cluster of sensors. Hence, to improve the scalability and robustness, it is expected to apply a localized infrastructure, in which each sensor exchanges information among its neighboring sensors and makes decision autonomously.

A distributed online SHM framework has been discussed in [10], in which each sensor distributively applying the AR-ARX method and communicates with a fusion center. In [9], the extended DLV method is used in each cluster; the cluster heads exchange damage information, make final decisions, and communicate to a central console. However, study of online SHM based on localized infrastructure is still in its beginning stage. In this paper, we focus on the online SHM in a localized manner based on the AR-ARX method, aiming at improving the scalability and robustness of a WSN under the constraints of bandwidth and energy consumption. The proposed localized information processing approach can be applied to other damage identification methods as well.

III. AR-ARX METHOD

The AR-ARX method is basically a statistic pattern recognition approach which is composed of a modeling stage where the structure is undamaged, and a decision-making stage where the damage state is unknown [2], [10]. The basic idea is to identify damages via comparing the statistics between baseline measurements and current measurements.

A. AR and ARX Models

Let $z = \{z_k\}$ be the time-series response of a structure at a specific sensor location. Assuming the response to be stationary, an AR model is used to fit the measurement data

$$z_k = \sum_{i=1}^{p} b_i^z z_{k-i} + r_k^z. \tag{2}$$

The response of the structure at sample time k, as denoted by z_k, is a function of p previous observations of the system response, plus a residual error term r_k^z. Weights on the previous observations of z_{k-i} are denoted by coefficients $\{b_i\}$.

Assume the residual error of the AR model r_k^z is influenced by an unknown excitation input to the system. As a result, an ARX model is chosen to model the relationship between the residual errors and the measured response of the system

$$z_k = \sum_{i=1}^{a} \alpha_i z_{k-i} + \sum_{j=0}^{b} \beta_j r_{k-j}^z + \epsilon_k^z. \tag{3}$$

Coefficients on past measurements and the residual errors of the AR model are $\{\alpha_i\}$ and $\{\beta_i\}$, respectively. The residual of the ARX model ϵ_k^z is a damage sensitive feature being used to identify the existence of damage in the structure. It is worth noting that the selection of model orders may influence the accuracy of the AR-ARX method [2].

In the modeling stage, i.e., the structure is known to be undamaged, the AR and ARX models are constructed under various ambient vibration levels. The coefficients of models, i.e., $\{b_i\}, \{\alpha_i\}$ and $\{\beta_i\}$, and the standard variances of the residuals, i.e., $\{\sigma^2(\epsilon_k^z)\}$, are stored in the database of each sensor, denoted as $\{b_i^{\mathrm{DB}}\}, \{\alpha_i^{\mathrm{DB}}\}, \{\beta_i^{\mathrm{DB}}\}$ and $\{\sigma^2(\epsilon_k^{\mathrm{DB}})\}$.

B. Statistical Pattern Recognition

In the decision-making stage, an AR model is fitted based on the response z_k of the structure in an unknown state (damage or undamaged). The coefficients of the fitted AR model are compared to the database of AR-ARX model pairs previously calculated for the undamaged structure. A match is determined by minimizing the Euclidian distance D of the newly derived AR model and the database AR model coefficients, b_i^z and b_i^{DB}, respectively. The Euclidian distance D is defined as

$$D = \sum_{i=1}^{p} \left(b_i^{\mathrm{DB}} - b_i^z\right)^2. \tag{4}$$

If no structural damage is experienced and the operational conditions of the two models are close to one another, the selected AR model from the database will closely approximate the measured response. If a damage has been sustained by the structure, even the closest AR model of the database will not approximate the measured structural response well.

