Time-reversal acoustics, virtual source imaging, and seismic interferometry

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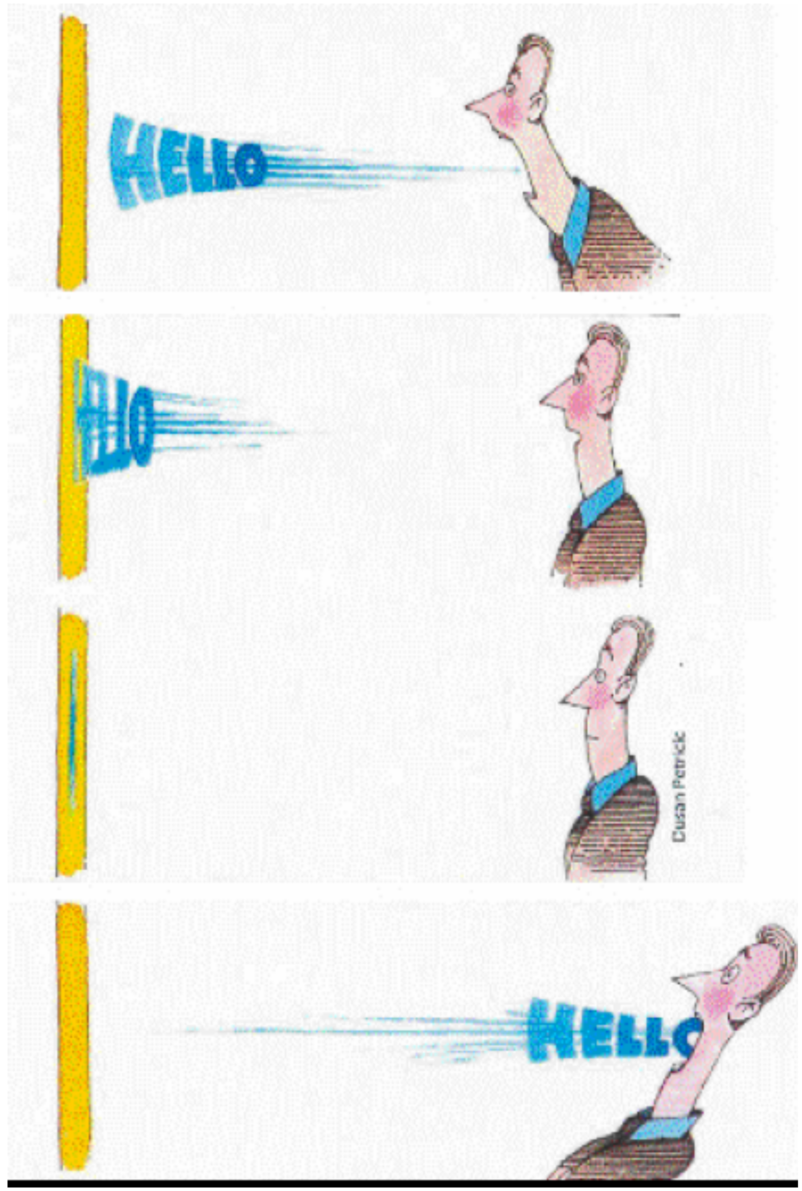
Time-reversal acoustics



Fink, 1997

$$div\left\{\frac{grad(p(\vec{r},t))}{\rho(\vec{r},p)}\right\}-\frac{1}{\rho(\vec{r},p)c^2(\vec{r},p)}\frac{\partial^2 p(\vec{r},t)}{\partial t^2}=0$$

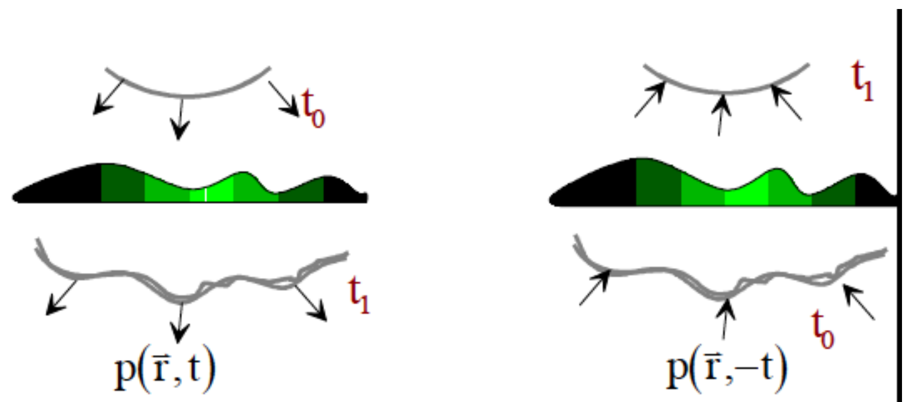
Spatial Reciprocity   Time Reversal Invariance

This equation contains only $\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})}{\partial \boldsymbol{t}^2}$

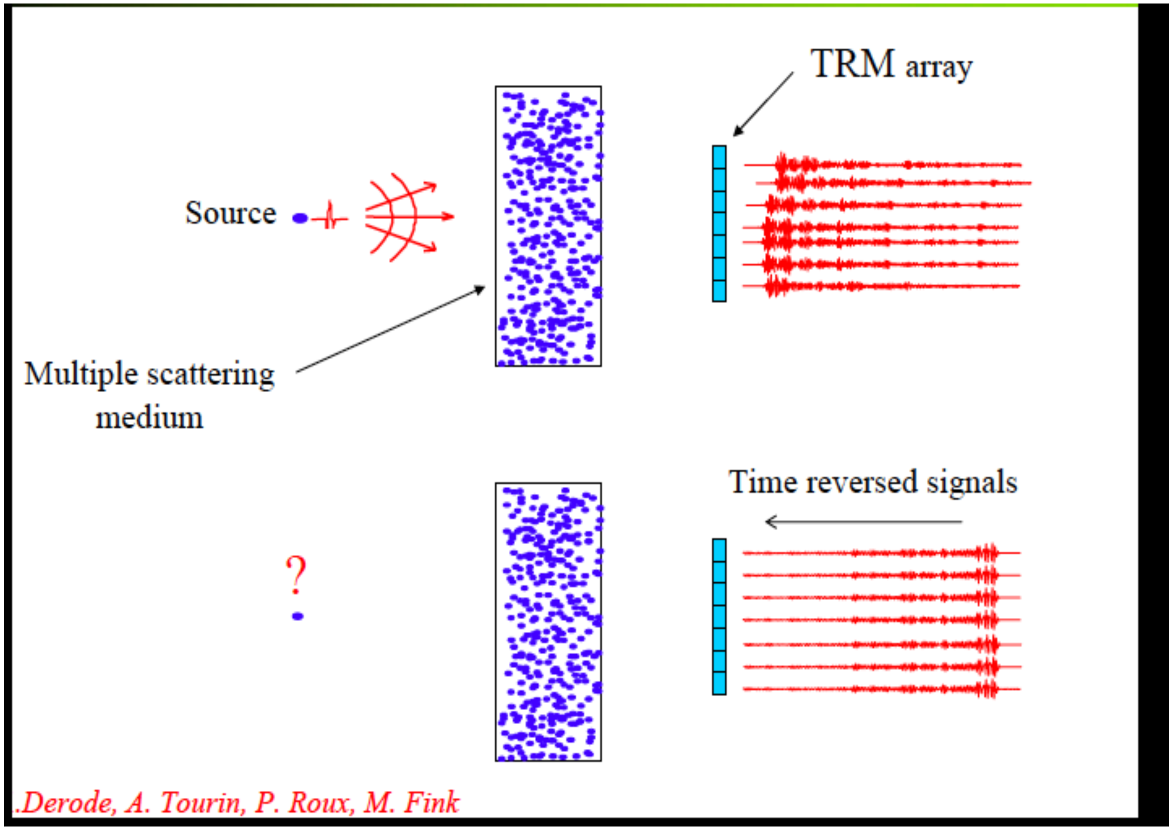
Then if $\boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})$ is a solution

$\boldsymbol{p}(\vec{\boldsymbol{r}},-\boldsymbol{t})$ is also a solution

because $\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})}{\partial \boldsymbol{t}^2}=\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},-\boldsymbol{t})}{\partial \boldsymbol{t}^2}$



Time-reversal and super resolution



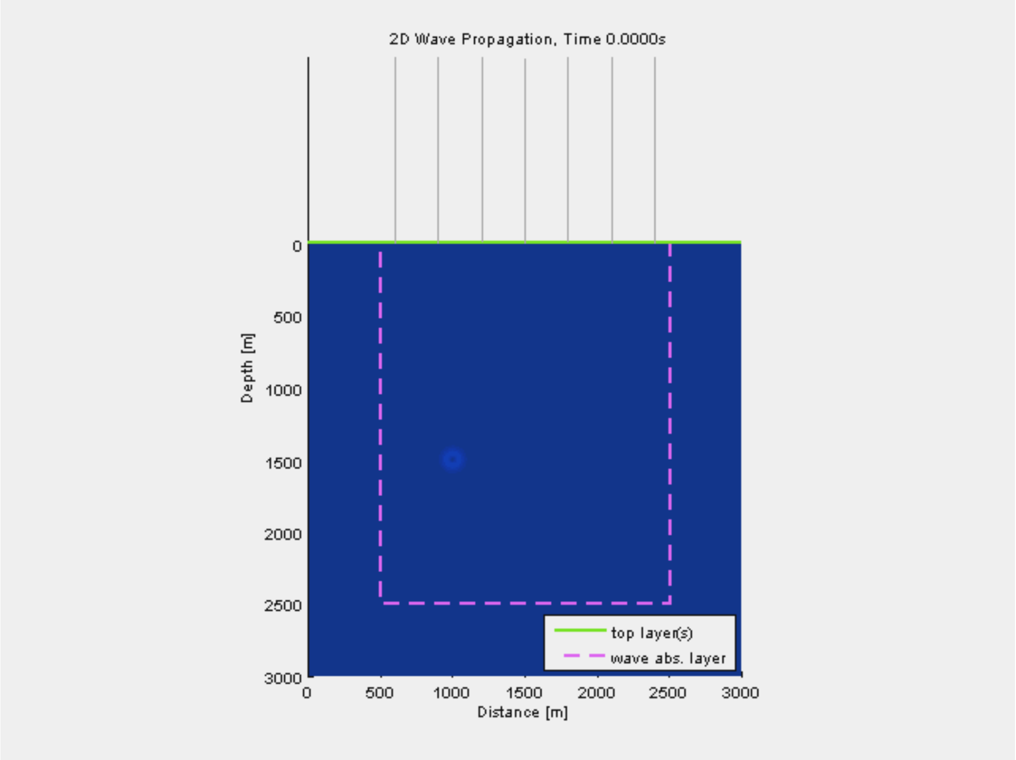
More heterogeneous and scattered the medium between the source and the TRM, the more focused the time-reversed signal.

