Joint event detection and environment perception in decentralised wireless sensor networks

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Abstract: In an event detection application, sensor nodes measure signals that are emitted from multiple events and attenuated by the environment, and then collaborate to estimate the locations and magnitudes of the events. Existing event detection algorithms often assume that the number of the events is known in advance and/or the attenuation coefficient of the environment is given. This paper considers the case that both the number of the events and the attenuation coefficient of the environment are unknown. Through exploiting the sparse nature of the events, we propose an ℓ-norm regularised least squares formulation that automatically estimates the number of the events as well as their locations and magnitudes; the attenuation coefficient of the environment also appears as an optimisation variable. We develop a decentralised algorithm and its accelerated variant to solve the joint event detection and environment perception problem using the alternating direction method of multipliers (ADMM).

Keywords: WSN; wireless sensor network; event detection; environment perception; decentralised optimisation.

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1 Introduction

In recent years, the rapid development of micro-electronics and wireless communication technologies has enabled the deployment of large-scale wireless sensor networks (WSNs) for event detection applications (Akyildiz and Kasimoglu, 2004; Yick et al., 2008). In a WSN, sensor nodes are able to collect measurements, process data locally, and communicate with their neighbours without the intervention of human operators. This characteristic makes WSNs fit for event detection applications in hazardous environments, such as nuclear radioactive source detection (Sundaresan et al., 2007) and contaminant source detection (Sun and Coyle, 2010). Further, WSNs are easy to deploy that motivates the applications of structural health monitoring (Lynch, 2007) and area surveillance (Wittenburg et al., 2010) where deploying wired sensor networks is often costly.

1.1 Related work

One of the main challenges in event detection is to estimate locations and magnitudes of an unknown number of events. When the number of events is known as a prior knowledge, traditional detection and estimation techniques, such as the maximum likelihood estimator or the expectation maximisation algorithm (Sheng and Hu, 2005; Mandel et al., 2010), are applicable. However, when the number of events is unknown in advance, the event detection becomes intractable since enumerating the possible numbers of events leads to unaffordable computation burden. To address this issue, an intuitive idea is to choose some candidate points in the sensing field and confine the events to occur at them. The candidate points can be sensor nodes (Ling and Tian, 2010; Yuan et al., 2015) or grid points (Meng et al., 2009; Bazerque and Giannakis, 2010). If the number of the candidate points is large enough, this approximation leads to high estimation accuracy at the risk of solving an underdetermined least squares problem. Nevertheless, since the events are sparsely occurring in the sensing field and the number of the events is often much smaller than that of the candidate points, the underdetermined least squares problem can be penalised by a sparsity-enhanced term that yields a unique solution (Ling and Tian, 2010; Yuan et al., 2015; Meng et al., 2009; Bazerque and Giannakis, 2010).

Another main challenge in event detection is to efficiently process data in WSNs. Traditionally an event detection algorithm is run on a fusion centre that collects measurements from distributed sensor nodes. However, this *centralised* data processing scheme suffers from energy-consuming multi-hop communication and is fragile to the failure of several critical relaying sensor nodes. To enable scalable data processing in a large-scale WSN, *decentralised* data processing has received extensive research interests recently (Predd et al., 2006; Wittenburg et al., 2012; Schmidt et al., 2012; Laitrakun and Coyle, 2013). In the decentralised data processing scheme, the sensor nodes no longer send their measurements to the fusion centre. By taking advantages of their computation abilities, the sensor nodes communicate with their one-hop neighbours and process their data locally. Through decentralised and iterative in-network data processing, sensor nodes achieve a consensus on the events occurring in the sensing field. Such a decentralised WSN involves energy-efficient one-hop communication and is robust to the failure of a portion of sensor nodes. Typical decentralised data processing algorithms include the decentralised subgradient method (Nedic and Ozdaglar, 2009), the decentralised dual averaging algorithm (Duchi et al., 2012), and the alternating direction method of multipliers (ADMM) (Bertsekas and Tsitsiklis, 1997; Schizas et al., 2008). Among these algorithms, the ADMM is a remarkable one due to its fast convergence, which means low communication cost in a decentralised WSN.

However, to the best of our knowledge most of the existing works assume that the attenuation coefficient of the environment is known as a prior knowledge, or has been estimated in advance. For example, the energy emitted from an acoustic or magnetic source is often assumed to be proportional to $\exp(-\theta d^2)$ where d is the distance between the source and the sensor node and θ is the known attenuation coefficient of the environment (Liu et al., 2012; Patwari et al., 2005). Since estimating the attenuation coefficient is often time-consuming and subject to the dynamic change of the environment, such an approach is unreliable and may lead to degradated event detection accuracy.

1.2 Our contributions

In light of the above discussions, in this paper we develop decentralised algorithms that simultaneously accomplish the multiple-event detection and environment perception tasks. Specifically, we confine that the events occur at a given set of candidate points and utilise the fact that the number of the events is much smaller than the number of the candidate points as in Ling and Tian (2010), Yuan et al. (2015), Meng et al. (2009) and Bazerque and Giannakis (2010). This way, the task of estimating the locations and magnitudes of multiple events can be formulated as recovering a sparse vector from its linear measurements. Different to Ling and Tian (2010), Yuan et al. (2015), Meng et al. (2009) and Bazerque and Giannakis (2010), in this paper the attenuation coefficient of the environment is no longer known as a prior knowledge, but an extra parameter to estimate. Since the two optimisation variables, the sparse vector that represents the locations and magnitudes of the events and the attenuation coefficient of the environment, are entangled together, the joint event detection and environment perception problem is nonconvex. We develop a decentralised joint event detection and environment perception algorithm based on the ADMM, which not only provides a skillful variable splitting technique to enable decentralised algorithm design, but also utilises the structure of the objective function such that each subproblem is tractable. To improve energy-efficiency of the WSN, we further develop a heuristic scheme that accelerates the proposed decentralised algorithm.

1.3 Paper organisation

The rest of this paper is organised as follows. Section 2 formulates the joint event detection and environment perception problem. Section 3 analyses the structure of the objective function that motivates a centralised algorithm through alternating minimisation. Section 4 proposes a decentralised joint event detection and environment perception algorithm based on the ADMM and then provides a heuristically accelerated version. Numerical experiments are given in Section 5 to verify the proposed algorithms. Section 6 concludes the paper.

2 Problem formulation

Suppose that we are monitoring a two-dimensional sensing area in which multiple event sources are emitting signals and a large-scale WSN is deployed to collect measurements. We make the following assumptions about how the signals emitted from the events attenuates in the environment and how the sensor nodes measure the received signals.

Assumption 1 (Attenuation function)**:** The signal emitted from any event attenuates following an attenuation function of the environment $f(d, \theta)$ where d is the distance from the event to the measurement point and θ is the attenuation coefficient of the environment. The attenuation function $f(d, \theta)$ is monotonically decreasing with respect to the distance d as well as normalised by $f(0, \theta) = 1$ and $f(\infty, \theta) = 0$.