The measured response z_k of the structure in the unknown state, and the residual errors r_k^z of the fitted AR model, are substituted into the database ARX model to determine the residual error ϵ_k^z of the ARX model

$$z_k = \sum_{i=1}^{a} \alpha_i^{\mathrm{DB}} z_{k-i} + \sum_{j=0}^{b} \beta_j^{\mathrm{DB}} r_{k-j}^z + \epsilon_k^z. \tag{5}$$

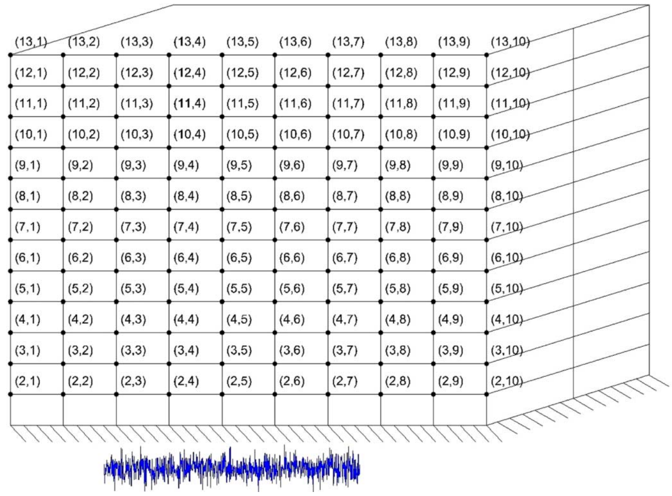

Fig. 2. Two-dimensional model of a steel frame structure with 12 stories and 9 bays. Gaussian random white noises are imposed to the foundation to simulate ambient vibrations.

The residual errors $\{\epsilon_k^z\}$ of the ARX model are the damage sensitive feature.

Here, we briefly discuss the statistics of the residual errors. Define the ratio of the variance of the residual errors to that in the database as

$$y = \frac{\sigma^2\left(\epsilon_k^z\right)}{\sigma^2\left(\epsilon_k^{\mathrm{DB}}\right)}. \tag{6}$$

Here, $\sigma^2(\epsilon_k^z)$ and $\sigma^2(\epsilon_k^{\mathrm{DB}})$ are variances of the residual errors. Suppose the models are accurate and the ambient excitations are Gaussian random variables with zero means. Hence, the residual errors are also Gaussian variables with zero means. If the structure is undamaged, i.e., the system models remain the same, and the noise level keeps invariant, then y follows F-distribution, and the degree-of-freedom is equal to the length of the measured data. If the structure is damaged, i.e., coefficients of the system models change, variance of the residual errors increases because of model mismatch. From this angle of view, y is a *damage-sensitive coefficient*.

C. Damage-Sensitive Coefficient

Assume a large-scale WSN is deployed in a sensing area. In the modeling stage, each sensor builds up a database for the measured responses. In the decision-making stage, sensors periodically perform monitoring tasks, firstly collecting batches of data and secondly generate damage-sensitive coefficients. It is of interest to study the spatial distribution of the damage-sensitive coefficients in the sensing area.

We consider a steel frame structure with 12 stories and 9 bays, simplified as a two-dimensional model, as illustrated in Fig. 2. Sensors are deployed in the joint points from the 2nd floor to the 13th floor, composing a grid network of 120 sensors in a two-dimensional sensing area. Gaussian random white noises are imposed to the foundation to simulate ambient vibrations. In each monitoring period of both modeling and decision-making stages, 1000 data points are sampled to generate the AR and ARX models. We introduce damages by reducing the stiffness of one or several columns. For example, if we reduce the stiffness

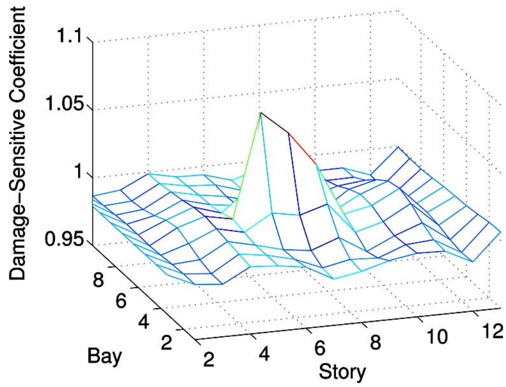

Fig. 3. Spatial distribution of the damage-sensitive coefficients after introducing damage to the column between sensors (6,5) and (7,5).

of the column between sensors (6,5) (located on the 6th floor, 5th bay) and (7,5) (located on the 7th floor, 5th bay), these two sensors should report damages.