Time-reversed seismic location

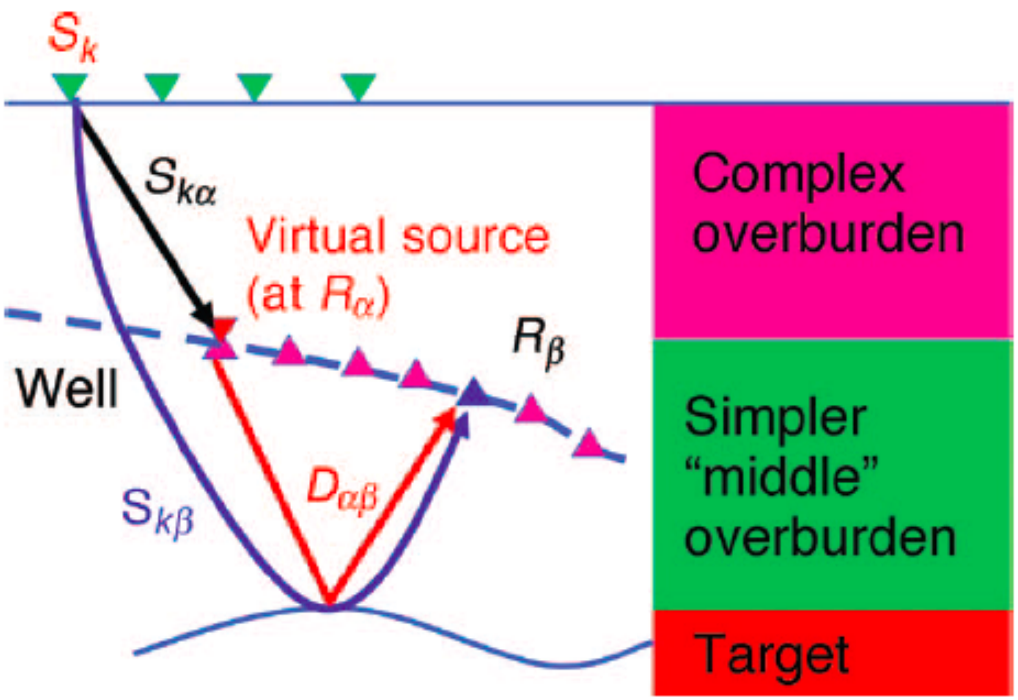
– No picking required

– Automatic (real-time) location

– Using full waveforms



Virtual source method

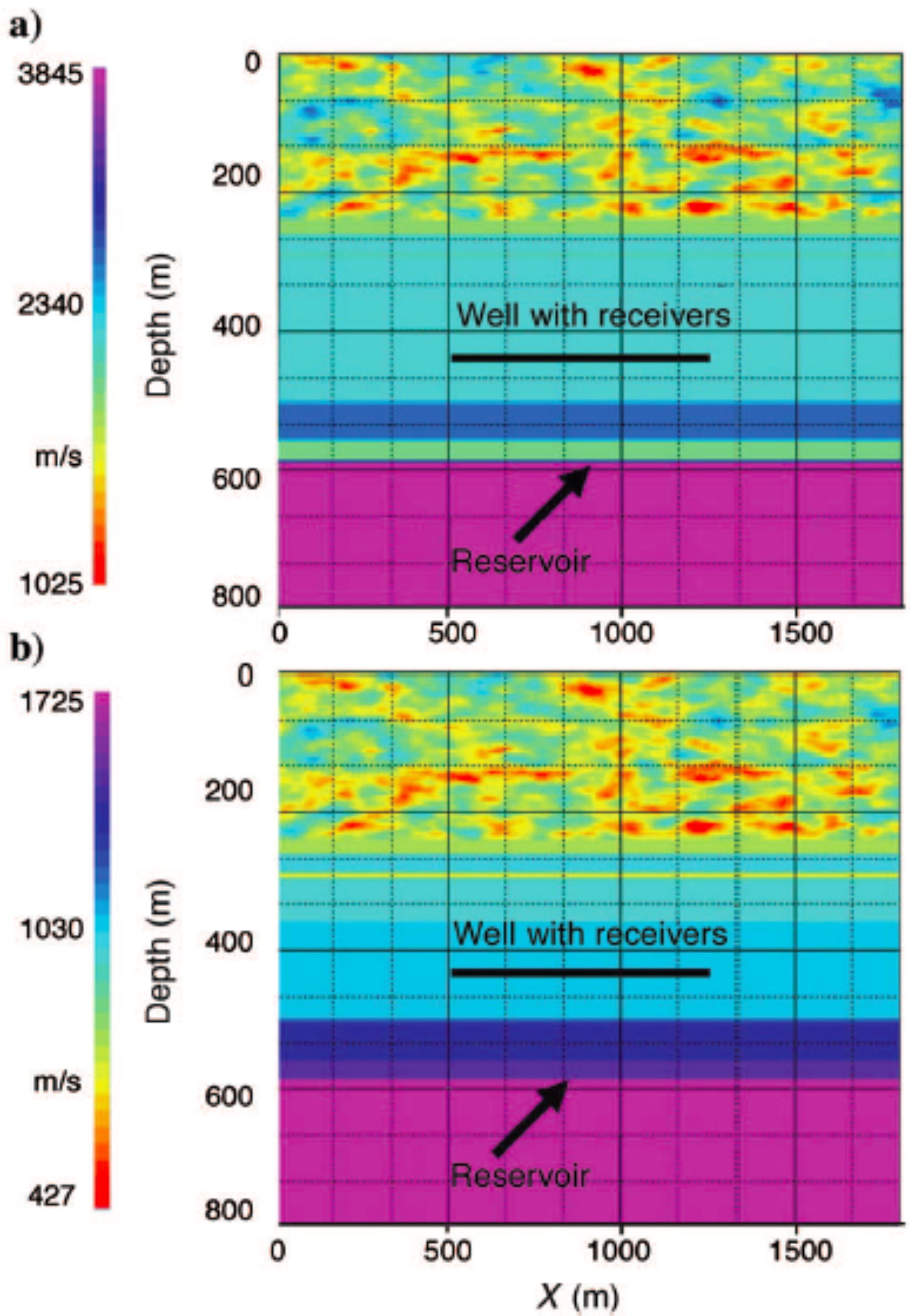


Sources S_k on the surface

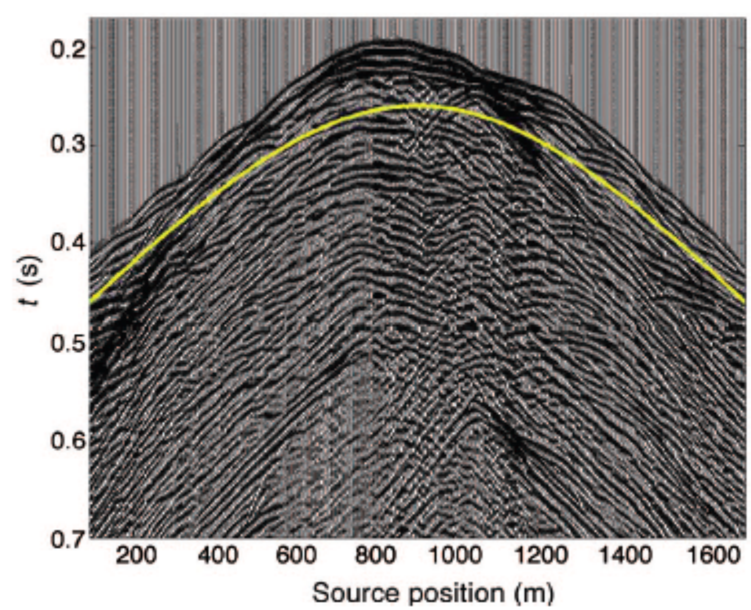
Receivers in the borehole

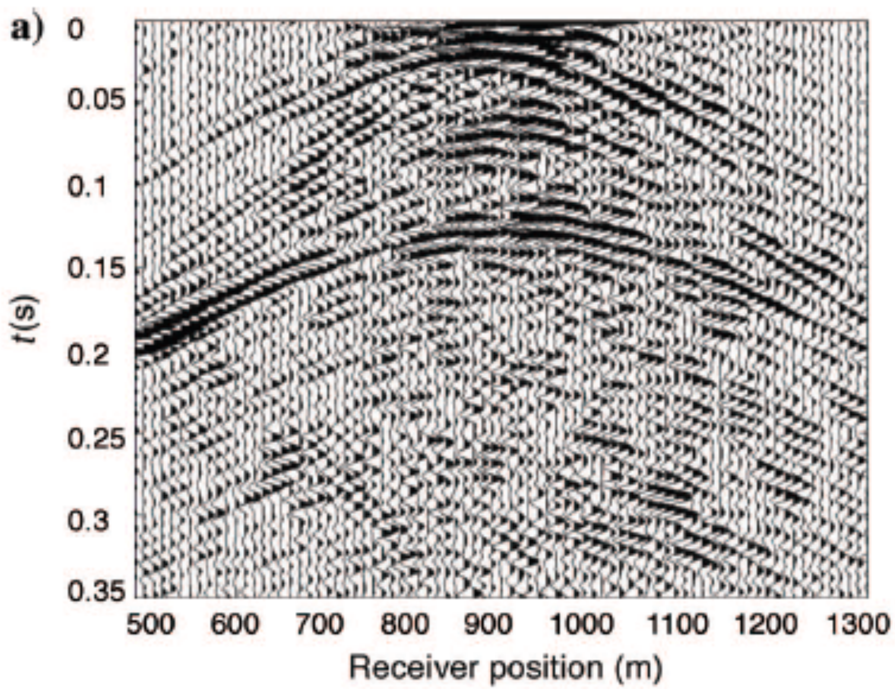
The goal is to construct seismic record from virtual source to receivers.

