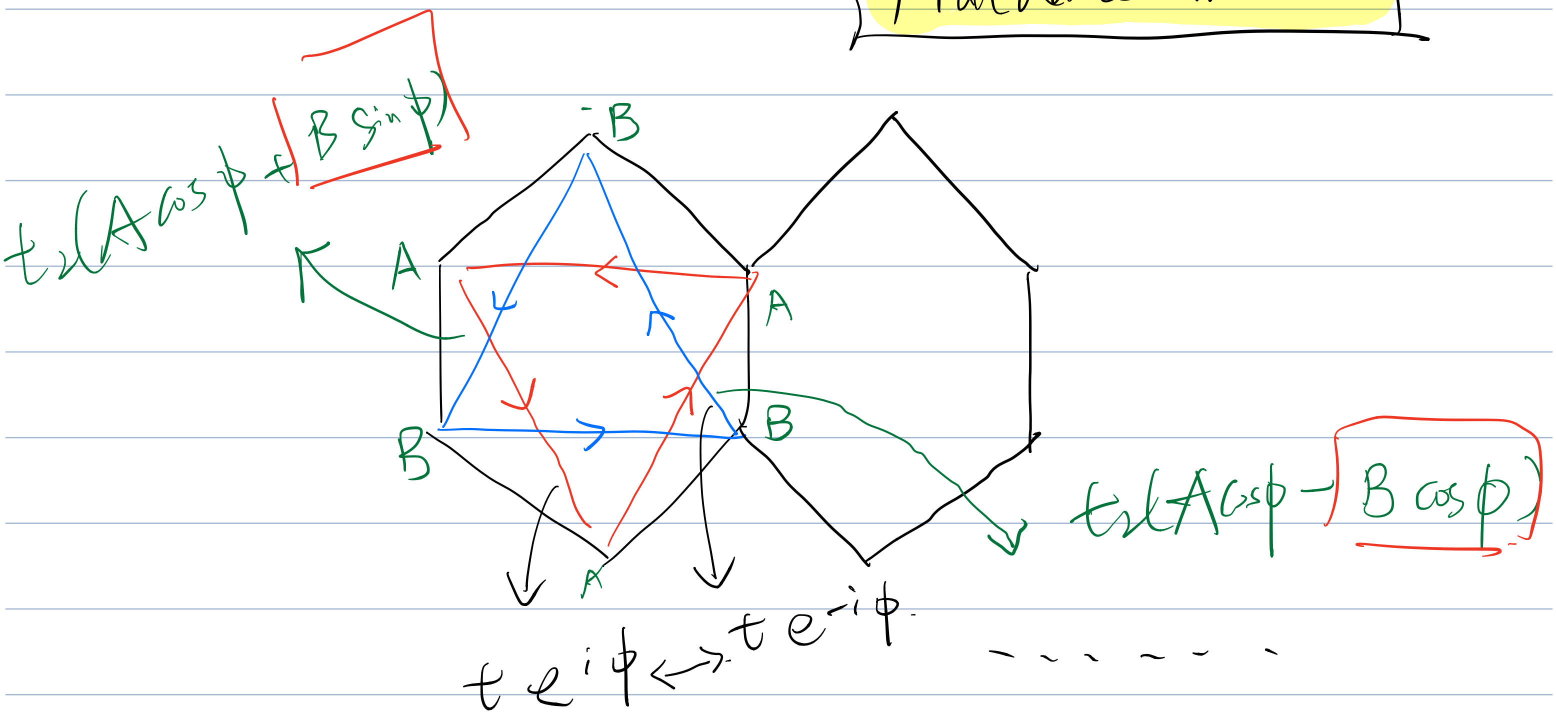
2021. 4. 22.

Haldane model.



$\Rightarrow$ Extended Dirac eq.

$$H = (\mu + \boxed{DR^2})\sigma_z + V(k_x\sigma_x + k_y\sigma_y).$$

重要!!!

One-point compactification. (单点紧致性)

$\mathbb{R}^2 \cup \{\infty\} \simeq S^2.$

Haldane model ~ 实验实现: 2005; 冷原子

Haldance paper.

1: 观点: $G = ne^2/h$ 本质 $\vec{A}$, 而非 $\vec{B}$.

