Semiconductor Superlattice Theory and Application

Introduction

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Superlattice is a periodic structure of layers of two or more materials. Typically the width of layers is orders of magnitude larger than the lattice constant, and is limited by the growth of the structure[1]. As shown in the figure below, it is a superlattice formed by alternating AlAs and GaAs layers. Because they have the similar lattice constant and different bandgap.

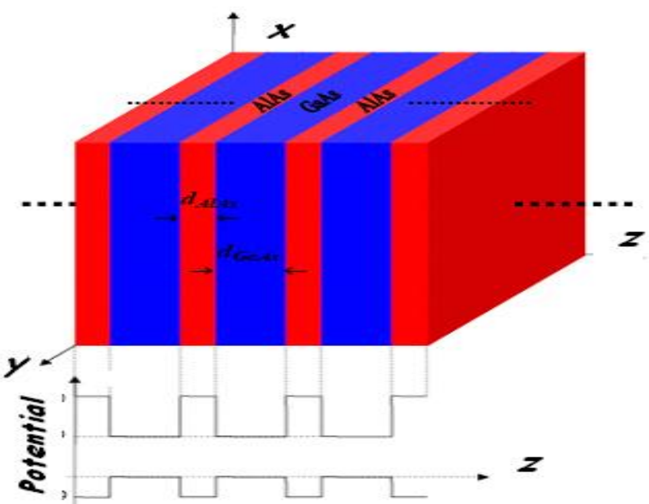


Fig. 1

Usually there are two different ways of forming superlattice structure[2]: periodic variation of donor or acceptor impurities, periodic variation of alloy composition introduced during the crystal growth. In the first method, the electrons and holes are confined in different locations, thus reducing the recombination possibility. However, this method has large thermal diffusion of impurities, so it is hard to maintain the periodic potential profile. So the second method is more popular.

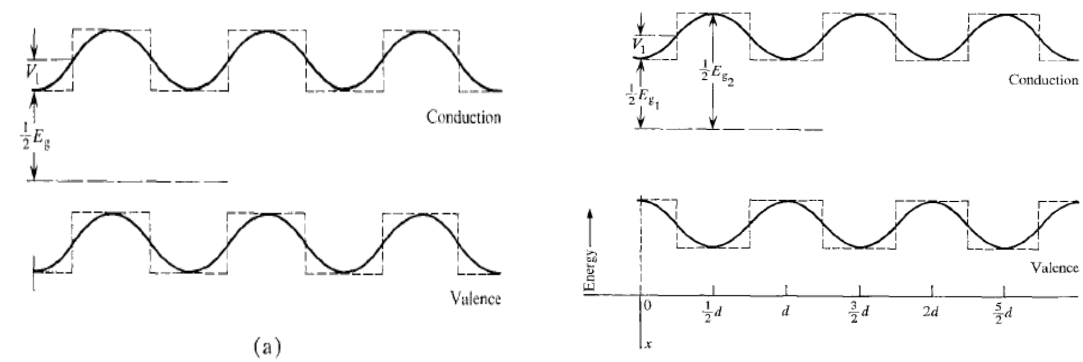


Fig. 2

It is worthy to make a difference between superlattice and multiple quantum well because they are very similar in their structure except that unlike the multiple quantum well, the superlattice barrier width is small enough that the different quantum wells are coupled with each other. As shown in the Fig. 3[5]

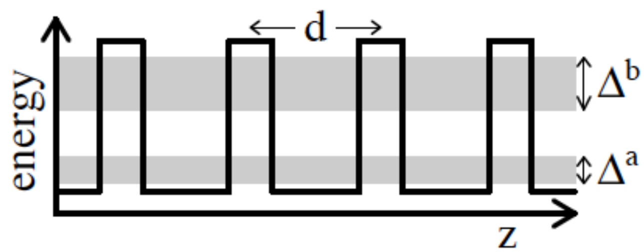


Fig. 3

In contrast, the quantum wells in multiple quantum well structure are highly localized. This difference will result in the difference in their carrier conduction mechanisms.