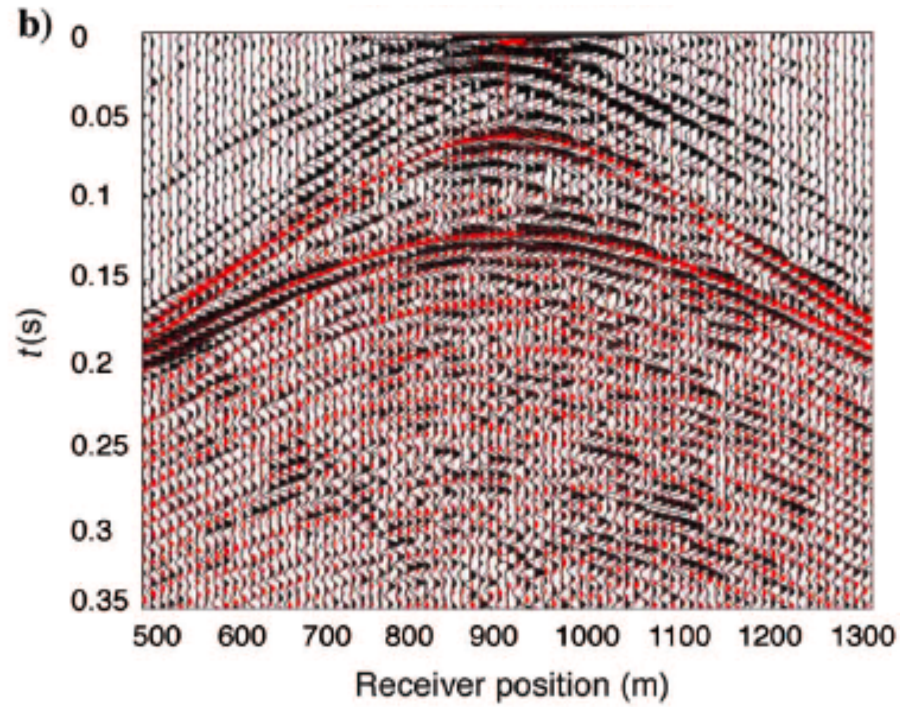
Bakulin and Calvert, 2006



Synthetic velocity models



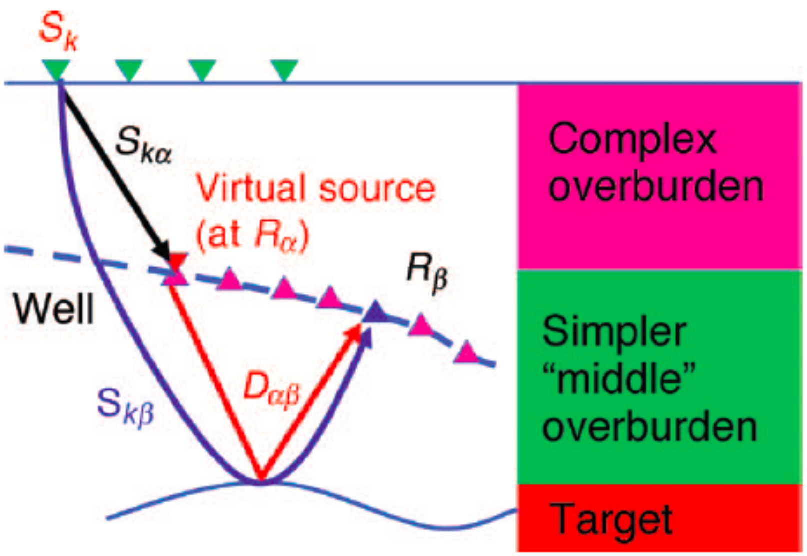




Virtual source gather (at 900 m)

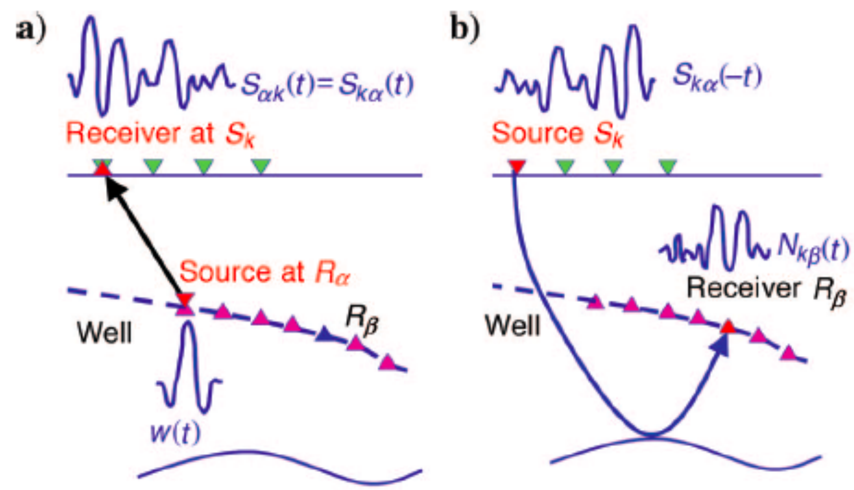
Real source gather (at 900 m)

Constructing virtual source (VS) data



$$D_{\alpha\beta}(t) = \sum_{k=1}^{N} S_{k\alpha}(-t) * S_{k\beta}(t)$$

$$D_{\alpha\beta}(t) = w(t) * w(-t) * \sum_{k=1}^{N} \tilde{S}_{k\alpha}(-t) * \tilde{S}_{k\beta}(t)$$



$$S_{k\beta}(t) = w_k(t) * \tilde{S}_{k\beta}(t)$$

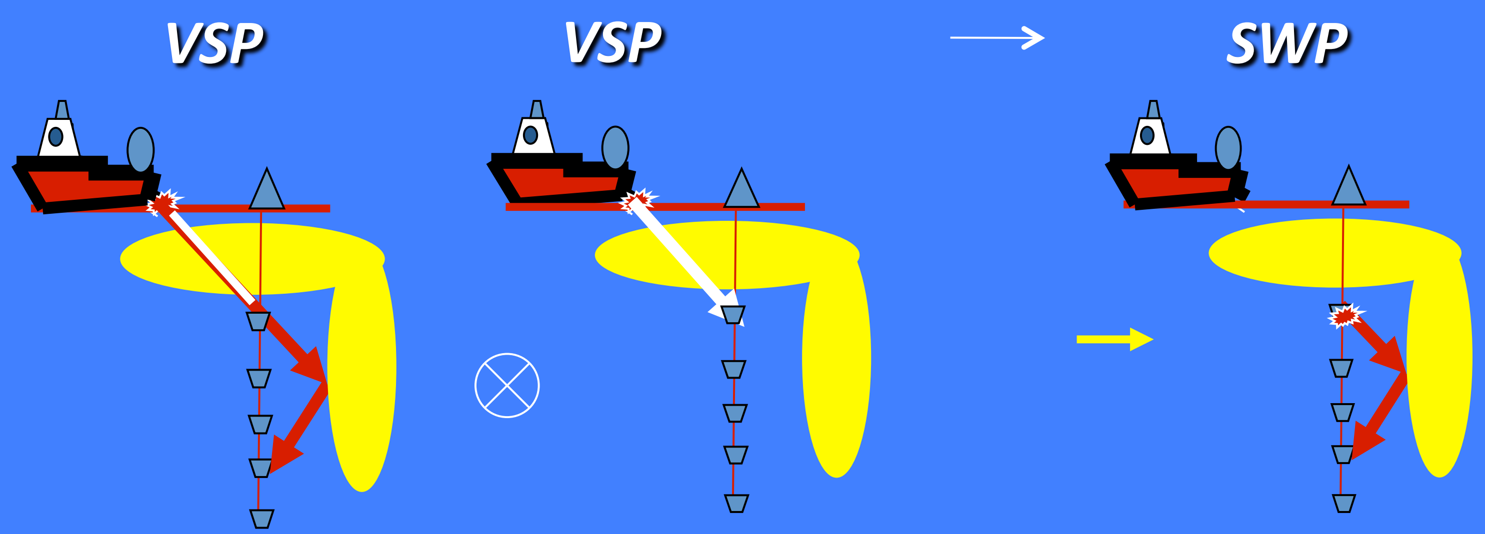
$$S_{ka}(t) = w_k(t)\tilde{S}_{ka}(t)$$

Bakulin and Calvert, 2006

Applications

Problem: ***Overburden+statics defocus VSP migration***

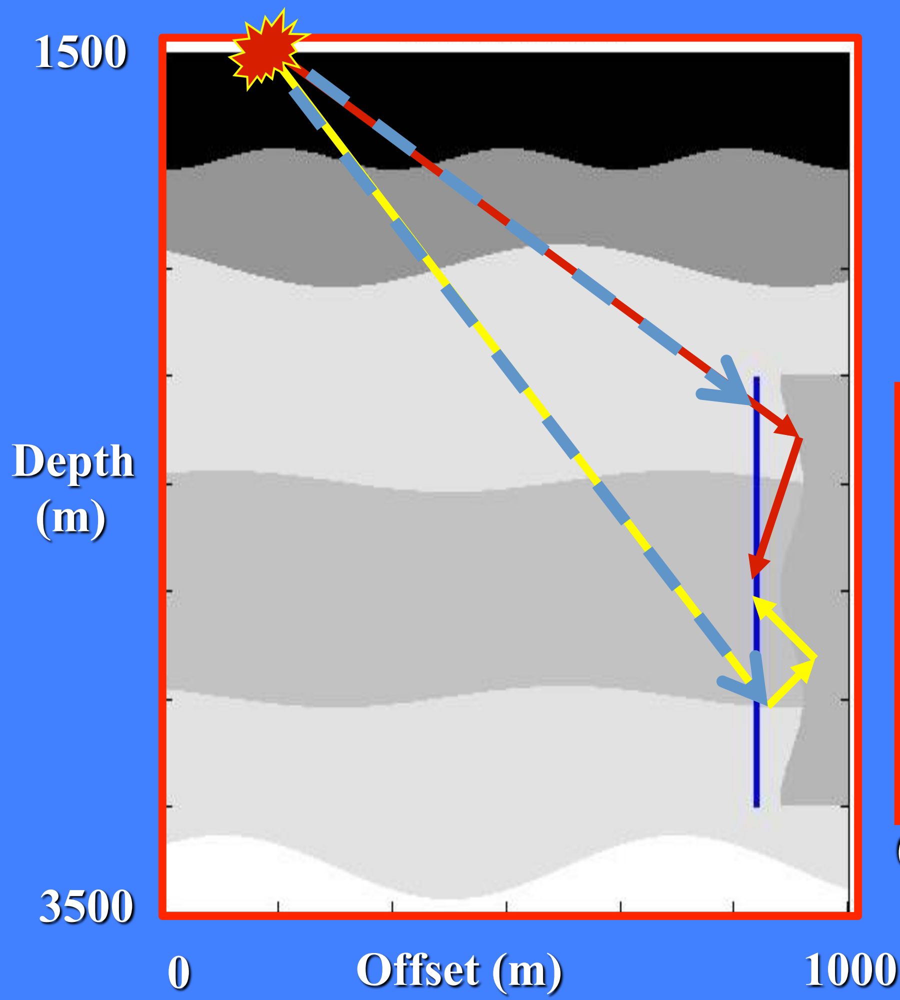
Solution: ***VSP -> SWP Transform (Calvert, Bakulin)***

