Review

1) Mathematica 技巧

2) RK方法 $\frac{dy}{dx} = f(x,y)$

3) ode、rk45 函数

Maxwell 方程

扩散方程 $\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2}$，

处理方法，在 x 方向上做离散化

$$\frac{\partial u(t, x_n)}{\partial t} = D\,\frac{u(t,x_{n+1}) + u(t,x_{n-1}) - 2u(t,x_n)}{(\Delta x)^2}$$

转化为可以用RK方法求解的问题.

$$\frac{dy}{dx} = f(y) \iff y_{n+1} = y_n + f(y_n)\,\Delta x$$

P.S. 相关问题

1) 数列 $\begin{cases} a_{n+1} = x a_n + y a_{n-1} \\ a_{n+1} = x a_n^2 + y a_{n-1} \end{cases}$

2) 极限 $\lim_{n\to\infty} y_n = ?$

$a_0 \to a_1 \to a_2 \to \cdots\cdots$

```
double a0, a1, a2

    a2 = x a1 + y a0
    output a2
    a0 = a1
    a1 = a2
```

伪代码.

问题特点：
- 不耗内存.
- 计算快.

⇑

很多问题在离散化后可以归为数列。

Roots and minimization.

1) 根，极值 ⟹ 可以互相转化.

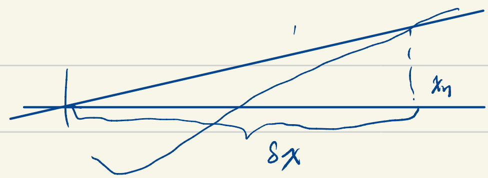
2) 迭代.

3) $f_i(\vec{x}) = 0 \Leftarrow F = \frac{1}{2}|f_i|^2 \to 0$

考虑一个一维的问题. $f(x)=0$.

$$x_{n+1} = x_n + b f^2(x_n)$$

如果极限存在 $\lim_{n\to\infty} x_n = x^*$



牛顿法求根 $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$f(x_n+\delta x) = f(x_n) + f'(x_n)\delta x = 0$$

$$\Rightarrow \delta x = -\frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} - x_n = -\frac{f(x_n)}{f'(x_n)}$$

在实际中往往需要求解很多方程的根.

$f_i(\vec{x}) = 0,\ i = 1, 2, \cdots, N$

$$f_i(x_{n+1}) = f_i(\vec{x}_n) + \frac{\partial f_i}{\partial x_j}(x^j_{n+1} - x^j_n) = 0$$

启发：$\vec{x}_{n+1} = \vec{x}_n + \vec{g}_n \Leftarrow$ 更改 gradient 函数.

求极小值. $\min F(\vec{x}) \Rightarrow F(\vec{x}+\delta\vec{x}) = F(\vec{x}) + \nabla F \cdot \delta\vec{x} + \frac{1}{2}\delta x^T\left(\frac{\partial^2 F}{\partial x_i \partial x_j}\right)\delta x$

$$F^{\prime\prime}(x+\delta x) = F + \underbrace{\frac{\partial F}{\partial x_i}}_{g_i}\delta x_i + \frac{1}{2}\delta x_i \delta x_j H_{ij}$$

$$\frac{\partial F}{\partial(\delta x_i)} = 0 \Rightarrow g_i + H_{ij}\delta x_j = 0$$

$$\Downarrow$$

$$g + H\delta x = 0 \Rightarrow \delta x = -H^{-1}g$$

作业.

应用

<1>

Baker's map. 不同情况下的动力学.

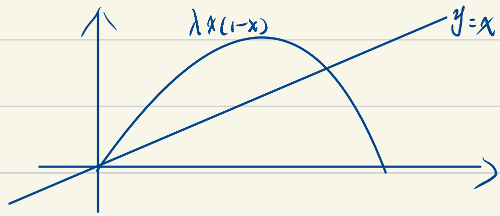
$$(x_{n+1}, y_{n+1}) = \begin{cases} (2x_n, \frac{y_n}{2}), & 0 \le x_n < \frac{1}{2} \\ (2-2x_n, 1-\frac{y_n}{2}), & \frac{1}{2} \le x_n < 1 \end{cases}$$

<2>

logistic map population dynamics.

$$x_{n+1} = \lambda x_n (1 - x_n)$$

极限是否存在？



<3>

kicked rotor model.

$$\mathcal{H} = \frac{p^2}{2} + K\cos(x)\sum_n \delta(t-n)$$

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial p} = p \\ \dot{p} = -\frac{\partial H}{\partial x} = -K\sin(x)\sum_n \delta(t-n) \end{cases}$$

$$\Rightarrow \begin{cases} x_{n+1} = x_n + p_n \\ p_{n+1} = p_n - K\sin(x_n) \end{cases}$$

讨论相应的物理.
混沌等性质.