Assumption 2 (Linearly superimposed measurement)**:** The measurement of any sensor node is the linear superposition of the signal strengths emitted from the events.

Assumption 1 is common in various event detection applications. For example, the energy emitted from an acoustic or magnetic source is often attenuated according to $f(d, \theta) = \exp(-\theta d^2)$. We assume a single attenuation coefficient θ for events that occurring at different locations, implying that the sensing field is homogeneous. In previous works θ is often assumed to be known in prior, while in this paper we will address the case that θ is unknown. Assumption 2 implies that the sensor nodes are calibrated and the measurement is free of distortion.

To enable tractable multiple-event detection, we choose some candidate points in the sensing field and confine the events to occur at them. The candidate points can be sensor nodes (Ling and Tian, 2010; Yuan et al., 2015) or grid points (Meng et al., 2009; Bazerque and Giannakis, 2010). Further, the events are sparse compared to the candidate points since it is uncommon to have a large number of events occurring simultaneously.

Assumption 3 (Candidate points and sparsity)**:** The events are confined to a set of candidate points in the sensing field. The number of the events is much smaller than the number of candidate points.

The WSN is composed of L sensor nodes; we denote the set of sensor nodes as $\mathcal{L}$. Sensor nodes have a communication range r_C which is the maximum distance within which two sensor nodes can directly communicate. For any sensor node v_i, it is able to communicate with another sensor node v_j if d_{ij}, the distance between v_i and v_j is no larger than r_C. The communication is bidirectional and v_i and v_j are called as neighbours. If d_{ij} is larger than r_C, then v_i and v_j are unable to communicate and they are not neighbours of each other. We denote the set of v_i's neighbours as $\mathcal{N}_i$ whose cardinality $|\mathcal{N}_i|$ is the number of v_i's neighbours. We require the WSN to be connected.

Assumption 4 (Network connectivity)**:** Given the communication range r_C of the sensor nodes, the WSN is bidirectionally connected.

Without loss of generality, we consider the case that the events occur at the sensor nodes. The derivation below can be straightforwardly extended to the case that the events occur at any other candidate points. For any possible event occurring at sensor node v_j, we denote its magnitude as x_j; $x_j = 0$ means that such an event does not exist and otherwise $x_j > 0$. Define $x = [x_1; x_2; \ldots; x_L]$ that represents the locations and magnitudes of the events. According to Assumption 1, its strength measured by sensor node v_i is $f(d_{ji}, \theta)$ where d_{ji} is the distance between v_j and v_i. According to Assumption 2, the measurement of v_i is the linear superposition of all possible events, i.e., $b_i = \sum_{v_j \in \mathcal{L}} f(d_{ji}, \theta) x_j$ in the noise-free case. Our objective is to jointly estimate x and θ from the measurements $\{b_i\}_{v_i \in \mathcal{L}}$ through decentralised in-network data processing.

If no prior knowledge of the vector x is available, a natural approach to solving x and θ from $\{b_i\}_{v_i \in \mathcal{L}}$ is to formulate a nonlinear least squares problem

$$\begin{aligned} \min_{x,\theta} \; & \sum_{v_i \in \mathcal{L}} \left(b_i - \sum_{v_j \in \mathcal{L}} f(d_{ji}, \theta) x_j \right)^2, \\ s.t. \; & x_i \geq 0, \quad \forall v_i \in \mathcal{L}. \end{aligned} \tag{1}$$

However, equation (1) is ill-posed. Suppose that θ is given and we only need to solve x from equation (1), there are L unknowns $x_1, x_2, \cdots, x_L$ and L linear measurements $b_1, b_2, \cdots, b_L$. Since θ is also unknown, the solutions to equation (1) is not unique. The same situation appears when we choose grid points other than sensor nodes as the candidate points, if the number of grid points is larger than the number of sensor nodes.

According to Assumption 3, we know that the number of events is much smaller than the number of sensor nodes. Hence there are only a smaller number of elements in x is nonzero; in another word, x is a sparse vector. This prior knowledge motivates us to penalise the ordinary least squares problem (1) with a sparsity-promoting ℓ_1-norm term

$$\min_{x,\theta} J(x,\theta) = \|x\|_1 + \frac{\lambda}{2}\sum_{v_i \in \mathcal{L}}\left(b_i - \sum_{v_j \in \mathcal{L}} f(d_{ji},\theta)x_j\right)^2, \quad \text{s.t.} x \geq 0. \tag{2}$$

In the ℓ_1-norm regularised nonlinear least squares problem (2), the ℓ_1-norm term $\|x\|_1 = \sum_{v_j \in \mathcal{L}} |x_j|$ helps find a sparse x. Through tuning the nonnegative weight λ, equation (2) achieves a tradeoff between the sparsity of x and the squared error. A large λ leads to small squared error but numerous false alarms, while a small λ results in a sparse solution of x but poor fitting to the measurements (Donoho, 2006; Candes et al., 2006).

3 Centralised algorithm

In the ℓ_1-norm regularised nonlinear least squares problem (2), the two optimisation variables x and θ are entangled in the squared error term. The objective function $J(x,\theta)$ is not necessarily convex for any nontrivial choice of $f(d,\theta)$. Therefore, designing an efficient centralised optimisation algorithm to solve equation (2) is a challenging task, not mentioning that our goal is to solve equation (2) in a decentralised manner.

Fortunately, the optimisation problem (2) has a nice structure that makes developing efficient centralised and decentralised algorithms possible. Observe that fixing θ, the x-subproblem minimises a ℓ_1-norm regularised linear least squares objective function subject to a nonnegativity constraint, and hence is a convex program (Boyd et al., 2004). On the other hand, fixing x, the θ-subproblem optimises over a one-dimensional variable θ. Though the θ-subproblem is often nonconvex, we are able to find an acceptable solution of θ using single-variable search methods.

Through investigating the problem structure of equation (2), we propose a centralised algorithm based on alternating minimisation (Gorski et al., 2007), which first fixes θ and minimises over x and then fixes x and minimises over θ. We summarise the centralised event detection and environment perception (CEEP) algorithm in Algorithm 1. Numerical experiments (cf. Section 5) demonstrate its satisfactory convergence properties, implying that the algorithm is robust to the nonconvexity of the θ-subproblem. Actually, we find that the final estimate of θ is very close to its true value in the numerical experiments.

Algorithm 1 Centralized Event detection and Environment Perception (CEEP)

Require: Measurement b, parameter λ, attenuation function $f(d,\theta)$.

1: Initialize $x(0) = 0$, $\theta(0) = 0$.
2: **for** $t = 0, 1, 2, \ldots$ **do**
3: Update $x(t+1)$ via

$$x(t+1) = \arg\min_{x \geq 0} \|x\|_1 + \frac{\lambda}{2}\sum_{v_i \in \mathcal{L}}\left(b_i - \sum_{v_j \in \mathcal{L}} f(d_{ji},\theta(t))x_j\right)^2. \tag{3}$$

4: Update $\theta(t+1)$ via

$$\theta(t+1) = \arg\min_{\theta} \frac{\lambda}{2}\sum_{v_i \in \mathcal{L}}\left(b_i - \sum_{v_j \in \mathcal{L}} f(d_{ji},\theta)x_j(t+1)\right)^2. \tag{4}$$

5: **end for**
6: Return $x(t+1)$ and $\theta(t+1)$.