$n \in \mathbb{Z}$. without LL. (朗道能级)

2. 破坏 A, B 简并性.

引入 $B_x \sin\phi\, \sigma_z$.

3. $H = v \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$

$\downarrow H(\vec{k} + e\vec{A})$.

(Dirac 点附近的 LL.)

$\hbar\omega(n+\frac{1}{2})$

$H^2 = \begin{pmatrix} 0 & \Pi_x + i\Pi_y \\ \Pi_x - i\Pi_y & 0 \end{pmatrix} \begin{pmatrix} 0 & \Pi_x + i\Pi_y \\ \Pi_x - i\Pi_y & 0 \end{pmatrix} \propto$

$= \begin{pmatrix} (\Pi_x + i\Pi_y)(\Pi_x - i\Pi_y) & 0 \\ 0 & (\Pi_x - i\Pi_y)(\Pi_x + i\Pi_y) \end{pmatrix}$

$\Rightarrow (\Pi_x + i\Pi_y)(\Pi_x - i\Pi_y) = \Pi_x^2 + \Pi_y^2 - i[\Pi_x, \Pi_y]$

$$[\pi_x, \pi_y] = [\underbrace{p_x + e\overbrace{A_x, \; p_y}} + eA_y].$$

$$= e(p_x A_y - p_y A_x)$$

$$= e(\nabla \times \vec{A})_z.$$

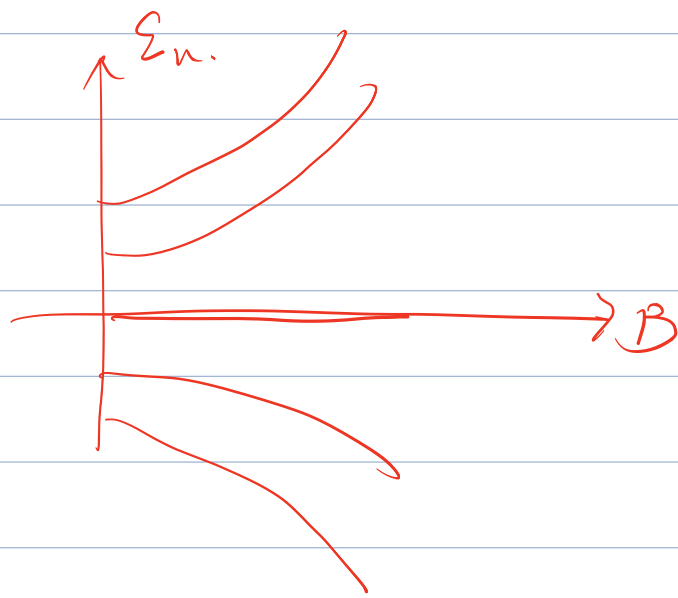
$$= eB$$

$$[\Pi_x, \Pi_y] = eB.$$

$$\begin{cases} \Pi_x = \sqrt{eB}\, a. \\ \Pi_y = \sqrt{eB}\, a^{+} \end{cases}$$

$$\varepsilon_{2n\pm} = \pm\sqrt{(mc^2)^2 + n\hbar(eB)c^2}$$

$n = 0, \quad \varepsilon_{20} = 0$



4. 能谱: $H(k) = \varepsilon(k) + d_x\sigma_x + d_y\sigma_y + d_z\sigma_z.$

$$= \varepsilon_0(k) + \begin{pmatrix} d_z & d_x - i d_y \\ d_x + i d_y & -d_z \end{pmatrix}$$

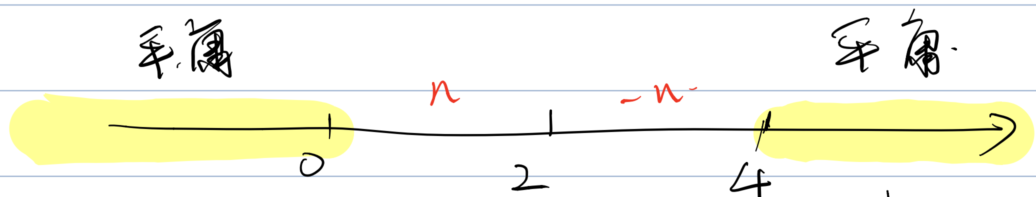
$$\varepsilon_k = \varepsilon_0 \pm |d|.$$