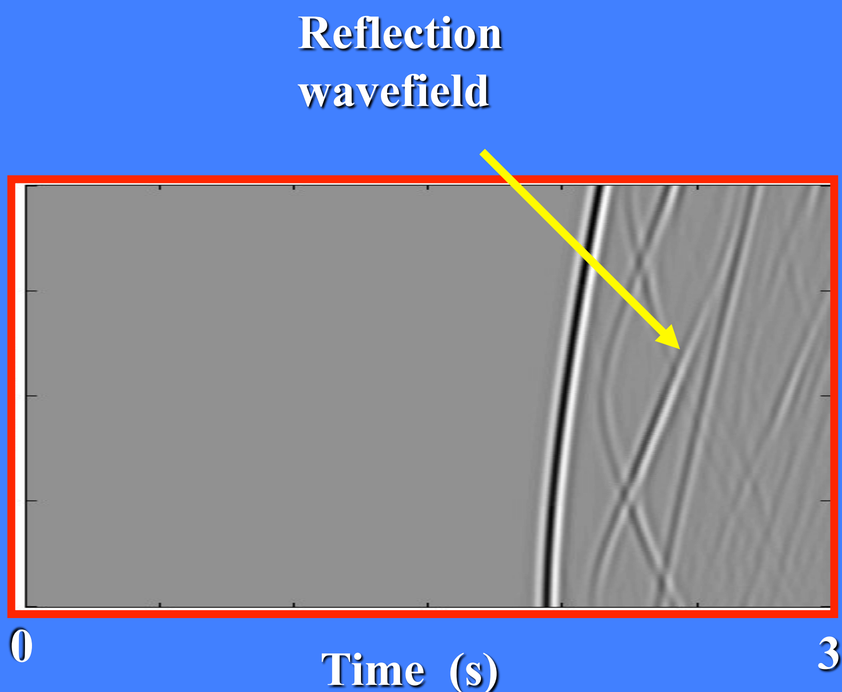


Redatum sources below overburden
Local VSP migration

Shuster, SEG 09

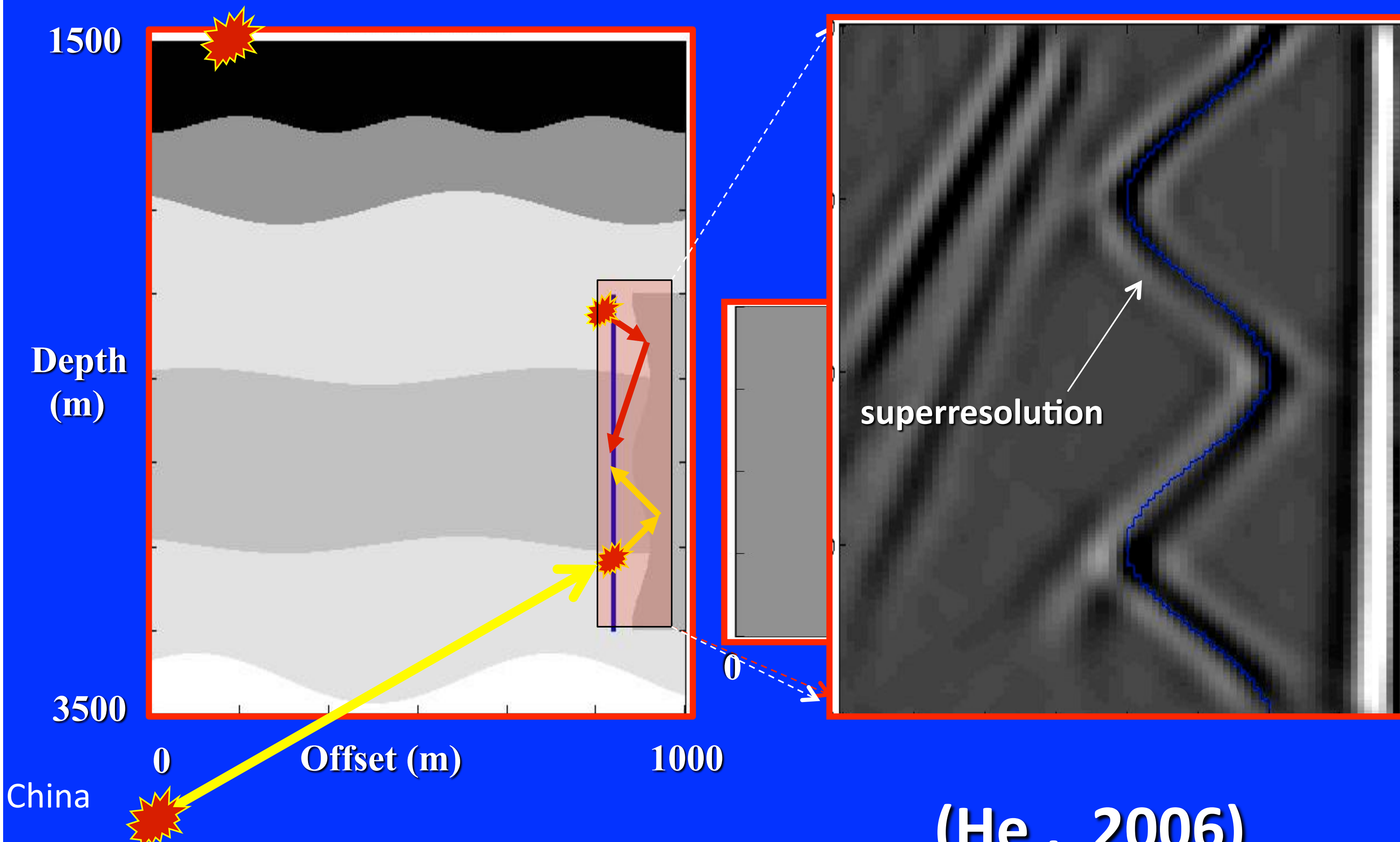
VSP Geometry





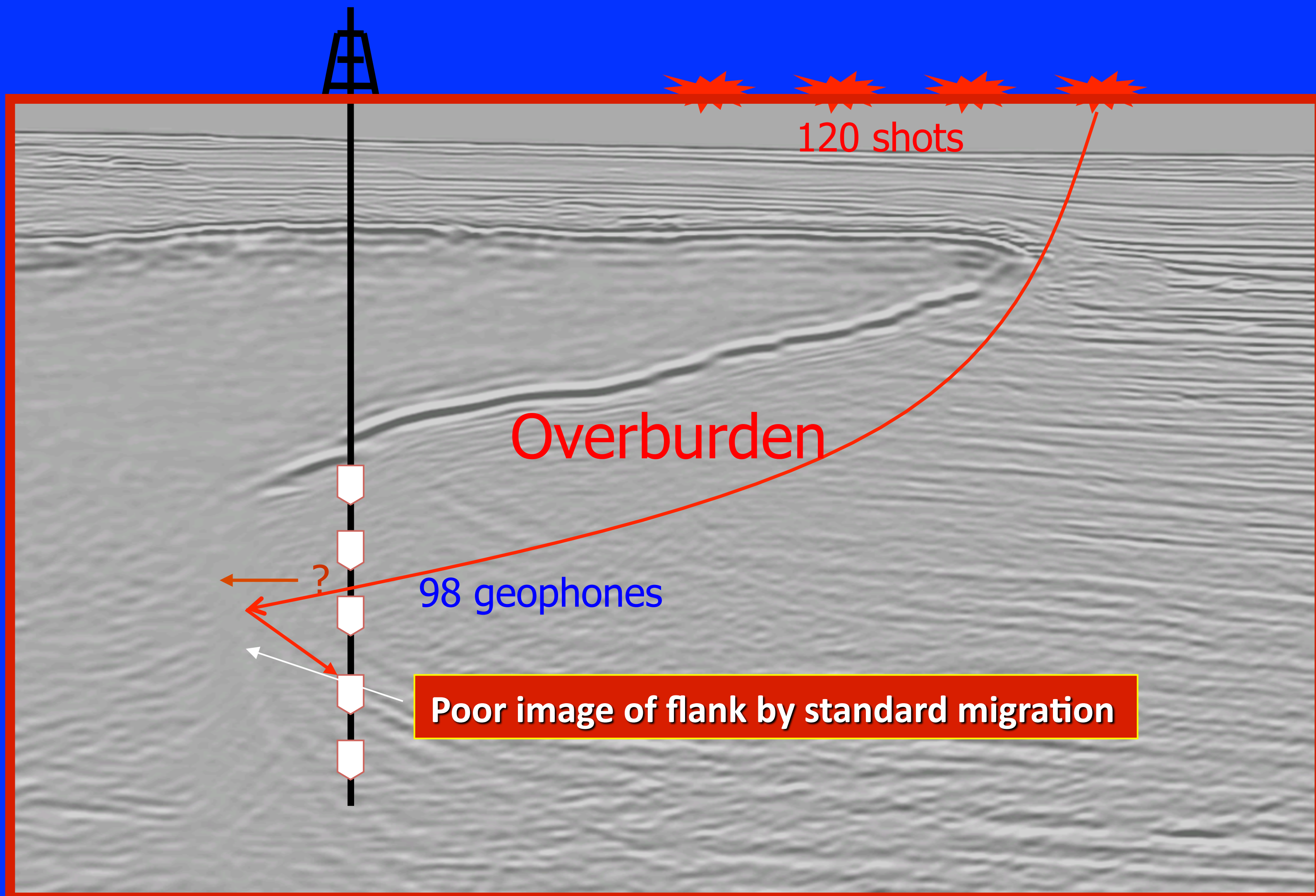
(He， 2006)