Superlattice Schrodinger Equation

For the square potential well, we could apply the Kronig-Penney model[3] to solve for the Schrodinger Equation (SE). And the sinusoidal potential, the wave function has the form of Mathieu's equation[4]. Here for simplicity, the Kronig-Penney model is considered.

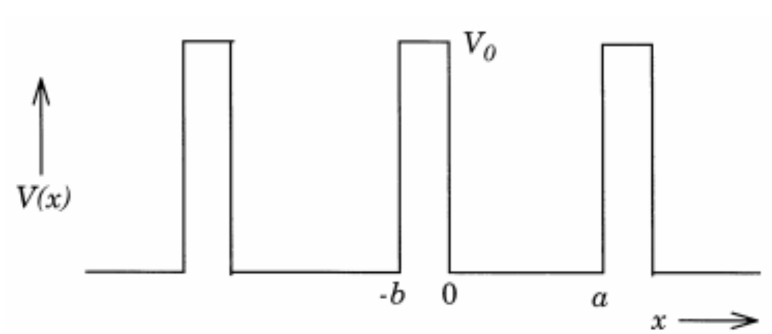


Fig. 4

Figure 4 shows the periodic potential with period $d = a + b$. It has the form

$$V(x) = \begin{cases} 0 & -b < x < 0 \\ V_o & 0 < x < a \end{cases}$$

So that the SE could be written

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad 0 < x < a$$
$$\frac{d^2\psi}{dx^2} + \frac{2m(E - V_o)}{\hbar^2}\psi = 0 \quad -b < x < a$$

Then the solutions would be

$$\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x} \quad 0 < x < a$$
$$\psi_2 = Ce^{\beta x} + De^{-\beta x} \quad -b < x < 0$$
$$\alpha = \frac{\sqrt{2mE}}{\hbar} \qquad \beta = \frac{\sqrt{2m(V_o - E)}}{\hbar}$$

Applying the Bloch theorem

$$\psi(x + d) = e^{ikd}\psi(x) \quad d = a + b$$

and the boundary condition

$$\psi_1(0) = \psi_2(0) \qquad \psi_1^{'}(0) = \psi_2^{'}(0)$$
$$\psi_1(a) = e^{ikd}\psi_2(-b) \quad \psi_1^{'}(a) = e^{ikd}\psi_2^{'}(-b)$$

we can get the final equation

$$\frac{\beta^2 - \alpha^2}{2\alpha\beta}\sinh(\beta b)\sin(\alpha a) - \cosh(\beta b)\cos(\alpha a) = \cos(k(a + b)) \tag{1}$$

Plotting the two sides of the equation,

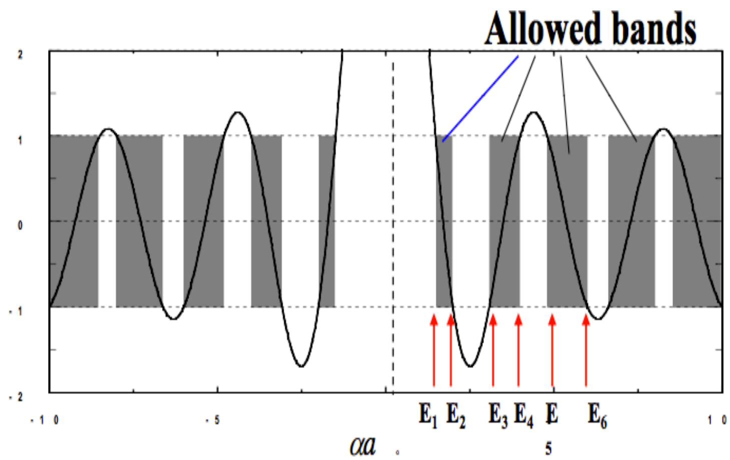


Fig. 5

The solutions suggest discrete energy bands separated by the forbidden bands just as the conventional crystal structure energy band. But the difference of superlattice and conventional crystal is that it is a man-made material that the energy band could be well controlled by the selected layers and with of different layers. Because of their thick alternating layers compared with the lattice constant, the energy bands form in the first brillouin zone of the well. As shown in the Fig. 6[2]

