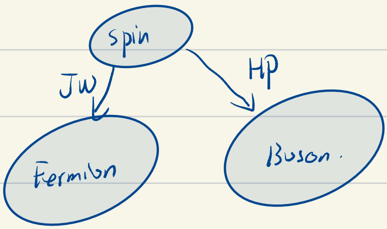
Review.

XXZ Model. Exact diag



☆ Symmetry 守恒量.

$k = k_0 \oplus k_1 \oplus \cdots$

Bethe-Ansatz 贝特解.

① Nagaosa, chap. 1.

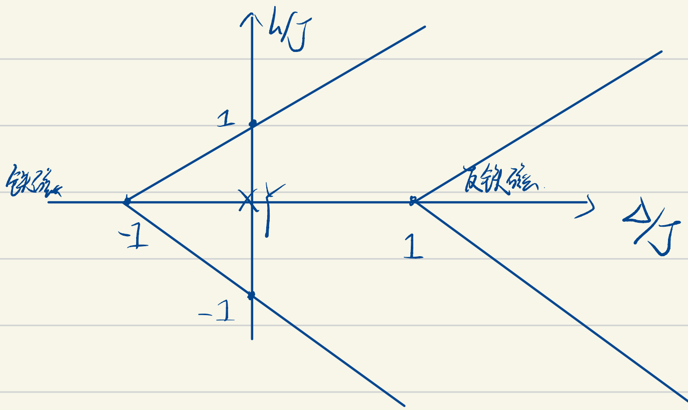
② On the theory of metals

③ C.N.Yang, Yang–Baxter equation.

④ Ting Ting Liu. "The Bethe-Ansatz solution of 1D Heisenberg Model"

$$H = \sum_i J(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta S_i^z S_{i+1}^z - h S_i^z$$

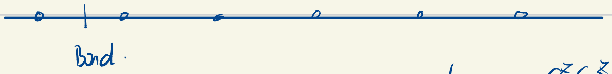
基态的相图



利用J-W变换.

$$H = \sum_i \frac{1}{2}(c_i^\dagger c_{i+1} + h.c) + \Delta(n_i - \tfrac{1}{2})(n_{i+1} - \tfrac{1}{2}) - h(n_i - \tfrac{1}{2})$$

$J=0$时. Ising Model.



$$L_{i+\frac{1}{2}} = S_i^z S_{i+1}^z$$

$$H = \sum_i J_z L_{i+\frac{1}{2}}, \quad J_z = \Delta$$

$$Z = \prod_i Z_i = \prod_i \left(e^{-\frac{\beta J_z}{4}} + e^{\frac{\beta J_z}{4}}\right)$$

则 $$f = -\frac{1}{\beta N}\ln Z = \frac{1}{\beta}\ln\left(2\cosh\left(\frac{1}{4}\beta J_z\right)\right) = \left[2\cosh\left(\frac{1}{4}\beta J_z\right)\right]$$

$$\langle \sigma_i \sigma_j \rangle = \left[\tanh\left(\frac{-1}{4}\beta J_z\right)\right]^N$$

$\Rightarrow$ 关联长度. $\xi^{-1} = \ln\left[\tanh\left(-\frac{1}{4}\beta J_z\right)\right]$

$\boxed{\text{作业. ，推导chapter 1}}$

考虑铁磁 $J<0$，则基态为 $|\uparrow\uparrow\uparrow\cdots\uparrow\rangle$ or $|\downarrow\downarrow\cdots\downarrow\rangle$

而由于XXZ模型保持自旋数，我们可以考虑不同自旋数的分块。

① 自旋翻转一次的空间.

第n个位置翻转.

设 $|\psi\rangle = \sum \frac{1}{\sqrt{N}}\psi_n |n\rangle$. $\quad |n\rangle \overset{def}{=} |\downarrow\downarrow\cdots\underline{\uparrow}\downarrow\cdots\downarrow\rangle$

而 $H|\psi\rangle = E|\psi\rangle$ 可以写为

$$\Rightarrow \frac{J}{2}(\psi_{n+1} + \psi_{n-1}) + E_0\psi_n = E\psi_n$$

利用傅利叶变换可以求解，有.

$$\underline{(E_0 - J_z + J\cos(k))\tilde{E}_k = E_k \tilde{E}_k}$$

② 自旋翻转二次的空间.

$$|\varphi\rangle = \sum_{nm} \varphi_{n,m} S_n^+ S_m^+ |\downarrow\downarrow\downarrow \cdots \downarrow\rangle$$

$$= \sum_{nm} \varphi_{nm} |n, m\rangle$$

令 $H = H_{xy} + H_z$ ，则

$$\begin{cases} m \neq n+1 & H|n,m\rangle = (E_0 - 2J_z)|n,m\rangle + \frac{J}{2}\left(|n-1,m\rangle + |n+1,m\rangle + |n,m+1\rangle + |n,m-1\rangle\right) \\ m = n+1, & H|n,n+1\rangle = (E_0 - 2J_z)|n,n+1\rangle + \frac{J}{2}\left(|n-1,n+1\rangle + |n,n+2\rangle\right) \end{cases}$$

则 本征方程 可以写为

$m \neq n+1$, $\quad -2J_z \varphi_{n,m} + \frac{J}{2}\left(\varphi_{n-1,m} + \varphi_{n+1,m} + \varphi_{n,m+1} + \varphi_{n,m-1}\right) = E\varphi_{n,m}$.

$m = n+1$, $\quad -2J_z \varphi_{n,m} + \frac{J}{2}\left(\varphi_{n+1,n+1} + \varphi_{n,n-2}\right)$