Fig. 3 shows a set of typical simulation results, from which we have following observations.

1) The damage-sensitive coefficient computed by each sensor is the superposition of a damaged state and an undamaged state. The undamaged state is a random variable with F-distribution, as discussed above.
2) The damaged state is also composed of two parts: damage effect of the current measurement point and damage effect of neighboring measurement points. Damage in one point results in a peak value in the corresponding point, while leads to attenuated effects on neighboring points. The effect decreases as the distance increases.

These two observations stimulate us to consider the damage identification problem in a localized pattern recognition framework. The damage-sensitive coefficients are the superposition of a series of basis functions and a random noise field. The objective of the localized SHM is hence to automatically and autonomously estimate the shape of the basis functions and filter out the random noises for the sensors, via only using local information exchange among neighboring sensors.

IV. Localized SHM Algorithm

In this section, we formulate the damage identification problem as a centralized nonlinear optimization problem and relax it to a centralized linear program. A localized algorithm is proposed to solve the linear program.

A. Problem Formulation

In a WSN, sensor j reports a damage-sensitive coefficient y_j as defined in (6). From the discussions in the previous section, y_j is the summation of a damage-related term s_j and a random variable e_j with F-distribution

$$y_j = s_j + e_j. \tag{7}$$

Here, s_j is decided by the cumulative effects of the damaged points. Let L be the set of sensors, without loss of generality, s_j can be written as

$$s_j = \sum_{i \in L} c_i f_i(j) \tag{8}$$

where c_i represents the severity of the damage point i, defined as *severity coefficient*, and the normalized basis function $f_i(j)$ represents the effect of damage i on point j. Based on the analysis above, the measurement y_j increases when a damage occurs somewhere, hence, $f_i(j) \geq 0$ and $c_i \geq 0$. Furthermore, a sensor near to the damage source is more influenced than a sensor far from the damage source, thus we can normalize $f_i(j)$ as $f_i(j) = 1$ when $i = j$ and $0 \leq f_i(j) < 1$ when $i \neq j$.

The random variable e_j is under F-distribution with degree-of-freedom (n, n), where n is nearly the number of sampling points. When n is large, the probability density function of e_j is almost symmetric with $e_j = 1$. For any given confidence coefficient, there is a threshold θ for the random variable e_j to pass the hypothesis testing with $|e_j - 1| \leq \theta$.

Our novel idea hinges on a key observation that damage is generally scarce in a structure, namely, the severity vector $\mathbf{c} = \{c_j \mid j \in L\}$ is sparse. The sparsity of $\mathbf{c}$ can be measured by its 0-norm $||\mathbf{c}||_0$. Hence, we have the following 0-norm minimization formulation $(\mathbf{P0})$:

$$\begin{aligned} \min_{\mathbf{c}} \quad & ||\mathbf{c}||_0, \\ \text{s.t.} \quad & |y_j - \sum_{i \in L} c_i f_i(j) - 1| \leq \theta, \forall j \in L, \\ \text{s.t.} \quad & c_j \geq 0, \forall j \in L. \end{aligned} \tag{9}$$

The problem $\mathbf{P0}$ has linear constraints and a nonlinear objective function. Hence, we relax it to a 1-norm minimization formulation $(\mathbf{P1})$

$$\begin{aligned} \min_{\mathbf{c}} \quad & ||\mathbf{c}||_1, \\ \text{s.t.} \quad & |y_j - \sum_{i \in L} c_i f_i(j) - 1| \leq \theta, \forall j \in L, \\ \text{s.t.} \quad & c_j \geq 0, \forall j \in L. \end{aligned} \tag{10}$$

$\mathbf{P1}$ is a standard linear programming problem, which can be easily solved in a centralized way. The relaxation to 1-norm in (10) still induces sparsity, while the conditions for its equivalence to the 0-norm solution has been investigated in the compressed sensing literature [11].