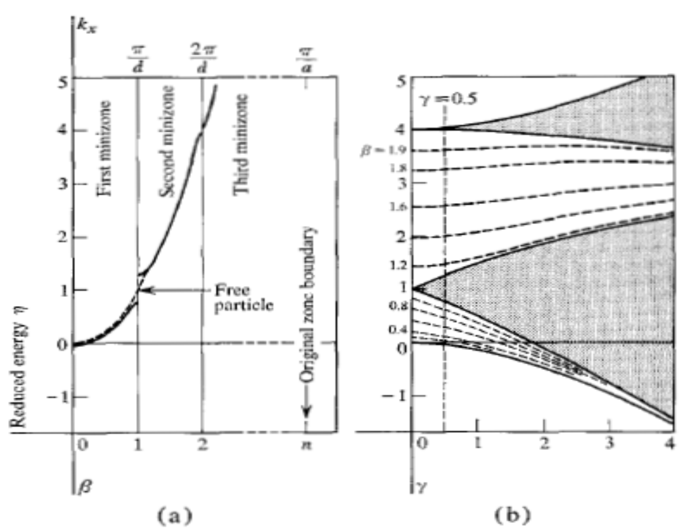


Fig. 6

The first brillouin zone boundary is $\frac{\pi}{a}$, and is subdivided into many minibands. And these minibands is the key feature of superlattice.

Superlattice Carrier Transport

In this part, the carrier transport in superlattice is considered. There are three types of transportation[5]: the miniband conduction, the Wannier-Stark hopping and the resonant sequential tunneling as shown in Fig. 7.

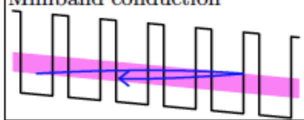
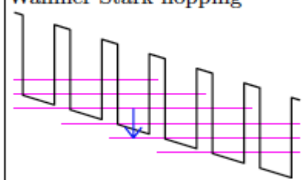
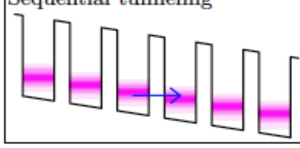
	coupling T_1	voltage drop eFd	scattering $\Gamma = \hbar/\tau$
Miniband conduction 	exact: miniband	acceleration	golden rule
Wannier-Stark hopping 	exact: Wannier Stark states		golden rule
Sequential tunneling 	lowest order	energy mismatch	"exact" spectral function

Fig. 7

Miniband Conduction

The equations of motion are[2]

$$\hbar \frac{dk_x}{dt} = eF, \; v_x = \frac{1}{\hbar}\frac{\partial E_x}{\partial k_x} \tag{2}$$

Assuming the superlattice grows in the x-direction. And the increment of velocity in a time interval would be

$$dv_x = \frac{eF}{\hbar^2}\frac{\partial^2 E_x}{\partial k_x^2} dt$$

Taking the scattering time into consideration

$$\begin{aligned} v_d &= \int_0^\infty \exp(-t/\tau) dv_x \\ &= \frac{eF}{\hbar^2}\int_0^\infty \frac{\partial^2 E_x}{\partial k_x^2}\exp(-t/\tau)dt \end{aligned}$$

Define $\gamma = \frac{V_o}{E_o}$, $E_o = \frac{\hbar^2 k_d^2}{2m}$, $k_d = \frac{\pi}{d}$, then

For large γ, the E-k relationship is approximately sinusoidal $E_x \approx \frac{E_1}{2}\left(1-\cos(k_x d)\right)$, and for small γ, the approximation is on longer valid and is approximated by two parabolas of opposite curvature, joined at the inflection point (E_i, k_i).