VSP Geometry

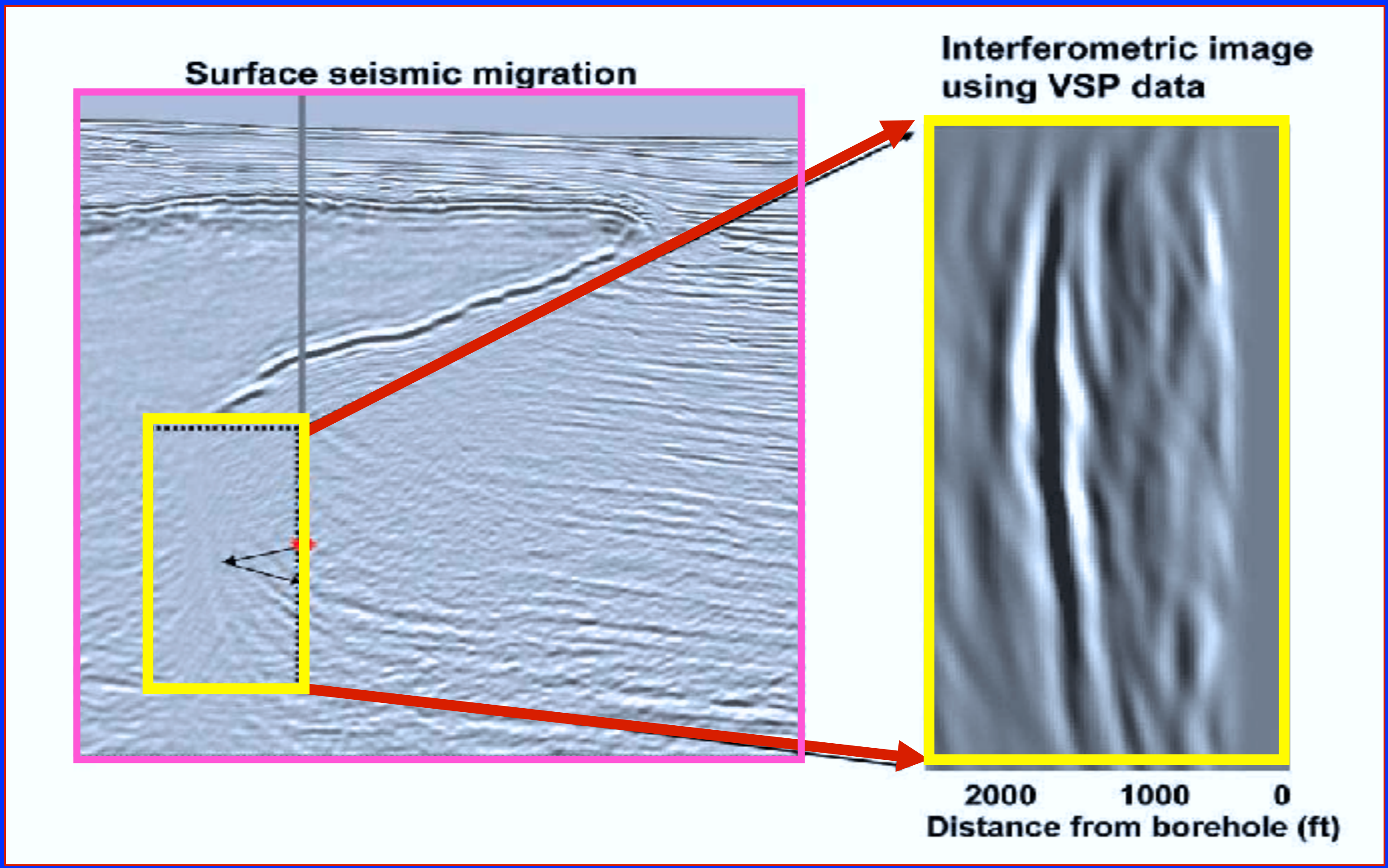


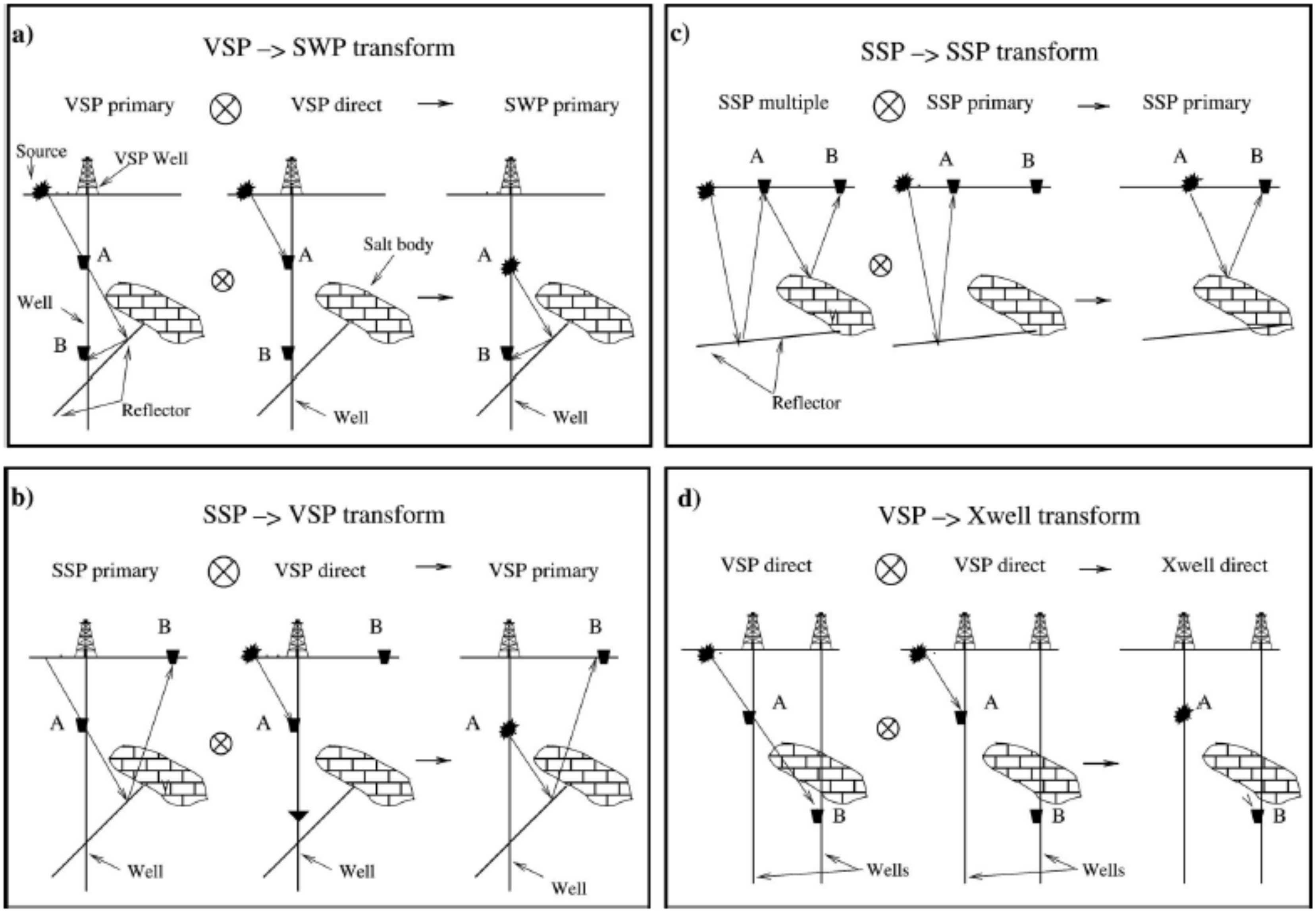
(He， 2006)

VSP Salt Flank Imaging
(Hornby & Yu, 2006)



Interferometric Migration Result





a)
VSP -> SWP transform
VSP primary
VSP direct
SWP primary
Source
VSP Well
A
Salt body
Well
B
Reflector
Well
Well
b)
SSP -> VSP transform
SSP primary
VSP direct
VSP primary
A
B
Well
Well
Well
c)
SSP -> SSP transform
SSP multiple
SSP primary
SSP primary
A
B
Reflector
d)
VSP -> Xwell transform
VSP direct
VSP direct
Xwell direct
A
B
Wells
Wells

Seismic Interferometry

- Interferometry is a family of techniques in which waves, are superimposed in order to extract information about the waves (from wikipedia).

- Seismic Interferometry involves cross correlating and stacking diffusive waves to extract Green's functions.

Cross-correlation and stacking

$$\Phi_a^{(s)}(t) = G_{sa}(t) \otimes e(t)$$

G_{sa}(t): Green's function from source *s* to *a*.
e(t): Source time function.