In the next section, we will introduce the ADMM to solve equation (2) in a decentralised manner, in which the separable structure of the problem still plays a key role.

4 Decentralised algorithms

This section applies the ADMM to solve the joint event detection and environment perception problem (2). Section 4.1 rewrites equation (2) to a form that can be handled by the ADMM and develops a decentralised algorithm. This algorithm is equivalent to a much simpler version given proper initialisation, as shown in Section 4.2. To improve energy-efficiency of the WSN, Section 4.3 further develops a heuristic scheme that efficiently accelerates the proposed decentralised algorithm.

4.1 The ADMM

The ADMM solves a constrained minimisation problem with two blocks of variables. In each iteration, the algorithm first minimises the augmented Lagrangian over one block of variables, then minimises the augmented Lagrangian over another block of variables, and finally updates the Lagrange multiplier (Bertsekas and Tsitsiklis, 1997; Boyd et al., 2010). Through reformulating equation (2) to a constrained form the ADMM is applicable, yielding a decentralised algorithm as we will demonstrate below.

We let each sensor node v_i keep a local copy of x (denoted by $x^{(i)}$) and a local copy of θ (denoted by $\theta^{(i)}$). We expect that the local copies of the sensor nodes are the same. To do so, for any two neighbours v_i and v_j we introduce auxiliary variables $z^{(ij)}$ and $w^{(ij)}$ and let $x^{(i)} = z^{(ij)} = x^{(j)}$ and $\theta^{(i)} = w^{(ij)} = \theta^{(j)}$. These way, equation (2) can be rewritten as

$$
\begin{aligned}
&\min \sum_{v_i\in\mathcal{L}} J_i(x^{(i)},\theta^{(i)})\\
&\quad = \sum_{v_i\in\mathcal{L}} \left[x_i^{(i)} + \tfrac{\lambda}{2}\left(b_i - \sum_{v_j\in\mathcal{L}} f(d_{ji},\theta^{(i)})x_j^{(i)}\right)^2\right],\\
&\text{s.t. } x^{(i)} = z^{(ij)}, x^{(j)} = z^{(ij)}, \quad \forall v_i\in\mathcal{L}, \forall v_j \in \mathcal{N}_i,\\
&\quad \theta^{(i)} = w^{(ij)}, \theta^{(j)} = w^{(ij)}, \quad \forall v_i\in\mathcal{L}, \forall v_j \in \mathcal{N}_i,\\
&\quad x^{(i)} \geq 0, \quad \forall v_i\in\mathcal{L}.
\end{aligned}
\tag{5}
$$

In equation (5) the optimisation variables are $\{x^{(i)}\}$, $\{\theta^{(i)}\}$, $\{z^{(ij)}\}$, $\{w^{(ij)}\}$ and the local objective function at sensor node v_i is defined as $J_i(x^{(i)},\theta^{(i)}) = x_i^{(i)} + \frac{\lambda}{2}(b_i - \sum_{v_j\in\mathcal{L}} f(d_{ji},\theta^{(i)})x_j^{(i)})^2$. The following proposition indicates that under Assumption 3, i.e., the WSN is bidirectionally connected, equations (2) and (5) are equivalent.

Proposition 2 (Equivalence of equations (2) and (5))**:** *If Assumption 3 holds, i.e., the WSN is bidirectionally connected, then equations* (2) *and* (5) *are equivalent in the sense that the solutions of x and θ to equation* (2) *are also the solutions of $x^{(i)}$ and $\theta^{(i)}$ to equation* (5) *for any $v_i \in \mathcal{L}$, and vise versa.*

Proof: We first show that the solutions of $x^{(i)}$ and $\theta^{(i)}$ to equation (5) should be the same as the solutions of $x^{(j)}$ and $\theta^{(j)}$ to equation (5), for any v_i and v_j. According to Assumption 3 the WSN is bidirectionally connected, and hence there is a path between any v_i and v_j. Since the equality constraints in equation (5) hold for any pair of neighbours along the path, the solutions of any local copies along the path should be the same. Therefore we have $x^{(i)} = \bar{x}$ and $\theta^{(i)} = \bar{\theta}$ for any $v_i \in \mathcal{L}$.

Note that $\bar{x}_i \geq 0$ for any $v_i \in \mathcal{L}$ due to the inequality constraints in (5). Further, $\sum_{v_i\in\mathcal{L}} J_i(\bar{x},\bar{\theta}) = J(\bar{x},\bar{\theta})$. Therefore, $(\bar{x},\bar{\theta})$ is also a minimiser to equation (2). On the other hand, a minimiser to equation (2) is also a part of a minimiser to equation (5). □

After reformulating equations (2)–(5), we separate the optimisation variables as two blocks, $\{x^{(i)}, w^{(ij)}\}$ and $\{\theta^{(i)}, z^{(ij)}\}$ such that the ADMM is applicable. The augmented Lagrangian function of equation (5) is

$$
\begin{aligned}
&h(\{x^{(i)}\},\{\theta^{(i)}\},\{z^{(ij)}\},\{w^{(ij)}\},\{p^{ij}\},\\
&\{q^{ij}\},\{m^{ij}\},\{n^{ij}\}) = \sum_{v_i\in\mathcal{L}} J_i(x^{(i)},\theta^{(i)})\\
&+\sum_{v_i\in\mathcal{L}}\sum_{v_j\in\mathcal{N}_i}\left([x^{(i)}-z^{(ij)}]^T p^{(ij)} + \frac{\tau}{2}\|x^{(i)}-z^{(ij)}\|^2\right)\\
&+\sum_{v_i\in\mathcal{L}}\sum_{v_j\in\mathcal{N}_i}\left([x^{(j)}-z^{(ij)}]^T q^{(ij)} + \frac{\tau}{2}\|x^{(j)}-z^{(ij)}\|^2\right)\\
&+\sum_{v_i\in\mathcal{L}}\sum_{v_j\in\mathcal{N}_i}\left([\theta^{(i)}-w^{(ij)}]^T m^{(ij)} + \frac{\tau}{2}\|\theta^{(i)}-w^{(ij)}\|^2\right)\\
&+\sum_{v_i\in\mathcal{L}}\sum_{v_j\in\mathcal{N}_i}\left([\theta^{(j)}-w^{(ij)}]^T n^{(ij)} + \frac{\tau}{2}\|\theta^{(j)}-w^{(ij)}\|^2\right),
\end{aligned}
\tag{6}
$$

subject to the constraints $x^{(i)} \geq 0, \forall v_i \in \mathcal{L}$. Here $p^{(ij)}, q^{(ij)} \in \mathbb{R}^L$ are Lagrange multipliers attached to the constraints $x^{(i)} = z^{(ij)}$ and $x^{(j)} = z^{(ij)}$, respectively; $m^{(ij)}, n^{(ij)} \in \mathbb{R}$ are Lagrange multipliers attached to the constraints $\theta^{(i)} = w^{(ij)}$ and $\theta^{(j)} = w^{(ij)}$, respectively; τ is a positive stepsize parameter of the ADMM. At iteration $t+1$, the algorithm works as follows.