B. Choice of Basis Functions

Choice of the basis function $f_i(j)$ relies on the understanding to the interrelationship between point i and point j in the sensing field. Generally speaking, the interrelationship is complicated and unavailable in a prior. However, simulation results in Section III-C suggest that the following two basis functions are good approximation to practical cases. The first basis function is a **Delta shape**

$$f_i(j) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{11}$$

which means that a damage in one point does not influence its neighboring points. Applying the Delta shape basis function in $\mathbf{P0}$, it leads to the trivial solution based on the threshold $\theta : c_i > 0$ if $|y_i - 1| > \theta$ and $c_i = 0$ if $|y_i - 1| \leq \theta$. The corresponding $\mathbf{P1}$ is feasible unless $y_i - 1 \leq -\theta$ for some i.

The second basis function is an **isotropic Gaussian shape**

$$f_i(j) = e^{-d_{ij}^2/\sigma_i^2} \tag{12}$$

where d_{ij} is the distance between i and j and σ_i is the impact coefficient of i. The isotropic Gaussian-shaped basis function assumes isotropic influence of a damage point, which is adopted for analytical convenience but still able to approximate practical cases. Substituting the isotropic Gaussian shape, $\mathbf{P1}$ is infeasible in two cases: 1) $y_i - 1 \leq -\theta$ for some i and 2) the measurement in the damage point is very large while the measurement in its neighboring point is near to 1. The first case reflects interdependence of the threshold and the noise level; and the second case reflects the matching ability of the basis function to the measured data.

C. Localized Solution Based on Random Gossip

Centralized solutions to the linear programming problem $\mathbf{P1}$ are mature, but do not fit for the distributed nature of WSNs. It is preferred to design localized algorithms, in which each sensor communicates with its neighboring sensors and make decisions autonomously. To this end, we assign the objective function and constraints in $\mathbf{P1}$ to individual sensors. For each sensor j, the optimization subproblem is

$$\begin{aligned} &\min_{c_j} \quad c_j, \\ &\text{s.t.} \quad |y_j - \sum_{i \in N_j} c_i f_i(j) - c_j - 1| \leq \theta, \\ &\text{s.t.} \quad c_j \geq 0. \end{aligned} \tag{13}$$

Here, N_j is the neighboring sensor set of j. Hence, each subproblem is a simple linear programming problem and only uses information from the neighboring sensors of j. Solution to (13) is $c_j = \max(0, y_j - \theta - \sum_{i \in N_j} c_i f_i(j) - 1)$ or null if $y_j + \theta - \sum_{i \in N_j} c_i f_i(j) - 1 < 0$.

Let us consider to update c_j as $c_j = \max(0, y_j - \theta - \sum_{i \in N_j} c_i f_i(j) - 1)$. Suppose that the neighboring set of each sensor j is large enough, namely, the difference between $\sum_{i \in N_j} c_i f_i(j) + c_j$ and $\sum_{i \in L} c_i f_i(j)$ is small enough. Then, if $\mathbf{P1}$ is feasible, iteratively applying the above-mentioned updating rule will approximately reach the optimal solution to $\mathbf{P1}$. When $\mathbf{P1}$ is infeasible, the iterative updating rule still converges to a fixed point.

Starting from the localized formulation (13), we have the following localized solution to $\mathbf{P1}$.

Step 1: Each sensor j holds a predefined common threshold θ, and sets the initial decision $c_j = 0$.

Step 2: Sensor j randomly wakes up and sends requests to its neighboring sensors in N_j. Upon receiving the request, each sensor $i, i \in N_j$ sends its current decision c_i and the basis function value $f_i(j)$ to j.

Step 3: Sensor j updates its decision with $c_j = \max(0, y_j - \theta - \sum_{i \in N_j} c_i f_i(j) - 1)$.