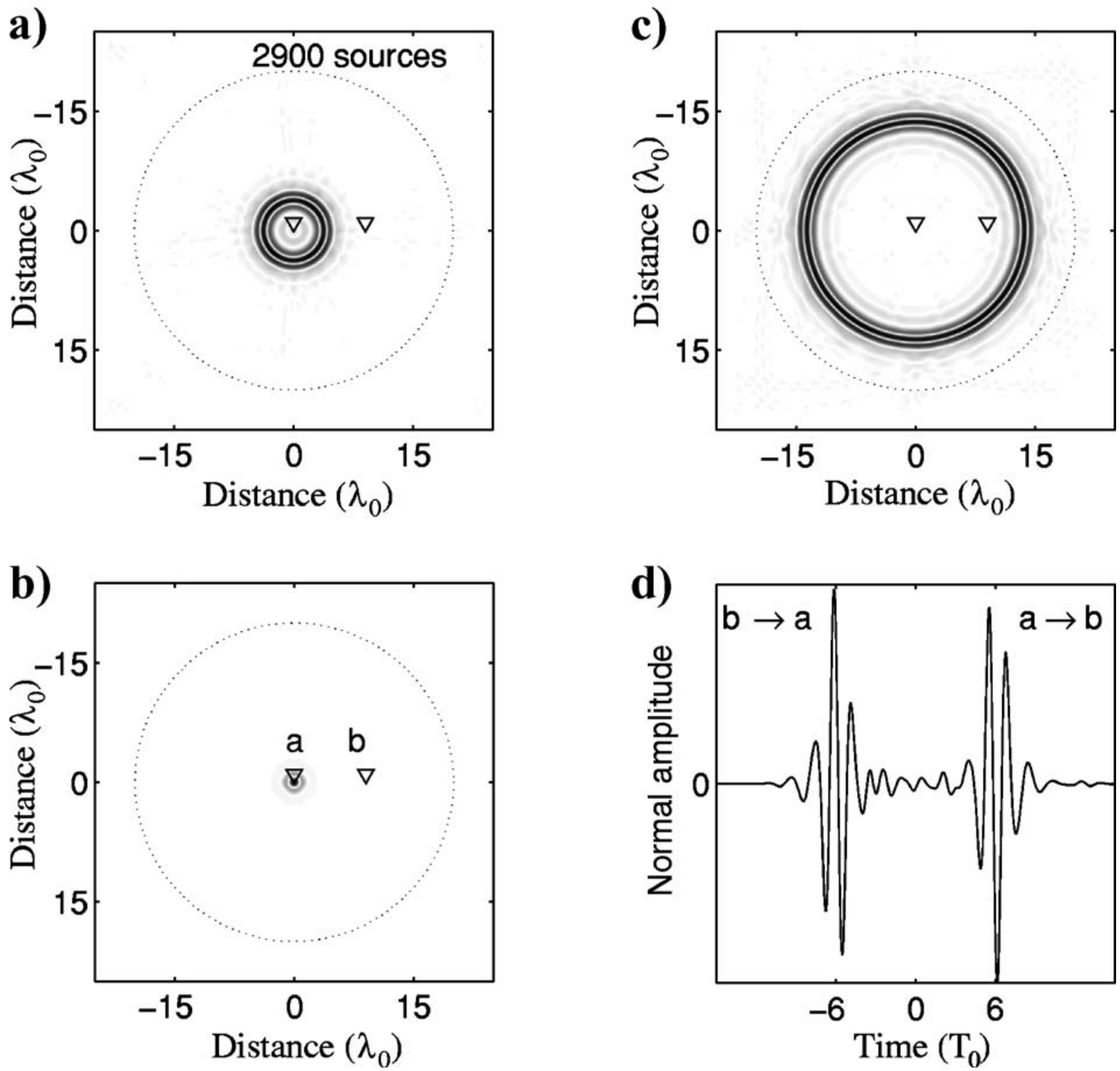
$$C_{ab}^{(s)}(\tau) = \int \Phi_a^{(s)}(t)\Phi_b^{(s)}(t+\tau)dt = \Phi_a^{(s)}(-\tau) \otimes \Phi_b^{(s)}(\tau).$$

Cross-correlation

$$C_{ab}(\tau) = \sum_s C_{ab}^{(s)}(\tau)$$

Stacking: source averaging

$$G(\mathbf{x}_B,\mathbf{x}_A,t) + G(\mathbf{x}_B,\mathbf{x}_A,-t) \propto \oint_{\partial D} G(\mathbf{x}_B,\mathbf{x},t) * G(\mathbf{x}_A,\mathbf{x},-t)\mathrm{d}^2\mathbf{x}$$

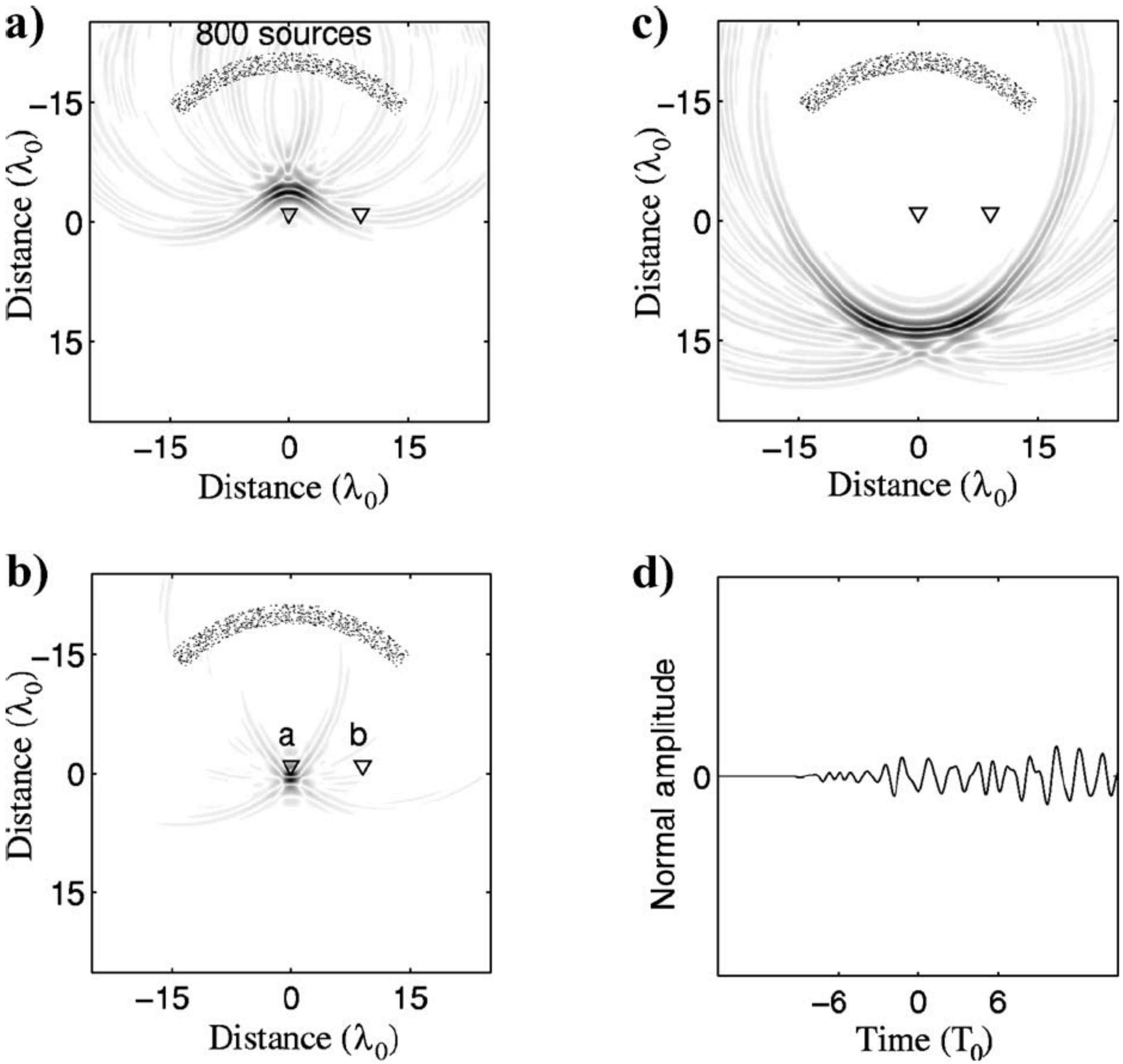


Homogeneous medium

Circular sources

a) $\tau = -3T_0$
b) $\tau = -T_0$
c) $T = 10T_0$

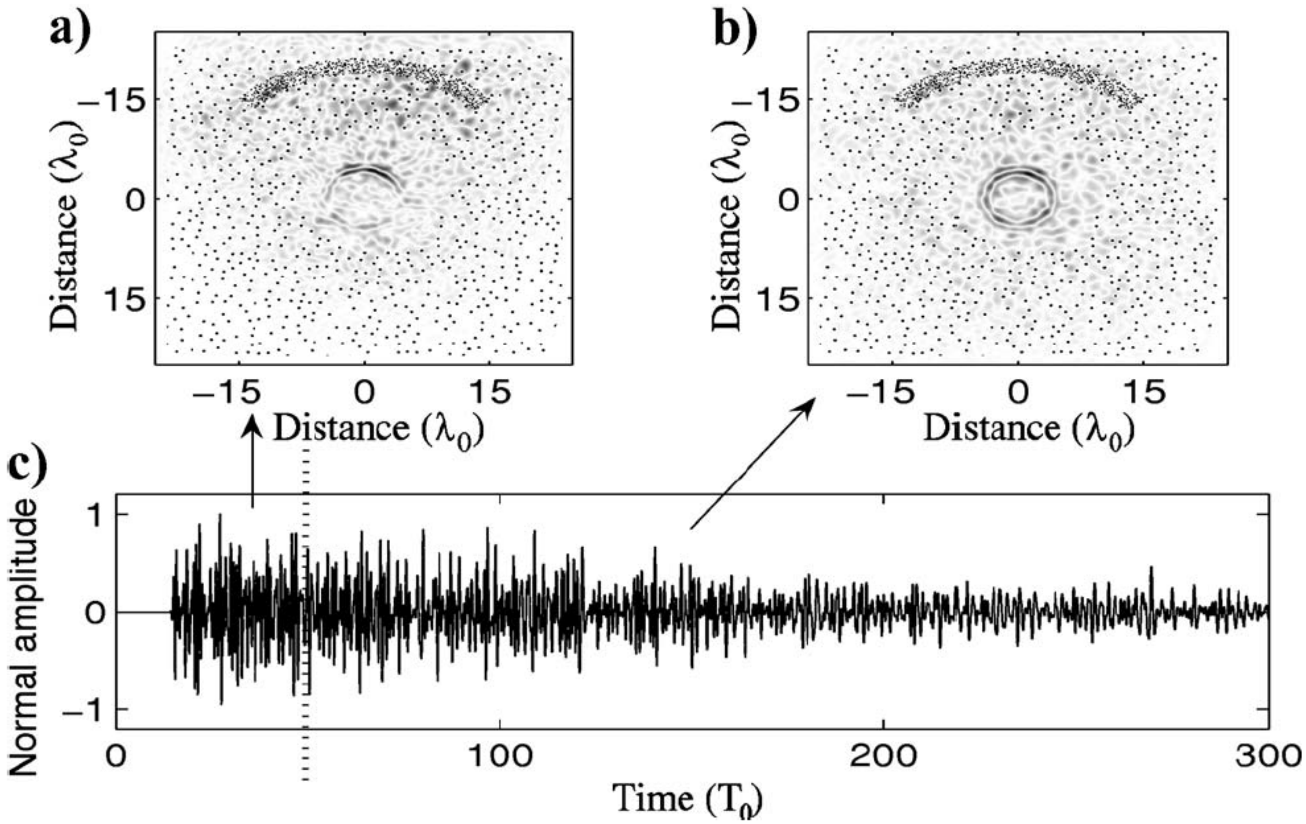
Larose et al., 2005



Homogeneous medium

800 sources (partially covered)

a) $\tau= -3T_0$
b) $\tau= -T_0$
c) $T=10T_0$



Heterogeous medium

800 sources (partially covered)