Step 1. Updates of $\{x^{(i)}, w^{(ij)}\}$. Fixing $\{\theta^{(i)}, z^{(ij)}\}$ as the current solutions and $\{p^{ij}\}, \{q^{ij}\}, \{m^{ij}\}, \{n^{ij}\}$ as the current Lagrange multipliers, the updates of $\{x^{(i)}, w^{(ij)}\}$ are

$$
\begin{aligned}
&\{x^{(i)}(t+1), w^{(ij)}(t+1)\}\\
&= \arg\min h(\{x^{(i)}\},\{\theta^{(i)}(t)\},\{z^{(ij)}(t)\},\{w^{(ij)}\},\\
&\qquad \{p^{ij}(t)\},\{q^{ij}(t)\},\{m^{ij}(t)\},\{n^{ij}(t)\}),\\
&\quad \text{s.t.} \quad x^{(i)} \geq 0, \quad \forall v_i \in \mathcal{L}.
\end{aligned}
\tag{7}
$$

Notice that equation (7) is separable to sensor nodes $\{v_i\}$ and pairs of neighbours $\{(v_i, v_j)\}$. Hence for any sensor node v_i

$$
\begin{aligned}
x^{(i)}(t+1) = \arg\min_{x^{(i)}\geq 0} & J_i(x^{(i)},\theta^{(i)}(t))\\
&+\sum_{v_j\in\mathcal{N}_i}[p^{(ij)}(t)\\
&+\tau\sum_{v_j\in\mathcal{N}_i}\|x^{(i)} - \tfrac{z^{(ij)}(t)+z^{(ji)}(t)}{2}\|^2.
\end{aligned}
\tag{8}
$$

And for any pair of neighbours (v_i, v_j)

$$
\begin{aligned}
w^{(ij)}(t+1) = &\tfrac{1}{2}[\theta^{(i)}(t) + \theta^{(j)}(t)]\\
&+\tfrac{1}{2\tau}[m^{(ij)}(t) + n^{(ij)}(t)].
\end{aligned}
\tag{9}
$$

Step 2. Updates of $\{\theta^{(i)}, z^{(ij)}\}$. Fixing $\{x^{(i)}, w^{(ij)}\}$ as the current solutions and $\{p^{ij}\}, \{q^{ij}\}, \{m^{ij}\}, \{n^{ij}\}$ as the current Lagrange multipliers, the updates of $\{\theta^{(i)}, z^{(ij)}\}$ are

$$
\begin{aligned}
&\{\theta^{(i)}(t+1), z^{(ij)}(t+1)\}\\
&= \arg\min h(\{x^{(i)}(t+1)\},\{\theta^{(i)}\},\{z^{(ij)}\},\\
&\quad \{w^{(ij)}(t+1)\},\{p^{ij}(t)\},\{q^{ij}(t)\},\{m^{ij}(t)\},\{n^{ij}(t)\}).
\end{aligned}
\tag{10}
$$

Again, notice that equation (10) is separable to sensor nodes $\{v_i\}$ and pairs of neighbours $\{(v_i, v_j)\}$. Hence for any sensor node v_i

$$
\begin{aligned}
\theta^{(i)}(t+1) = \arg\min_{\theta^{(i)}} & J_i(x^{(i)}(t+1),\theta^{(i)})\\
&+\sum_{v_j\in\mathcal{N}_i}[m^{(ij)}(t) + n^{(ji)}(t)]^T\theta^{(i)}\\
&+\tau\sum_{v_j\in\mathcal{N}_i}\|\theta^{(i)} - \tfrac{w^{(ij)}(t+1)+w^{(ji)}(t+1)}{2}\|^2.
\end{aligned}
\tag{11}
$$

And for any pair of neighbours (v_i, v_j)

$$
\begin{aligned}
z^{(ij)}(t+1) = &\tfrac{1}{2}[x^{(i)}(t+1) + x^{(j)}(t+1)]\\
&+\tfrac{1}{2\tau}[p^{(ij)}(t) + q^{(ij)}(t)].
\end{aligned}
\tag{12}
$$

Step 3. Updates of $\{p^{ij}\}, \{q^{ij}\}, \{m^{ij}\}, \{n^{ij}\}$. Fixing $\{x^{(i)}, \theta^{(i)}, z^{(ij)}, w^{(ij)}\}$ as the current solutions, for any pair of neighbours (v_i, v_j) its corresponding Lagrange multipliers $p^{ij}, q^{ij}, m^{ij}, n^{ij}$ are updated as

$$
\begin{aligned}
&p^{(ij)}(t+1) = p^{(ij)}(t) + \tau[x^{(i)}(t+1) - z^{(ij)}(t+1)],\\
&q^{(ij)}(t+1) = q^{(ij)}(t) + \tau[x^{(j)}(t+1) - z^{(ij)}(t+1)],\\
&m^{(ij)}(t+1) = m^{(ij)}(t) + \tau[\theta^{(i)}(t+1) - w^{(ij)}(t+1)],\\
&n^{(ij)}(t+1) = n^{(ij)}(t) + \tau[\theta^{(j)}(t+1) - w^{(ij)}(t+1)].
\end{aligned}
\tag{13}
$$

The algorithm proposed above is fully decentralised. At time $t+1$, each sensor node v_i collects $\theta^{(j)}(t)$ and $z^{ji}(t)$ from each of its neighbours v_j, solves $x^{(i)}(t+1)$ according to equation (8), and updates $w^{(ij)}(t+1)$ according to equation (9). Then each sensor node v_i collects $w^{(ji)}(t+1)$ and $x^{(j)}(t+1)$ from each of its neighbours v_j, solves $\theta^{(i)}(t+1)$ according to equation (11), and updates $z^{(ij)}(t+1)$ according to equation (12). Finally each sensor node v_i updates its Lagrange multipliers $p^{ij}(t+1)$, $q^{ij}(t+1)$, $m^{ij}(t+1)$, $n^{ij}(t+1)$ according to equation (13) using its local information.

Since equation (5) is not a convex program, the proposed algorithm cannot guarantee convergence to the global optimal solution. Nevertheless, the convex program (8) is tractable and the nonconvex program (11) can be approximately solved by various single-variable optimisation methods. This nice characteristic endows the proposed algorithm based on the ADMM with satisfactory empirical convergence properties (see the similar applications of the ADMM in nonconvex optimisation, e.g., Boyd et al. (2010); Mardani et al. (2013)), which will be demonstrated through numerical experiments in Section 5.