Step 4: Repeat steps 2 and 3 until reach the maximum optimization time.

The proposed localized algorithm shares similarity with the random gossip method [21] which has successful applications in consensus problems. Effectiveness of the localized solution is validated in the following numerical simulations.

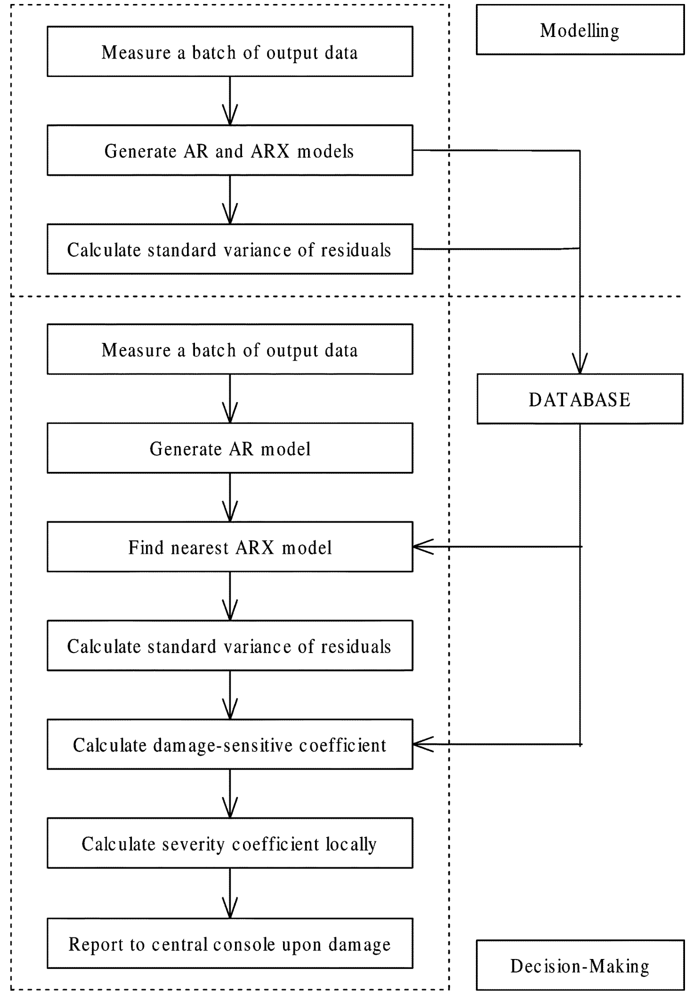

Fig. 4. Procedures of the localized SHM algorithm.

Procedures of the whole localized SHM algorithm, which is composed of a modeling stage and a decision-making stage, is schematically depicted in Fig. 4.

V. Simulation Results

In this section, we provide extensive simulation results to illustrate the effectiveness of the proposed localized online SHM algorithm based on a two-dimensional structure model.

A. General Settings

We adopt the general simulation settings as in Section III-C. A grid network of 120 sensors is deployed in the joint points of a steel frame structure with 12 stories and 9 bays. The width of a bay is 24 feet and the height of a floor is 14 feet. Ambient vibrations are imposed to the foundation with Gaussian white noises. Response of the structure is analyzed by the finite-element program OpenSees [22]. In each monitoring period of both modeling and decision-making stages, 1000 acceleration output data points are sampled to generate the AR and ARX models. The order of the AR model is set as 30 and the orders of the ARX model are set as 5 and 5. Damage patterns are introduced by reducing the stiffness of one or several columns.

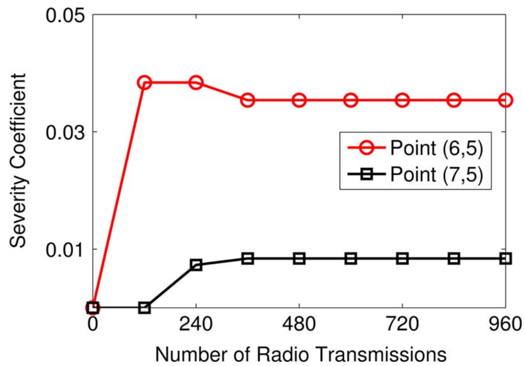

Fig. 5. Convergence property of the *Localized* algorithm with respect to the total number of radio transmissions for all 120 sensors.