$\boxed{\text{Qi-Wu-Zhang}}$ PRB. 74. 085308 (2006).

$$\begin{cases} d_x = \sin k_y. \\ d_y = -\sin k_x \\ d_z = 2[2-\cos k_x - \cos k_y - e_s] \end{cases}$$

$$\varepsilon_k = \pm |d|$$

$$\varepsilon_k = 0 \Rightarrow k_x, k_y = 0, \pi, \quad e_s = 0, 2, 4.$$



可以 $e_s \to \pm\infty$，来理解.

$$H = \left(\begin{array}{c|c} d_z & d_x - i d_y \\ \hline d_x + i d_y & -d_z. \end{array}\right)$$

$$H|\psi\rangle = -|d|\,|\psi\rangle$$

$$|\psi\rangle = \begin{pmatrix} x \\ y \end{pmatrix}.$$

$$d_z x + (d_x - i d_y) y = -|d|\, x$$

$$\Downarrow$$

$$(d_z + |d|)\, x = (i d_y - d_x)\, y = 0$$

$$\boxed{|\psi\rangle \sim R \begin{pmatrix} i d_y - d_x \\ d_z + |d| \end{pmatrix}}$$

$$|\psi\rangle = \underset{\uparrow\text{归一化.}}{R}\begin{pmatrix} u \\ v \end{pmatrix} \nearrow 1.$$

$$\vec{A} = (u^*, v^*)R(\nabla R)\begin{pmatrix} u \\ v \end{pmatrix} + (u^*, v^*)R^2\begin{pmatrix} \nabla u \\ \nabla v \end{pmatrix}$$

$$\vec{A} = \langle\psi|\nabla|\psi\rangle$$

$$= \underbrace{\nabla \ln R}_{\int \sim dk = 0} + \underbrace{\boxed{\left(\frac{u^*\nabla u + v^*\nabla v}{|u|^2 + |v|^2}\right)}}_{\int \sim dk \neq 0.} \quad ☆☆☆$$

$k_x, k_y \sim 0. \Rightarrow \begin{cases} d_x = k_x \\ d_y = +k_y \\ d_z = A + Bk^2 \end{cases}$

计算：

① $\oint A_x dk_x + A_y dk_y \xrightarrow{②} \oint B\, dk_x dk_y$

$B = (\partial_x A_y - \partial_y A_x)$

$\downarrow$ ③.

求积分. $k_x, k_y \to k, \theta$. (极坐标)

HW.

用 model. ① . k_x, k_y, $A + Bk^2$. $\underbrace{(-\infty, +\infty)}_{k_x\ \ k_y} \to \underbrace{(0, +\infty)}_{k,\ \theta}$

② . Qwz. $(k_x, k_y \in BZ)$

$\downarrow$

$(-\pi, \pi)$

SSH model 1d. 1976 Soliton → 分数化. 聚乙炔

↓实现了-R.

Jackin-Rebbi model 1d. 场论. 't Hooft.

Su. Wu. Pei
Sehrieffer (BCS).
Heeger.

ref: PRB. 22. 2099 (1980)
PRL. 42. 1698 (1979).
PRL. 46. 738 (1981).

$\Downarrow$

S-S-H. 1D 是不稳定, 会形成聚乙炔,
从金属 → 绝缘体 → 能量更低

$$\begin{pmatrix} m(x) & -i\frac{\partial}{\partial x} \\ -i\frac{\partial}{\partial x} & -m(x) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = 0.$$

$$\begin{cases} mu - i\frac{\partial}{\partial x} v = 0 \\ -i\frac{\partial}{\partial x} u - mv = 0 \end{cases} \Leftarrow |\psi\rangle \sim \begin{pmatrix} a \\ b \end{pmatrix} e^{\theta(x)}$$

$\Downarrow$

$$mae^{\theta} - i\, be^{\theta}(\theta') = 0$$

$$\begin{cases} ma - ib\,\theta' = 0 \\ -ia\,\theta' - mb = 0 \end{cases} \Rightarrow \begin{cases} ma = ib\theta' \\ -ia\theta' = mb \end{cases}$$

$$-ia^2 m\theta' = im\theta' b^2 \Rightarrow a^2 = -b^2.$$

$$\therefore |\psi\rangle \sim \begin{pmatrix} 1 \\ \pm i \end{pmatrix} e^{-\int_0^x m(x')dx'}$$

Δ

$\int_0^x$ 为何这里取?