In the proposed algorithm, at time $t+1$ sensor node v_i needs to collect $\theta^{(j)}(t)$, $z^{(ji)}(t)$, $w^{(ji)}(t+1)$, and $x^{(j)}(t+1)$ from each of its neighbours v_j. In the next subsection, we will show that with proper initialisation the proposed algorithm can be simplified to an equivalent form that reduces 50% of communication cost.

4.2 Decentralised joint event detection and environment perception

Substituting the update of $z^{(ij)}(t+1)$ in equation (12) into the updates of $p^{(ij)}(t+1)$ and $q^{(ij)}(t+1)$ in equation (13), we have

$$\begin{aligned} p^{(ij)}(t+1) &= \frac{1}{2}[p^{(ij)}(t) - q^{(ij)}(t)] \\ &\quad + \frac{\tau}{2}[x^{(i)}(t+1) - x^{(j)}(t+1)], q^{(ij)}(t+1) \\ &= \frac{1}{2}[q^{(ij)}(t) - p^{(ij)}(t)] \\ &\quad + \frac{\tau}{2}[x^{(j)}(t+1) - x^{(i)}(t+1)]. \end{aligned} \tag{14}$$

Apparently, if we initialise $p^{(ij)}(0) = -q^{(ij)}(0) = -p^{(ji)}(0) = q^{(ji)}(0)$, then for $t \geq 0$ we have $p^{(ij)}(t) = -q^{(ij)}(t) = -p^{(ji)}(t) = q^{(ji)}(t)$. Hence The updates of $p^{(ij)}(t+1)$ and $q^{(ji)}(t+1)$ are

$$\begin{aligned} p^{(ij)}(t+1) &= p^{(ij)}(t) + \tfrac{\tau}{2}[x^{(i)}(t+1) - x^{(j)}(t+1)], \\ q^{(ji)}(t+1) &= q^{(ji)}(t) + \tfrac{\tau}{2}[x^{(i)}(t+1) - x^{(j)}(t+1)]. \end{aligned} \tag{15}$$

For v_i, summing the two equations in equation (15) up over all neighbours $v_j \in \mathcal{N}_i$ and defining a new Lagrange multiplier $\alpha^{(i)}(t) = \sum_{v_j \in \mathcal{N}_i}[p^{(ij)}(t) + q^{(ji)}(t)]$, we have

$$\begin{aligned} \alpha^{(i)}(t+1) &= \alpha^{(i)}(t) \\ &\quad + \tau \sum_{v_j \in \mathcal{N}_i}[x^{(i)}(t+1) - x^{(j)}(t+1)]. \end{aligned} \tag{16}$$

Applying the equality $p^{(ij)}(t) = -q^{(ij)}(t)$ in equation (12) leads to $z^{(ij)}(t+1) = \frac{1}{2}[x^{(i)}(t+1) + x^{(j)}(t+1)]$. If we further initialise $z^{(ij)}(0) = \frac{1}{2}[x^{(i)}(0) + x^{(j)}(0)]$, then for $t \geq 0$ we have $z^{(ij)}(t) = \frac{1}{2}[x^{(i)}(t) + x^{(j)}(t)]$.

Substituting $\alpha^{(i)}(t) = \sum_{v_j \in \mathcal{N}_i}[p^{(ij)}(t) + q^{(ji)}(t)]$ and $z^{(ij)}(t) = \frac{1}{2}[x^{(i)}(t) + x^{(j)}(t)]$ into equation (8), the update can be equivalently rewritten as

$$\begin{aligned} x^{(i)}(t+1) &= \arg\min_{x^{(i)} \geq 0} J_i(x^{(i)}, \theta^{(i)}(t)) \\ &\quad + [\alpha^{(i)}(t)]^T x^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|x^{(i)} - \tfrac{x^{(i)}(t) + x^{(j)}(t)}{2}\|^2. \end{aligned} \tag{17}$$

On the other hand, summing up the updates of $m^{(ij)}(t+1)$ and $n^{(ij)}(t+1)$ in equation (13), we have

$$\begin{aligned} m^{(ij)}(t+1) + n^{(ij)}(t+1) &= m^{(ij)}(t) + n^{(ij)}(t) \\ &\quad + \tau[\theta^{(i)}(t+1) + \theta^{(j)}(t+1)] - 2\tau w^{(ij)}(t+1). \end{aligned} \tag{18}$$

Substituting the update of $w^{(ij)}(t+1)$ in equations (9), (18) can be simplified to

$$\begin{aligned} & m^{(ij)}(t+1) + n^{(ij)}(t+1) \\ &= \tau[\theta^{(i)}(t+1) + \theta^{(j)}(t+1)] - \tau[\theta^{(i)}(t) + \theta^{(j)}(t)]. \end{aligned} \tag{19}$$

If we initialise $m^{(ij)}(0) + n^{(ij)}(0) = \tau[\theta^{(i)}(0) + \theta^{(j)}(0)] - \tau[\theta^{(i)}(-1) + \theta^{(j)}(-1)]$, then for $t \geq 0$

$$\begin{aligned} & m^{(ij)}(t) + n^{(ij)}(t) \\ &= \tau[\theta^{(i)}(t) + \theta^{(j)}(t)] - \tau[\theta^{(i)}(t-1) + \theta^{(j)}(t-1)]. \end{aligned} \tag{20}$$

Substituting equation (20) into equation (9), we have

$$\begin{aligned} w^{(ij)}(t+1) &= [\theta^{(i)}(t) + \theta^{(j)}(t)] \\ &\quad - \tfrac{1}{2}[\theta^{(i)}(t-1) + \theta^{(j)}(t-1)], \end{aligned} \tag{21}$$

and hence

$$\begin{aligned} & w^{(ij)}(t+1) + w^{(ji)}(t+1) \\ &= 2[\theta^{(i)}(t) + \theta^{(j)}(t)] - [\theta^{(i)}(t-1) + \theta^{(j)}(t-1)]. \end{aligned} \tag{22}$$

Defining a new Lagrange multiplier $\beta^{(i)}(t) = \sum_{v_j \in \mathcal{N}_i}[m^{(ij)}(t) + n^{(ji)}(t)]$, from equation (13) we know

$$\begin{aligned} & \beta^{(i)}(t+1) = \beta^{(i)}(t) \\ & + \tau \sum_{v_j \in \mathcal{N}_i}[2\theta^{(i)}(t+1) - w^{(ij)}(t+1) - w^{(ji)}(t+1)]. \end{aligned} \tag{23}$$

Substituting equation (22) into equation (23) yields

$$\begin{aligned} \beta^{(i)}(t+1) &= \beta^{(i)}(t) + \tau \sum_{v_j \in \mathcal{N}_i}[2\theta^{(i)}(t+1) - 2\theta^{(i)}(t) \\ &\quad - 2\theta^{(j)}(t) + \theta^{(i)}(t-1) + \theta^{(j)}(t-1)]. \end{aligned} \tag{24}$$