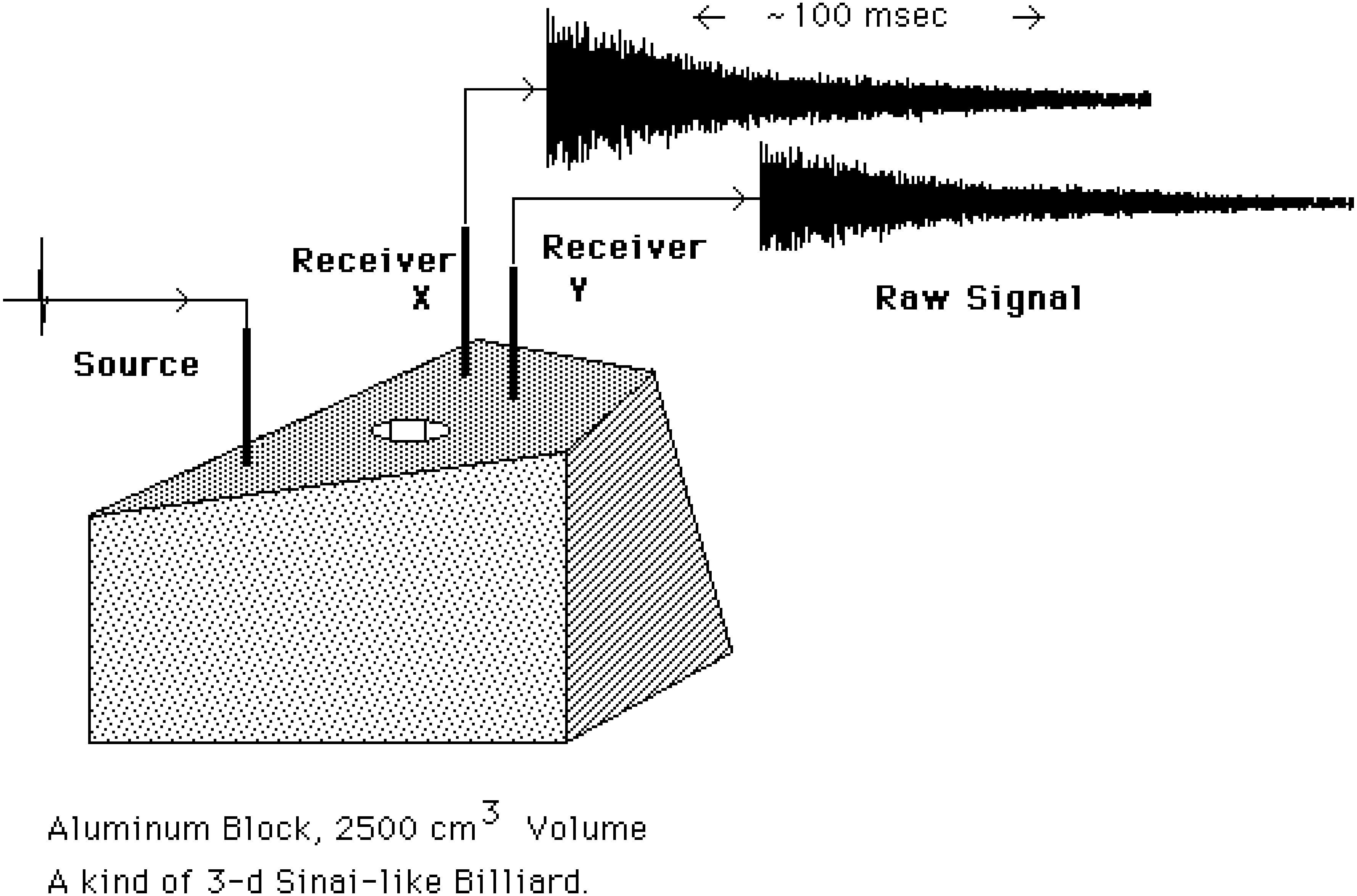
a) first-part coda
b) late coda

Time-reversal interpretation of the correlation process

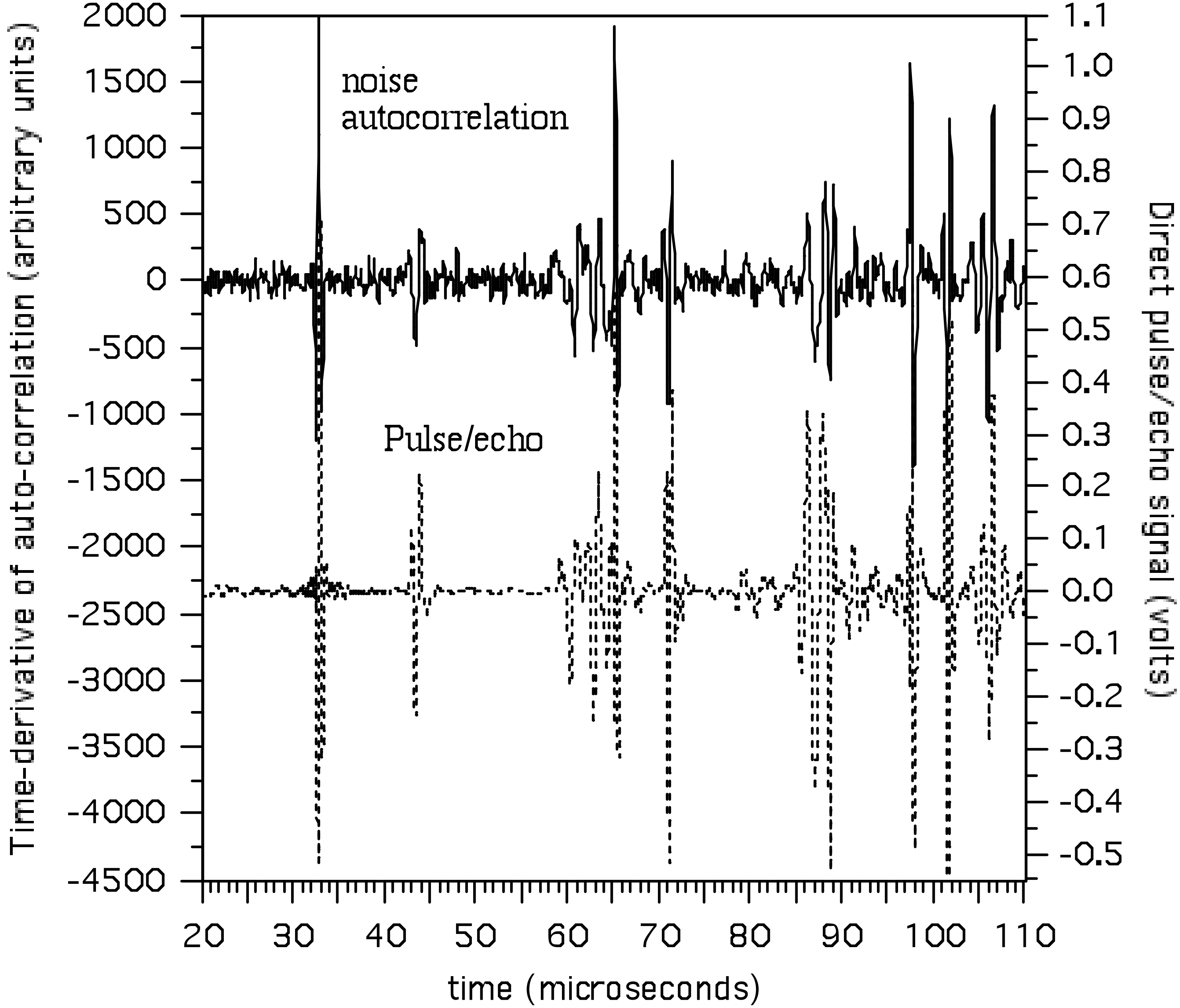
$$C_{ab}^{(s)}(t) = G_{sa}(-\ \tau) \otimes G_{sb}(\tau) = G_{as}(-\ \tau) \otimes G_{sb}(\tau)$$

Treat *a* as the virtual source

$$C_{ab}(\tau) = \sum_{s} C_{ab}^{(s)}(\tau)$$



(Weaver and Lobkis, Ultrasonics, **40**, 435-439, 2002)



(Weaver and Lobkis, Ultrasonics, **40**, 435-439, 2002)

