今天的话题，FDTD，有限时域差分方法。

Def：The Finite-Difference Time Domain Method (FDTD).

回顾：Lindblad方程推导以及计算

$$i\dot{\rho} = -i[H,\rho] + \mathcal{L}(\rho)$$

其中 $\mathcal{L}(\rho) = \sum_i \gamma_i \left(2L_i\rho L_i^+ - \rho L_i^+L_i - L_i^+L_i\rho\right)$

$\Downarrow$

化为线性方程。

$\exists$ $|\psi\rangle \to |\psi\rangle + |d\psi\rangle$

EM求解 $\begin{cases} \text{FDTD} & \text{finite-difference time-domain} \\ \text{FETD} & \text{finite-element time domain} \end{cases}$

$\begin{cases} \text{EM方程} \\ \text{流体力学} \end{cases}$

内容
① FDTD方法 1d
② Courant（库朗）Condition
③ Leapfrog method
④ 推广到 2d, 3d ……边界条件

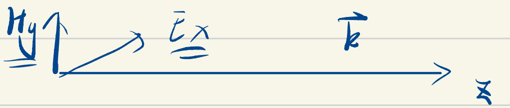
History：历史. Kane Yea. 1951 家人移民美国. 1966年 FDTD

Maxwell eq.

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0}\nabla\times\vec{H},\quad \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0}\nabla\times\vec{E}$$

电场空间分布 ⇒ 磁场
磁场空间分布 ⇒ 电场
⇒ 数值方法，体现了这种关系。

1d system ⇒



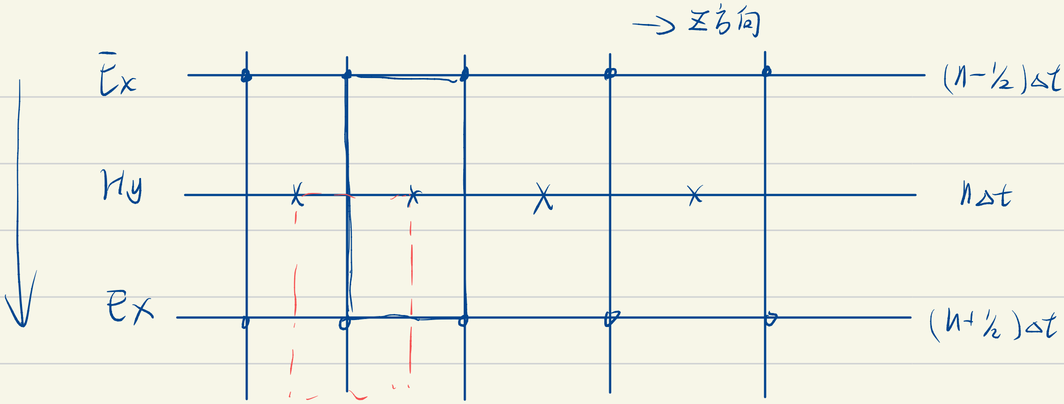
$$\begin{cases} \dfrac{\partial E_x}{\partial t} = -\dfrac{1}{\varepsilon_0}\dfrac{\partial H_y}{\partial z} \\[2ex] \dfrac{\partial H_y}{\partial t} = -\dfrac{1}{\mu_0}\dfrac{\partial E_x}{\partial z} \end{cases} \Rightarrow \boxed{\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 E_x}{\partial z^2}}$$ ← 波动方程.

中点差分法.

$$f'(x) = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$, f 和 f′ 出现在不同的格子上. 具有更好的数值精度.

Yee 元胞.

$$\begin{cases} \dfrac{E_{x,k}^{n+1/2} - E_{x,k}^{n-1/2}}{\Delta t} = -\dfrac{1}{\varepsilon_0}\,\dfrac{H_{y,k+1/2}^{n} - H_{y,k-1/2}^{n}}{\Delta z} \\[3ex] \dfrac{H_{y\,k+1/2}^{n+1} - H_{y\,k+1/2}^{n}}{\Delta t} = -\dfrac{1}{\mu_0}\,\dfrac{E_{x\,k+1}^{n+1/2} - E_{x\,k}^{n+1/2}}{\Delta z} \end{cases}$$



$(n-\frac{1}{2})\Delta t$ 时刻的 E_x $\xrightarrow{\text{给出}}$ $n\Delta t$ 时刻 H_y → $(n+\frac{1}{2})\Delta t$ E_x

↓

...... ← $(n+1)\Delta t$ H_y

问 $\Delta x, \Delta t$ 如何取，光速 $c = 3\times 10^8 m$

$$\begin{cases} \Delta x \ll \lambda \\ \Delta t \ll T \end{cases}$$

可以有一个收敛的结果.