Substituting $\beta^{(i)}(t)$ and equation (22) into equation (11), the update can be equivalently rewritten as

$$\begin{aligned} \theta^{(i)}(t+1) &= \arg\min_{\theta^{(i)}} J_i(x^{(i)}(t+1), \theta^{(i)}) + [\beta^{(i)}(t)]^T \theta^{(i)} \\ &\quad + \tau \sum_{v_j \in \mathcal{N}_i} \|\theta^{(i)} - \theta^{(i)}(t) - \theta^{(j)}(t) \\ &\quad + \frac{\theta^{(i)}(t-1) + \theta^{(j)}(t-1)}{2}\|^2. \end{aligned} \tag{25}$$

In summary, the simplified algorithm works as follows. At time $t+1$, sensor node v_i collects $x^{(j)}(t)$ and $\theta^{(j)}(t)$ from each of its neighbours v_j. Upon receiving all of them, v_i updates $x^{(i)}(t+1)$ and $\theta^{(i)}(t+1)$ according to equations (17) and (25), respectively. Then v_i updates $\alpha^{(i)}(t+1)$ and $\beta^{(i)}(t+1)$ according to equations (16) and (24), respectively. The algorithm is decentralised and equivalent to the one developed in the previous subsection given initialisation $p^{(ij)}(0) = -q^{(ij)}(0) = -p^{(ji)}(0) = q^{(ji)}(0)$, $z^{(ij)}(0) = \frac{1}{2}[x^{(i)}(0) + x^{(j)}(0)]$, and $m^{(ij)}(0) + n^{(ij)}(0) = \tau[\theta^{(i)}(0) + \theta^{(j)}(0)] - \tau[\theta^{(i)}(-1) + \theta^{(j)}(-1)]$, while reduces 50% communication cost since transmission of $\{z^{ji}\}$ and $\{w^{(ji)}\}$ is no longer needed. We name the algorithm as decentralised event detection and environment perception (DEEP) that is outlined in Algorithm 2.

Algorithm 2 Decentralized Event detection and Environment Perception (DEEP) at v_i

---

Require: Measurement b_i, parameter λ, attenuation function $f(d,\theta)$, neighbor set $\mathcal{N}_i$, distance d_{ji} for all $v_j \in \mathcal{N}_i$.
Require: Stepsize τ.
1: Initialize $x^{(i)}(0) = 0$, $\theta^{(i)}(0) = 0$, $\theta^{(i)}(-1) = 0$, $\alpha^{(i)}(0) = 0$, and $\beta^{(i)}(0) = 0$.
2: Initialize $x^{(j)}(0) = 0$, $\theta^{(j)}(0) = 0$, and $\theta^{(j)}(-1) = 0$ for all $v_j \in \mathcal{N}_i$.
3: **for** $t = 0, 1, 2, \ldots$ **do**
4: Update $x^{(i)}(t+1)$ via (17)
$$x^{(i)}(t+1) = \arg\min_{x^{(i)} \geq 0} J_i(x^{(i)}, \theta^{(i)}(t)) + [\alpha^{(i)}(t)]^T x^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|x^{(i)} - \tfrac{x^{(i)}(t)+x^{(j)}(t)}{2}\|^2. \quad (26)$$
5: Update $\theta^{(i)}(t+1)$ via (25)
$$\theta^{(i)}(t+1) = \arg\min_{\theta^{(i)}} J_i(x^{(i)}(t+1), \theta^{(i)}) + [\beta^{(i)}(t)]^T \theta^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|\theta^{(i)} - \theta^{(i)}(t) - \theta^{(j)}(t) + \tfrac{\theta^{(i)}(t-1)+\theta^{(j)}(t-1)}{2}\|^2. \quad (27)$$
6: Transmit $x^{(i)}(t+1)$ and $\theta^{(i)}(t+1)$ to and receive $x^{(j)}(t+1)$ and $\theta^{(j)}(t+1)$ from all neighbors $v_j \in \mathcal{N}_i$.
7: Update $\alpha^{(i)}(t+1)$ via (16)
$$\alpha^{(i)}(t+1) = \alpha^{(i)}(t) + \tau \sum_{v_j \in \mathcal{N}_i} [x^{(i)}(t+1) - x^{(j)}(t+1)]. \quad (28)$$
8: Update $\beta^{(i)}(t+1)$ via (24)
$$\beta^{(i)}(t+1) = \beta^{(i)}(t) + \tau \sum_{v_j \in \mathcal{N}_i} [2\theta^{(i)}(t+1) - 2\theta^{(i)}(t) - 2\theta^{(j)}(t) + \theta^{(i)}(t-1) + \theta^{(j)}(t-1)]. \quad (29)$$
9: **end for**
10: Return $x^{(i)}(t+1)$ and $\theta^{(i)}(t+1)$.

4.3 *Heuristic acceleration*

To accelerate the DEEP algorithm and mitigate the communication cost, next we introduce two heuristic acceleration techniques.

The first technique comes from the observation that if the measurement b_i of sensor node v_i is small, it is very likely that there is no event occurring at the sensor point v_i. To see so, recall that according to Assumption 1, the attenuation function $f(d,\theta)$ is monotonically decreasing with respect to the distance d as well as normalised by $f(0,\theta) = 1$ and $f(\infty, \theta) = 0$. If there is a significant event occurring at the sensor point v_i such that x_i is large enough, then $b_i = \sum_{v_j \in \mathcal{L}} f(d_{ji}, \theta) x_j \geq x_i$ in the noise-free case. Unless the random noise is large enough to cancel the effect of the event, b_i should be large enough. This fact motivates us to simply set $x_i = 0$ during the optimisation process if b_i is smaller than a certain threshold ϵ. Through truncating the intermediate estimates of x and force those small elements to be zero, the algorithm can concentrate on those large elements that are likely corresponding to the true events.

Algorithm 3 Accelerated Decentralized Event detection and Environment Perception (ADEEP) at v_i