According to these approximations we could get

$$v_d = \begin{cases} g(\xi)\dfrac{\hbar k_d}{m(0)} & \text{small } \gamma, g(\xi) = \dfrac{\xi}{1+\pi^2\xi^2} \quad \xi = \dfrac{eF\tau}{\hbar k_d} \\ f(\xi)\dfrac{\hbar k_d}{m(0)} & \text{large } \gamma, f(\xi) = \xi\left[1 + \dfrac{2k_d}{k_d - k_i}\dfrac{\sinh(k_i / k_d\xi)}{\exp(2/\xi)-1} - \dfrac{k_d}{k_d-k_i}\exp(-k_i / k_d\xi)\right] \end{cases} \tag{3}$$

Plotting the magnitude function $g(\xi), f(\xi)$ in figure 8.

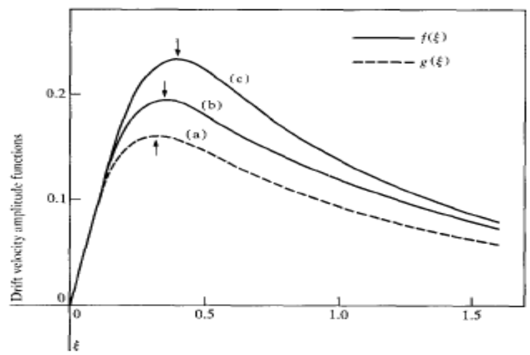


Fig. 8

There is an interesting behavior here, which shows that at certain electric field, the drift velocity reaches the maximum, and then decreases with the increasing electric field. Because the drift velocity is directly proportional to the current, the miniband transport shows the negative differential conductance.

Sequential Resonant Tunneling

In sequential resonant tunneling[6], the wave functions of different quantum wells are weakly coupled that the miniband transport is not applicable. One schematic illustration is shown in Fig. 9.

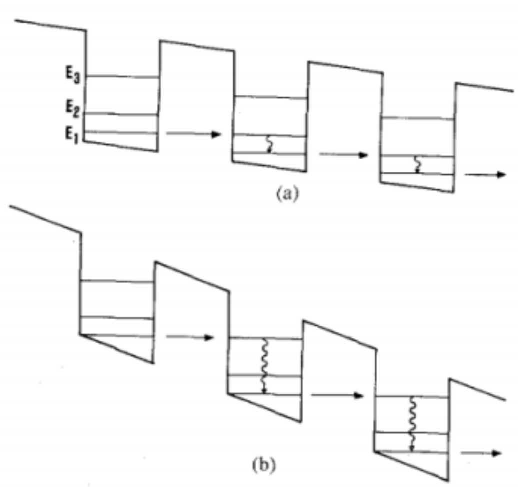


Fig. 9

If the applied electric field is just to make the E_1 level in the nth quantum well matches the E_2 level in the $n+1$th quantum well, then the resonant sequential tunneling reaches a maximum. If the applied electric field is increased further, that the E_1 level in the nth quantum well matches the E_3 level in the $n+1$th quantum well, then the tunneling reaches another maximum. These process could be shown in Fig. 10

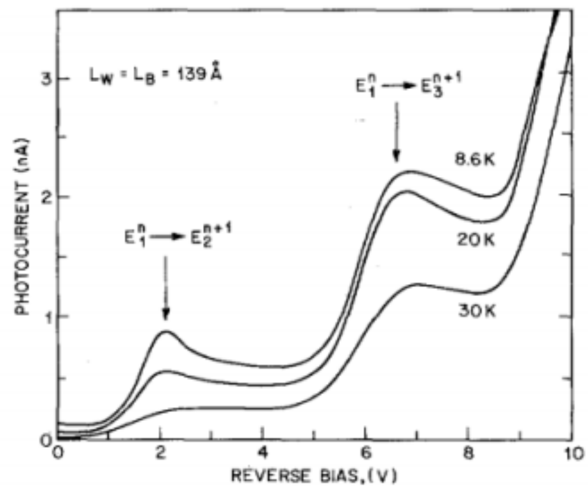


Fig. 10

The photocurrent reaches maximum when the miniband levels match.