回顾
$$\begin{cases} E_{x\,k}^{n+1/2} - E_{x\,k}^{n-1/2} = \dfrac{\Delta t}{\varepsilon_0 \Delta z}\left(H_{y\,k-1/2}^{n} - H_{y\,k+1/2}^{n}\right) \\ H_{y,k+1/2}^{n+1} - H_{y,k-1/2}^{n} = \dfrac{\Delta t}{\mu_0 \Delta z}\left(E_{x\,k}^{n+1/2} - E_{x\,k+1}^{n+1/2}\right) \end{cases}$$

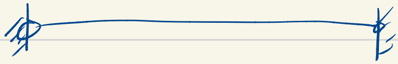
取 $\hat{E}_x = \sqrt{\dfrac{\varepsilon_0}{\mu_0}}\, E$

→ ½ ~ ⅓

$$\Rightarrow \begin{cases} \hat{E}_{x\,k}^{n+1/2} - \hat{E}_{x\,k}^{n-1/2} = \dfrac{\Delta t}{\sqrt{\varepsilon_0 \mu_0}\,\Delta z}\left(H_{y\,k-1/2}^{n} - H_{y\,k+1/2}^{n}\right) \\ H_{y\,k+1/2}^{n+1} - H_{y\,k+1/2}^{n} = \dfrac{\Delta t}{\sqrt{\varepsilon_0 \mu_0}\,\Delta z}\left(\hat{E}_{x\,k}^{n+1/2} - \hat{E}_{x,k+1}^{n+1/2}\right) \end{cases}$$

边界条件：完全吸收边界条件

$$E_{x1}^{n+1/2} = E_{x2}^{n-2+1/2}$$



$$E_{x,L}^{n+1/2} = E_{x,L-1}^{n-2+1/2}$$

有电介质的物理

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0\varepsilon_r}\nabla\times\vec{H} - \underline{\frac{\sigma}{\varepsilon_0\varepsilon_r}\vec{E}}$$

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu_0}\nabla\times\vec{E}$$

$$\downarrow$$

$$-\frac{\sigma}{\varepsilon_0\varepsilon_r}\cdot\left(E_{xk}^{n-1/2}\right)$$

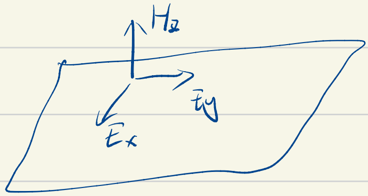
求有 $-\frac{\sigma}{\varepsilon_0\varepsilon_r}\cdot\left(E_{xk}^{n+1/2} + E_{xk}^{n-1/2}\right)$

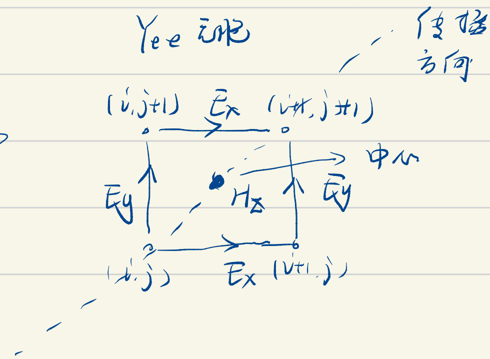
$$\Rightarrow \quad E_{xk}^{n+1/2} = \frac{1-\frac{\sigma\Delta t}{2\varepsilon_r\varepsilon_0}}{1-\frac{\sigma\Delta t}{2\varepsilon_r\varepsilon_0}} E_{xk}^{n-1/2} + \frac{\frac{\Delta t}{C\Delta z}\frac{1}{\sqrt{\varepsilon_r}}}{1+\frac{\sigma\Delta t}{2\varepsilon_0\varepsilon_r}}\left(H_{y\,k-1/2}^{n} - H_{y\,k+1/2}^{n}\right)$$

另外一个分量不变化

Yee 元胞

2D, 3D 会怎样？





☆ leapfrog 蛙跳

牛顿方程

$$\begin{cases} \dot{x} = v \\ \dot{v} = f(x,t) \end{cases}$$

Maxwell 方程

$$\begin{cases} \dot{E}_x \sim \partial_z H_y \\ \dot{H}_y \sim \partial_z E_x \end{cases}$$

$$\begin{cases} x_{n+1} - x_n = v_{n+1/2}\, h \\ v_{n+1/2} - v_{n-1/2} = f(x_n, t_n)\, h \end{cases}$$

☆ 库朗数 $C = \frac{v\Delta t}{\Delta x}$ or $\frac{D\Delta t}{2\Delta x^2}$

$$\left.\begin{array}{l} \\ C < 1 \quad \text{or} \quad C \ll 1 \end{array}\right\} \text{收敛.}$$

<u>East FDTD</u>，一个软件.