---

Require: Measurement b_i, parameter λ, attenuation function $f(d,\theta)$, neighbor set $\mathcal{N}_i$, distance d_{ji} for all $v_j \in \mathcal{N}_i$.
Require: Stepsize τ, threshold ϵ, ratio ρ.
1: Initialize $x^{(i)}(0) = 0$, $\theta^{(i)}(0) = \hat{\theta}^{(i)}(0) = \theta^{(i)}(-1) = 0$, $\alpha^{(i)}(0) = 0$, $\beta^{(i)}(0) = 0$, $r^{(i)}(0) = 0$, and $\eta^{(i)}(0) = 0$.
2: Initialize $x^{(j)}(0) = 0$, $\theta^{(j)}(0) = 0$, and $\theta^{(j)}(-1) = 0$ for all $v_j \in \mathcal{N}_i$.
3: **for** $t = 0, 1, 2, \ldots$ **do**
4: If $b_i \geq \epsilon$, update $x^{(i)}(t+1)$ via
$$x^{(i)}(t+1) = \arg\min_{x^{(i)} \geq 0} J_i(x^{(i)}, \hat{\theta}^{(i)}(t)) + [\alpha^{(i)}(t)]^T x^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|x^{(i)} - \tfrac{x^{(i)}(t)+x^{(j)}(t)}{2}\|^2, \quad (30)$$
otherwise
$$x^{(i)}(t+1) = \arg\min_{x^{(i)} \geq 0, x_i^{(i)} = 0} J_i(x^{(i)}, \hat{\theta}^{(i)}(t)) + [\alpha^{(i)}(t)]^T x^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|x^{(i)} - \tfrac{x^{(i)}(t)+x^{(j)}(t)}{2}\|^2. \quad (31)$$
5: Update $\theta^{(i)}(t+1)$ via (25)
$$\theta^{(i)}(t+1) = \arg\min_{\theta^{(i)}} J_i(x^{(i)}(t+1), \theta^{(i)}) + [\beta^{(i)}(t)]^T \theta^{(i)} + \tau \sum_{v_j \in \mathcal{N}_i} \|\theta^{(i)} - \theta^{(i)}(t) - \theta^{(j)}(t) + \tfrac{\theta^{(i)}(t-1)+\theta^{(j)}(t-1)}{2}\|^2. \quad (32)$$
6: Transmit $x^{(i)}(t+1)$ and $\theta^{(i)}(t+1)$ to and receive $x^{(j)}(t+1)$ and $\theta^{(j)}(t+1)$ from all neighbors $v_j \in \mathcal{N}_i$.
7: Update $\alpha^{(i)}(t+1)$ via (16)
$$\alpha^{(i)}(t+1) = \alpha^{(i)}(t) + \tau \sum_{v_j \in \mathcal{N}_i} [x^{(i)}(t+1) - x^{(j)}(t+1)]. \quad (33)$$
8: Update $\beta^{(i)}(t+1)$ via (24)
$$\beta^{(i)}(t+1) = \beta^{(i)}(t) + \tau \sum_{v_j \in \mathcal{N}_i} [2\theta^{(i)}(t+1) - 2\theta^{(i)}(t) - 2\theta^{(j)}(t) + \theta^{(i)}(t-1) + \theta^{(j)}(t-1)]. \quad (34)$$
9: Calculate primal residual $r^{(i)}(t+1) = \|\theta^{(i)}(t+1) - \hat{\theta}^{(i)}(t)\|$.
10: **if** $r^{(i)}(t+1) < \rho r^{(i)}(t)$ **then**
11: $\eta^{(i)}(t+1) = \frac{1+\sqrt{1+4(\eta^{(i)}(t))^2}}{2}$ and $\hat{\theta}^{(i)}(t+1) = \theta^{(i)}(t+1) + \frac{\eta(t)-1}{\eta(t+1)}[\theta^{(i)}(t+1) - \theta^{(i)}(t)]$.
12: **else**
13: $\eta^{(i)}(t+1) = 1$ and $\hat{\theta}^{(i)}(t+1) = \theta^{(i)}(t+1)$.
14: **end if**
15: **end for**
16: Return $x^{(i)}(t+1)$ and $\theta^{(i)}(t+1)$.

This heuristic acceleration technique can be viewed as using support detection to enhance the performance of a sparse optimisation algorithm. To solve a sparse optimisation problem, the key issue is to find the nonzero locations of the optimal solution, defined as supports. Once the supports are correctly detected, the algorithm reduces to solving a low-dimensional ordinary nonlinear least squares problem and converges much faster.

The second technique is to use Nesterov acceleration to expedite the ADMM. Specifically, for each sensor node v_i define an acceleration factor $\eta^{(i)}$ that updates via

$$\eta^{(i)}(t) = \tfrac{1+\sqrt{1+4(\eta^{(i)}(t-1))^2}}{2}. \tag{35}$$

Then in the $x^{(i)}$-update equation (17), replace $\theta^{(i)}(t)$ by the accelerated one

$$\hat{\theta}^{(i)}(t) = \theta^{(i)}(t) + \frac{\eta^{(i)}(t-1)-1}{\eta^{(i)}(t)}[\theta^{(i)}(t) - \theta^{(i)}(t-1)], \tag{36}$$

if the current primal residual reduces sufficiently from the previous value. This acceleration technique is motivated by goldstein et al. (2012) that accelerates on both the primal and dual variables. In our algorithm we only accelerate on the primal variables $\theta^{(i)}$, because through preliminary numerical experiments we find that for our specific optimisation problem accelerating on the dual variables may lead to numerical instability. Our conjecture is that the joint event detection and environment perception problem is nonconvex, and is thus sensitive to the dual acceleration strategy.

The algorithm using the acceleration techniques, named as accelerated decentralised event detection and environment perception (ADEEP), is shown in Algorithm 3. The modifications alleviate the interference of those false supports to the optimisation process and accelerate on the primal variables. The effectiveness of the acceleration techniques will be demonstrated in the numerical experiments.

5 Numerical experiments

In the numerical experiments we assume that $L = 100$ sensor nodes are deployed as a 10×10 grid in a two dimensional sensing area $[1, 10] \times [1, 10]$. Four events occur at points $(1, 10)$, $(2, 2)$, $(2, 5)$ and $(4, 6)$ with amplitudes 2, 2, 1 and 2, respectively. The influence function $f(d, \theta) = \exp(-\theta d^2)$. In the joint event detection and environment perception problem (2), we choose the weight parameter $\lambda = 100$ that balances the sparsity of the events and the estimation error.

We compare the following four algorithms in the numerical experiments:

1 CEEP that is introduced in Section 3

2 DEEP that is introduced in Section 4.2 with $\tau = 1$

3 ADEEP that is introduced in Section 4.3 with $\tau = 1$, $\epsilon = 0.1$, and $\rho = 0.99$

4 D-Lasso that is proposed in Bazerque and Giannakis (2010).

Here CEEP is centralised, while DEEP, ADEEP, and D-Lasso are decentralised. CEEP, DEEP, and ADEEP are joint event detection and environment perception algorithms, in which the one-dimensional environment perception subproblems are approximately solved by the 0.618 method (Miller, 2000). D-Lasso assumes that the attenuation coefficient θ is known in advance.

Next we briefly discuss energy consumption of the decentralised algorithms. At each iteration of DEEP and ADEEP, sensor node v_i broadcasts an L-dimensional vector $x^{(i)}$ and a scalar $\theta^{(i)}$ to and receives an L-dimensional vector $x^{(j)}$ and a scalar $\theta^{(j)}$ from all neighbours $v_j \in \mathcal{N}_i$. Suppose that powers of broadcasting and receiving a scalar are p_b and p_r, respectively. Therefore, the energy consumption of sensor node v_i is $(L+1)\left[p_b + p_r|\mathcal{N}_i|\right]$ and the energy consumption of the overall network is $(L+1)\sum_{v_i \in \mathcal{L}} \left[p_b + p_r|\mathcal{N}_i|\right]$ per iteration. D-Lasso assumes that the attenuation coefficient θ is fixed and not optimised, hence the energy consumption of the overall network is $L\sum_{v_i \in \mathcal{L}} \left[p_b + p_r|\mathcal{N}_i|\right]$ per iteration.