Superlattice Application

Superlattice has many applications. Here only two applications are presented: the Bloch Oscillator and the Quantum Cascade Laser.

Bloch Oscillator

The equation of motion is presented in equation (2), suppose tight bonding model, the E-k relationship is sinusoidal

$$E = A\cos ak \Rightarrow v(k) = -\frac{Aa}{\hbar}\sin ak$$

Then

$$x(t) = \int v(k(t))dt = -\frac{A}{eF}\cos\left(\frac{aeF}{\hbar}t\right)$$

it shows that the position is oscillating at a period

$$T = \frac{2\pi\hbar}{aeF}$$

where a is the lattice constant. Before the superlattice, because a is very small, so that the period is large compared with the scattering time, so that it is hard to observe the Bloch Oscillation. Now with superlattice, the periodic potential period d could be made relatively big, then the oscillation period is smaller than the scattering time, so the Bloch Oscillator is possible. If we have $d = 100\overset{o}{A}, F = 10^3 V/cm$, then the oscillation frequency is $250GHz$. That is in the THz range. So superlattice could also be used as THz source generator.

Quantum Cascade Laser

Another application of superlattice is the Quantum Cascade Laser[7]. It is different than the traditional laser in that it uses the intrasubband transitions instead of interband transitions used in the traditional lasers. So the quantum cascade laser is usually used in mid- to far-infrared portion of the electromagnetic spectrum. Figure 11 illustrates this process

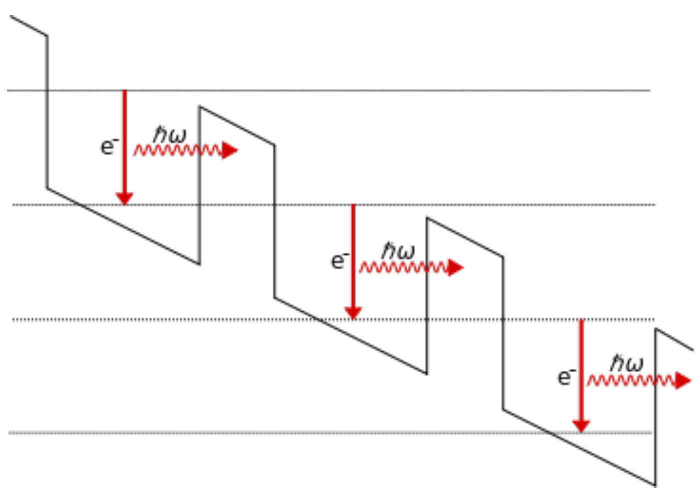


Fig. 11

In Fig. 11, it shows the intrasubband transitions. And another important property is that in the intrasubband transitions, the carriers are not recombined and disappear, instead they still reside in the quantum well and could be further transported.

Conclusion

In this project, the superlattice is reviewed. Two different formation methods are introduced and comparison is made between them. Morever, the Kronig-Penney model is applied to solve the Schrodinger Equation, and we get the minibands in the original first Brillouin Zone. And three different carrier transport mechanisms are introduced. And the minibands transport and sequential resonant tunneling are examined in detail. Interestingly, the negative differential conductance behavior is found. Lastly, two applications of superlattice are presented. The Bloch Oscillator is very important, and so is the quantum cascade laser.

Reference

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[7] Wikipedia:http://en.wikipedia.org/wiki/Quantum_cascade_laser