Here, we compare the performance of three algorithms.

1) *Threshold*: The centralized 1-norm minimization algorithm of (10) with a Delta-shaped basis function. To avoid the potential infeasibility problem we simply use a threshold to identify damages thus it can be implemented in both centralized and localized ways.
2) *Centralized*: The centralized 1-norm minimization algorithm of (10) with an isotropic Gaussian-shaped basis function and $\sigma_i = 14$, namely, the height of a floor.
3) *Localized*: The localized solution to (13) with an isotropic Gaussian shape basis function and $\sigma_i = 14$. Transmission range of each sensor is slightly larger than 24 feet, that is, each sensor is able to communicate with four neighboring sensors. Initial decision variable c_i is 0 in each sensor i.

Throughout the simulations the threshold is set as $\theta = 0.04$.

We adopt two performance criteria: false-negative rate (neglecting a damage when it occurs) and false-positive rate (reporting a damage when it does not exist).

B. Single-Damage Pattern

We firstly consider a single-damage pattern, in which the *centralized* algorithm is feasible, to illustrate the convergence rate of the *Localized* algorithm. The data has been shown in Fig. 3, with stiffness reduction as 57% in a single column between (6,5) and (7,5). The *Threshold* algorithm identifies the damages in points (6,5) and (7,5). The *Centralized* algorithm also identifies the damages, and provides the relative severity coefficients as 0.0354 in (6,5) and 0.0081 in (7,5). For the *Localized* algorithm, convergence rate is depicted in Fig. 5. For asynchronous random gossip, there is only one radio transmission per iteration for the entire network of 120 sensors. It is shown that both of the severity coefficients converge to the solutions of the *Centralized* algorithm after 360 iterations, which indicates that each sensor only needs to wake up and transmit locally for three times on average.

We secondly still consider the single-damage pattern discussed above with four more data sets. Damage identification results are shown in Table I. The *Centralized* algorithm is sometimes infeasible, as we have discussed above. The *Threshold* algorithm tends to generate some false-positive alarms. The *Localized* algorithm reduces the false-positive alarms by considering the effect of the damage in its neighboring area.

TABLE I
PERFORMANCE OF THE THREE ALGORITHMS FOR A SINGLE-DAMAGE PATTERN

Data Set	Algorithm	Feasibility	False-Negative	False-Positive
1	*Threshold*	Yes	0	0
	Centralized	Yes	0	0
	Localized	Yes	0	0
2	*Threshold*	Yes	0	1
	Centralized	No	–	–
	Localized	Yes	0	0
3	*Threshold*	Yes	1	1
	Centralized	No	–	–
	Localized	Yes	1	0
4	*Threshold*	Yes	0	1
	Centralized	Yes	0	0
	Localized	Yes	0	0

TABLE II
PERFORMANCE OF THE THREE ALGORITHMS FOR A MULTIPLE-DAMAGE PATTERN

Data Set	Algorithm	Feasibility	False-Negative	False-Positive
1	*Threshold*	Yes	0	15
	Centralized	No	–	–
	Localized	Yes	0	2
2	*Threshold*	Yes	0	4
	Centralized	No	–	–
	Localized	Yes	0	1
3	*Threshold*	Yes	1	9
	Centralized	No	–	–
	Localized	Yes	1	1
4	*Threshold*	Yes	1	1
	Centralized	No	–	–
	Localized	Yes	1	1

C. Multiple-Damage Pattern

Consider damage of the column beneath the sensor (1,1) and damage of the column between sensors (6,5) and (7,5), both with stiffness reduction as 57%. Table II provides simulation results on 4 sets of data, in which each data set represents a random realization of the damage model and contains 1000 sampling points. The *Centralized* algorithm greatly suffers from the infeasibility problem, while the *Threshold* algorithm generates many false-positive alarms. On the contrary, the *Localized* algorithm both tackles the infeasibility problem and reduces the false-positive alarms.