The numerical experiments use root mean square error (RMSE) of event detection as the performance criterion. Suppose that the true value of the signal is x^o, the centralised estimate is x, and the decentralised estimate of sensor node v_i is $x^{(i)}$. RMSE is defined as $\|x - x^o\|_2$ or $\frac{1}{L}\sum_{v_i \in \mathcal{L}} \|x^{(i)} - x^o\|_2$. We consider two different cases for the measurements, one is noise-free and another is noise-polluted.

5.1 The noise-free case

Consider the noise-free case first. Set the attenuation coefficient $\theta = 2$ that means fast decay of signal strength. We suppose that D-Lasso has the correct knowledge of θ. Figure 1 compares the four algorithms with respect to the RMSE of event detection. It is interesting that the three joint event detection and environment perception algorithms achieve the same RMSE as that of D-Lasso, which exactly knows the attenuation coefficient θ in advance. The centralised algorithm CEEP converges to a stationary point quickly. The decentralised algorithm DEEP converges to the same stationary point within 500 iterations. Its accelerated version ADEEP reaches the same RMSE within 400 iterations. The observation shows that the acceleration techniques are effective.

We also demonstrate the process of environment perception in Figure 2 for CEEP, DEEP, and ADEEP. In DEEP and ADEEP, since each sensor node holds a local estimate of θ, we choose one random local estimate as an representative. The estimates of θ all converge to the true value and the convergence time is the same as that of event detection.

Now we impose variation to the environment such that the true attenuation coefficient θ is changed to 1, which means that signal strength decays slower. D-Lasso still assumes the true value of θ is 2. As shown in Figure 3, D-Lasso fails because the prior knowledge of θ is wrong. As a comparison, the three joint event detection and environment perception algorithms CEEP, DEEP, and ADEEP work well due to the assistance of the environment perception step. DEEP and ADEEP converge much slower than those in Figure 1. The reason is that if the true θ is smaller, then signal strength decays slower and thus each event couples more sensor nodes. Therefore, more sensor nodes must reach consensus on the estimates of the events in the algorithm.

Figure 1 RMSE with respect to event detection in CEEP, DEEP, ADEEP, and D-Lasso. The measurements are noise-free. The true attenuation coefficient $\theta = 2$, which is exactly known by D-Lasso in advance (see online version for colours)

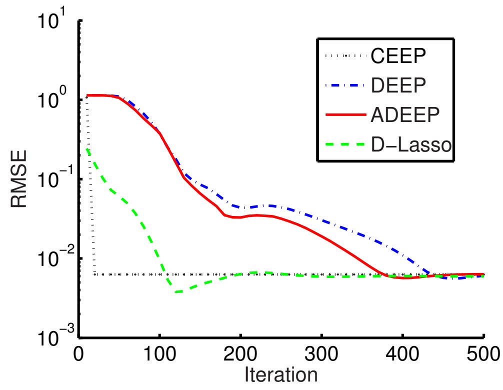


Figure 2 Estimate of θ for environment perception in CEEP, DEEP, and ADEEP. The measurements are noise-free. The true attenuation coefficient $\theta = 2$ (see online version for colours)

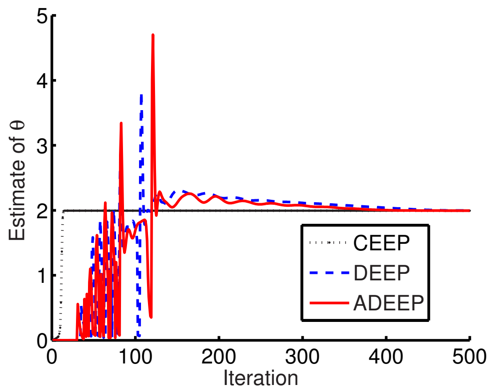
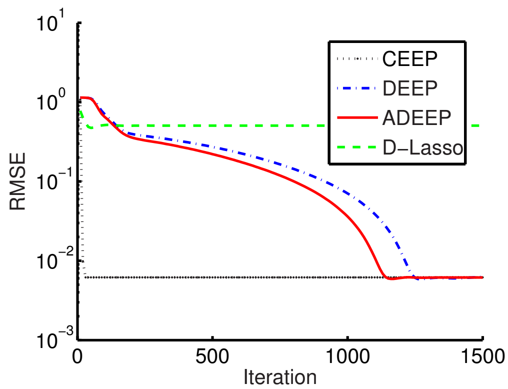


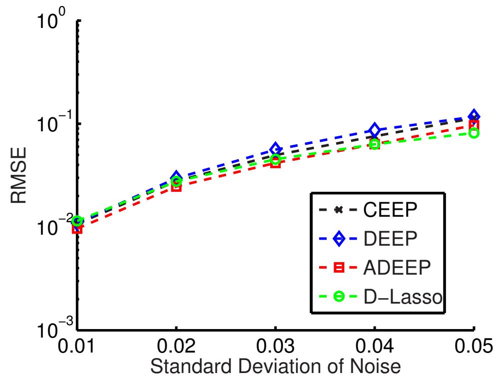
Figure 3 RMSE with respect to event detection in CEEP, DEEP, ADEEP, and D-Lasso. The measurements are noise-free. The true attenuation coefficient $\theta = 1$ while D-Lasso assumes its value is 2 (see online version for colours)



5.2 *The noise-polluted case*

Second we consider the noise-polluted case. Suppose that the measurements are polluted by the Gaussian noise with different standard deviations. Set the attenuation coefficient $\theta = 2$ that is exactly known by D-Lasso in advance. Figure 4 illustrates the RMSE with respect to event detection for the four algorithms. The algorithms all demonstrate robustness to the measurement noise.

Figure 4 RMSE with respect to event detection in CEEP, DEEP, ADEEP, and D-Lasso. The measurements are polluted by the Gaussian noise with different standard deviations. The true attenuation coefficient $\theta = 2$ that is known by D-Lasso in advance (see online version for colours)



6 Conclusion

This paper addresses the joint event detection and environment perception problem in a WSN. Through exploiting the sparse nature of the events, we propose an ℓ_1-norm regularised least squares formulation that automatically estimates the number of the events as well as their locations and magnitudes; the attenuation coefficient of the environment is also an optimisation variable in the formulation. We develop a decentralised algorithm based on the ADMM. Through exploiting structures of the problem, the decentralised algorithm in each node boils down to three steps: an event detection step that is a convex program; an environment perception step that is a nonconvex one-dimensional optimisation problem; a multiplier update step that contains only algebraic operations. To improve energy-efficiency of the WSN, we further develop a heuristic scheme that accelerates the proposed decentralised algorithm. Numerical experiments demonstrate the effectiveness of the decentralised joint event detection and environment perception algorithms.

In DEEP and ADEEP, we assume that the communication among sensor nodes is synchronised through implementing existing synchronisation techniques (Ye et al., 2008). Though synchronous optimisation is common in designing decentralised algorithms, asynchronous optimisation has attracted increasing research interest in recent years (Nedic, 2011). One of our future research directions is to develop asynchronous joint event detection and environment perception algorithms in decentralised networks.

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