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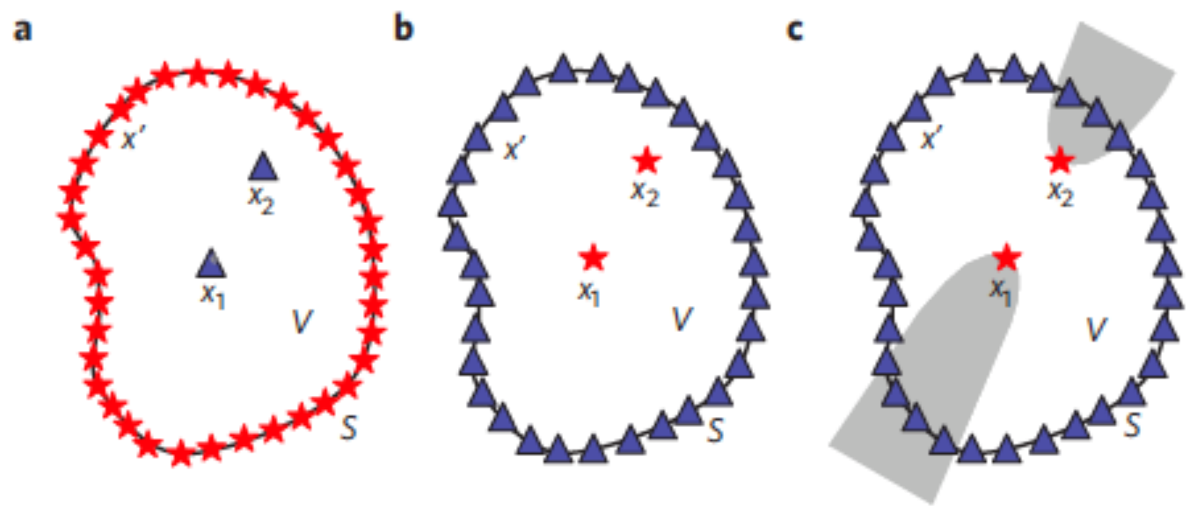
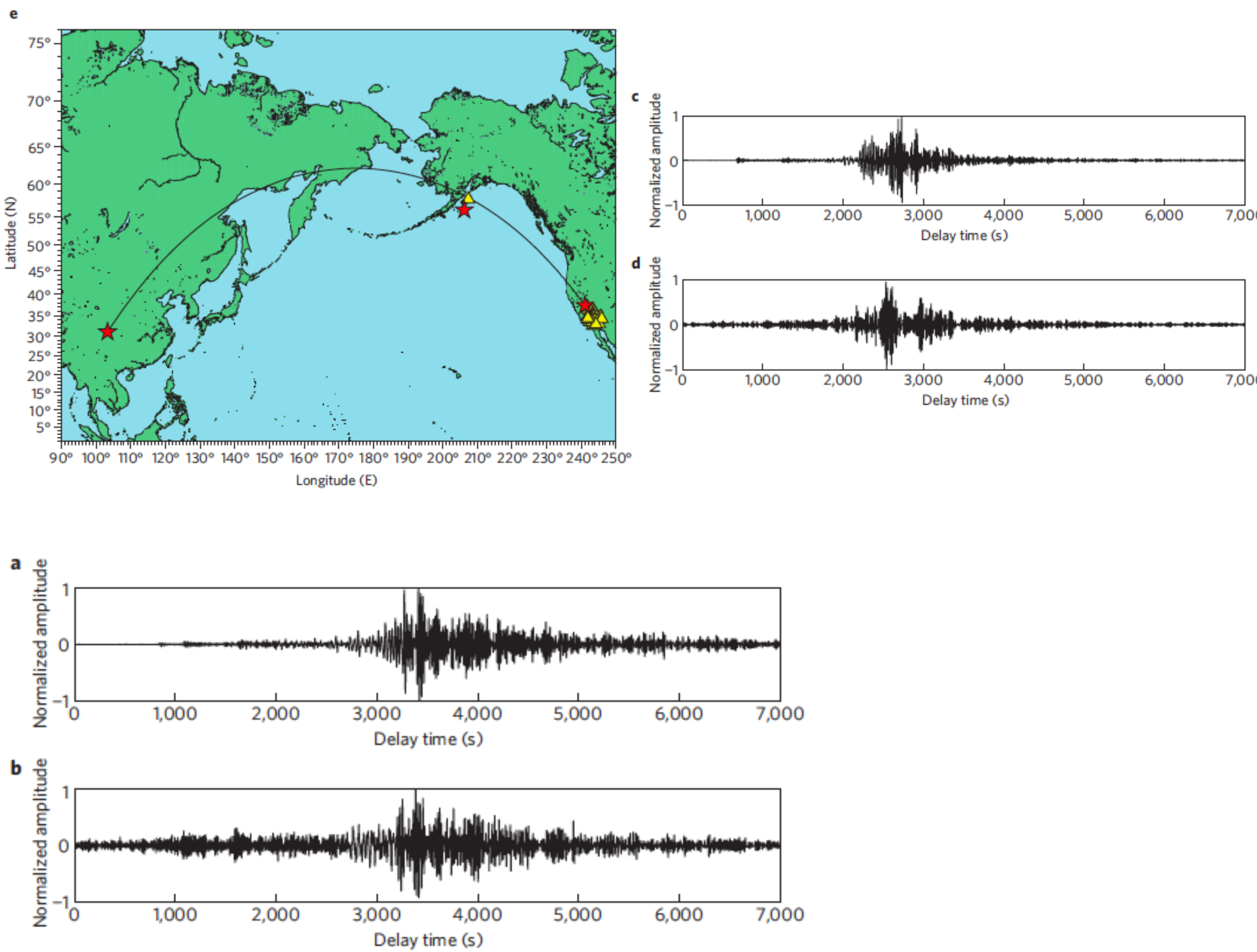
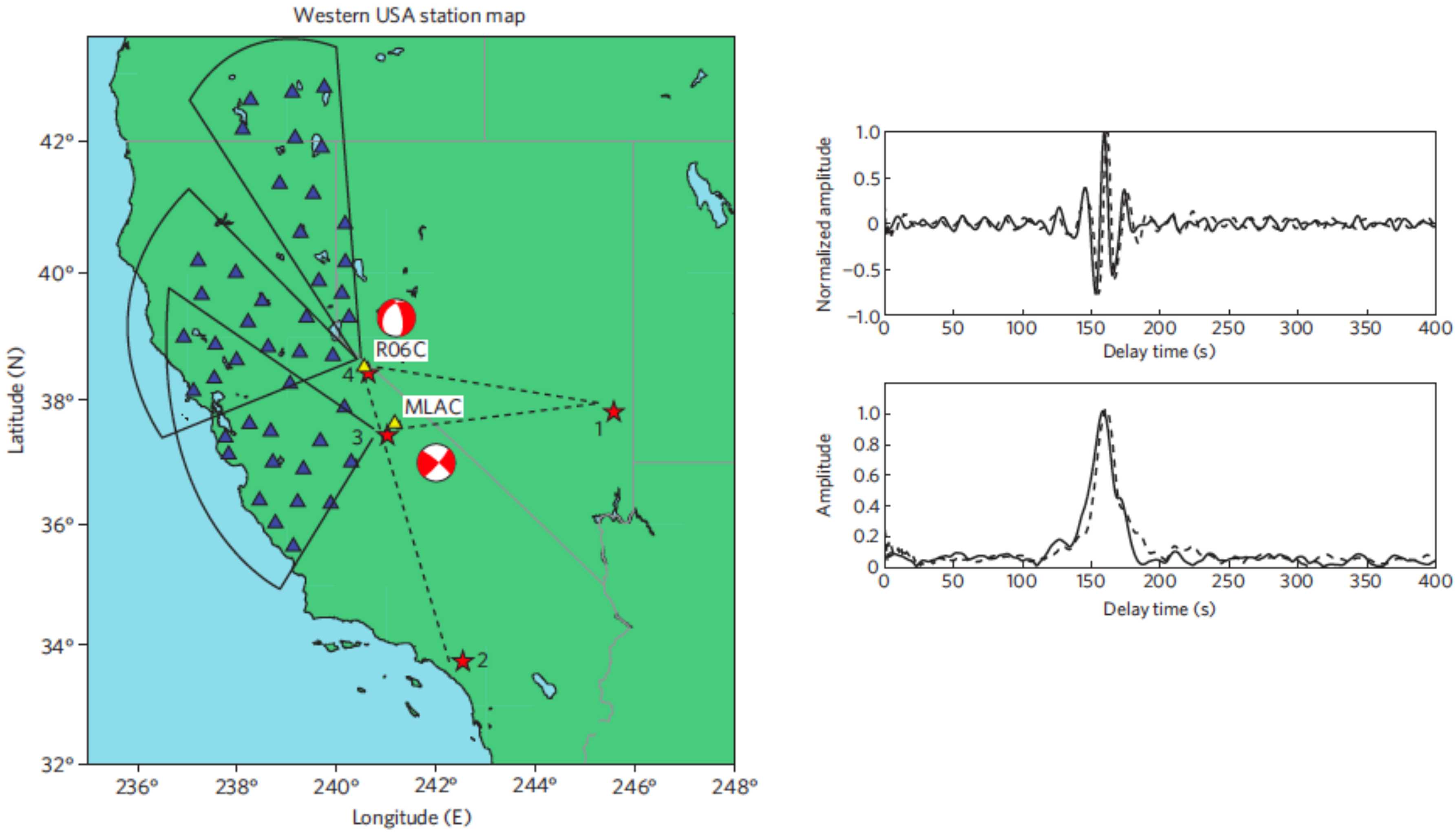
nature geoscience

Virtual seismometers in the subsurface of the Earth from seismic interferometry

Andrew Curtis[1,2]*, Heather Nicolson[1,2,3], David Halliday[1,2,4], Jeannot Trampert[5] and Brian Baptie[2,3]




e
Latitude (N)
75°
70°
65°
60°
55°
50°
45°
40°
35°
30°
25°
20°
15°
10°
5°
90° 100° 110° 120° 130° 140° 150° 160° 170° 180° 190° 200° 210° 220° 230° 240° 250°
Longitude (E)
c
Normalized amplitude
1
0
−1
0 1,000 2,000 3,000 4,000 5,000 6,000 7,000
Delay time (s)
d
Normalized amplitude
1
0
−1
0 1,000 2,000 3,000 4,000 5,000 6,000 7,000
Delay time (s)
a
Normalized amplitude
1
0
−1
0 1,000 2,000 3,000 4,000 5,000 6,000 7,000
Delay time (s)
b
Normalized amplitude
1
0
−1
0 1,000 2,000 3,000 4,000 5,000 6,000 7,000
Delay time (s)



Western USA station map
R06C
MLAC
1
2
3
4
Latitude (N)
Longitude (E)
42°
40°
38°
36°
34°
32°
236°
238°
240°
242°
244°
246°
248°
Normalized amplitude
Amplitude
Delay time (s)
1.0
0.5
0
−0.5
−1.0
0.8
0.6
0.4
0.2
50
100
150
200
250
300
350
400