D. Damage Severity

Now, we discuss the ability of the localized SHM algorithm to evaluate damage severity. By setting stiffness reduction as different levels, relative severity coefficients are shown in Fig. 6. For each sensor point, the severity coefficient increases as the stiffness reduction increases, therefore the proposed algorithm is able to not only localize damage position but also quantify damage severity. It should be noted that the severity coefficient of point (6,5) is always larger than that of point (7,5). This common phenomenon in the AR-ARX method indicates that damage in a column has larger effect on the upper part of a structure than that on the lower part.

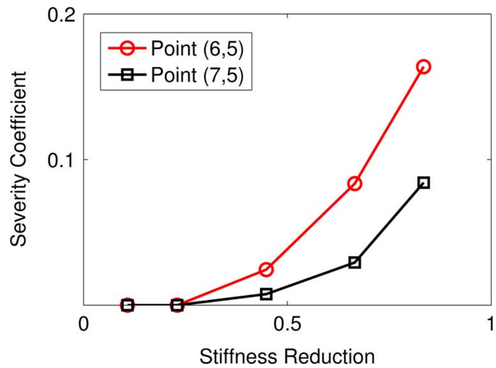

Fig. 6. Sensitivity of the *Localized* algorithm to stiffness reduction.

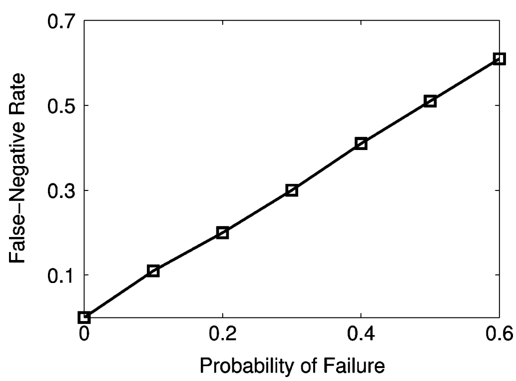

Fig. 7. Robustness of the *Localized* algorithm to sensor failure.

E. Network Robustness

In practical applications, WSNs are often subject to sensor failure. For the hierarchical network structure, failure of a cluster head may lead to malfunction of all sensors in the cluster. With localized information exchange and decision-making, network robustness is substantially improved in the localized network structure. To evaluate network robustness, we adopt the same single-damage pattern settings in Section V-B. A random portion of sensors are set to be in failure state. The false-negative rate, as shown in Fig. 7, is proportional to the probability of node failure for both damage point (6,5) and (7,5). The localized algorithm utilizes the intrinsic sparsity of damages, hence achieves high robustness of the network.

VI. CONCLUSION

In a wireless sensor network setup, this paper discusses the online structural health monitoring problem on three levels: identification of damage, localization of damage and quantification of damage severity. The AR-ARX method, which classifies the damaged pattern via comparing the statistics between baseline measurements and current measurements, is applied as the embedded damage identification approach. After damage identification, sensors generate a map of damage-sensitive coefficients, which are composed of damage information and random noises. Assuming the shape of the damage information based on theoretical and empirical analysis, we develop a centralized 0-norm minimization formulation, and relax it to a centralized 1-norm linear programming problem. An iterative localized optimization framework, which shares similarity with the random gossip algorithm, is proposed to achieve fast convergence to a near-optimal solution.

By using a larger set of basis functions in combination with optimized function parameters, we will be able to depict more complicated sensing maps and identify diverse types of damages. Starting from this point, the localized SHM algorithm is applicable to a much broader area of localized curve-fitting, localized pattern recognition and localized learning.

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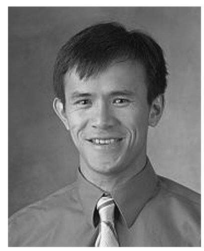

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