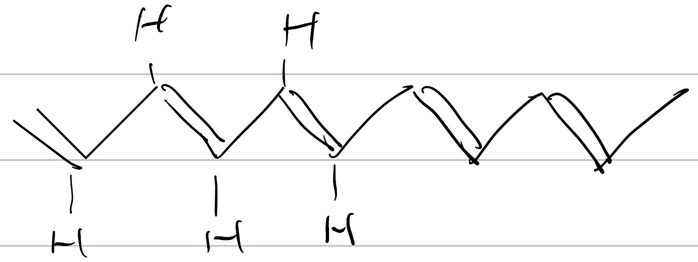
$$H^2 = \begin{pmatrix} m(x) & -i\frac{\partial}{\partial x} \\ -i\frac{\partial}{\partial x} & -m(x) \end{pmatrix} = \begin{pmatrix} m^2 - \frac{\partial^2}{\partial x^2} & \\ & m^2 - \frac{\partial^2}{\partial x^2} \end{pmatrix}$$

$$-i\frac{\partial}{\partial x} m + mi\frac{\partial}{\partial x} = -i\left(\frac{\partial}{\partial x} m\right) = -ig\,\delta(x)$$

$\Rightarrow H^2 = m^2 - \frac{\partial^2}{\partial x^2} \pm g\,\delta(x)$ 有束缚态 $g<0$

$\sqrt{H^2} \to H \to e^{-\int m(x)dx}$

SSH.



① SSH. 2个简并基态.

②. From J-R model, 2个简并基态 $\Rightarrow$ soliton, in gap

$$SSH = \sum_n \left(t_{n+1,n} C^\dagger_{n+1} C_n + h.c \right)$$
$$+ \sum_n \frac{1}{2} k (u_{n+1} - u_n)^2 + \frac{1}{2} M \dot{u}_n^2.$$

↑ phonon.

$$t_{n+1,n} = t_0 - \alpha (u_{n+1} - u_n).$$

How to do ???

(1). 量子力学处理. 量子化. → 行不通.

(2) 经典处理 $\begin{cases} u_n = (-1)^n u. & \text{常数}, u' = 0 \\ u_n = u \cos(\frac{2}{3}\pi n + \theta) \end{cases}$

$$t_{n+1,n} = t_0 - \alpha (u_{n+1} - u_n)$$
$$= t_0 + 2\alpha u (-1)^n$$
$$= \begin{cases} t_0 + t_1 \\ t_0 - t_1 \end{cases} \quad t_1 = 2\alpha u.$$

$$\sum_n \frac{1}{2} k (u_{n+1} - u_n)^2$$
$$\sim \frac{1}{2} k u^2 \times 4, \quad \boxed{N = 2Nku^2} \leftarrow \text{声子}$$

$\therefore H \Rightarrow (t_0+t_1) C^{\dagger}_{iA} C_{iB} + (t_0-t_1) C^{\dagger}_{iB} C_{i+1,A}$.

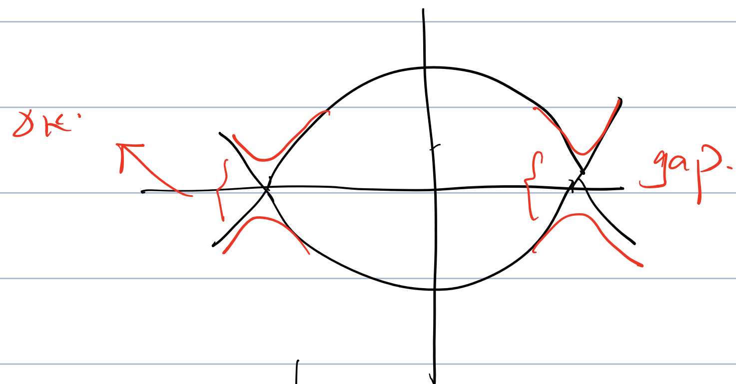
$E_k = \pm\sqrt{\varepsilon_k^2 + \sigma_k^2}$

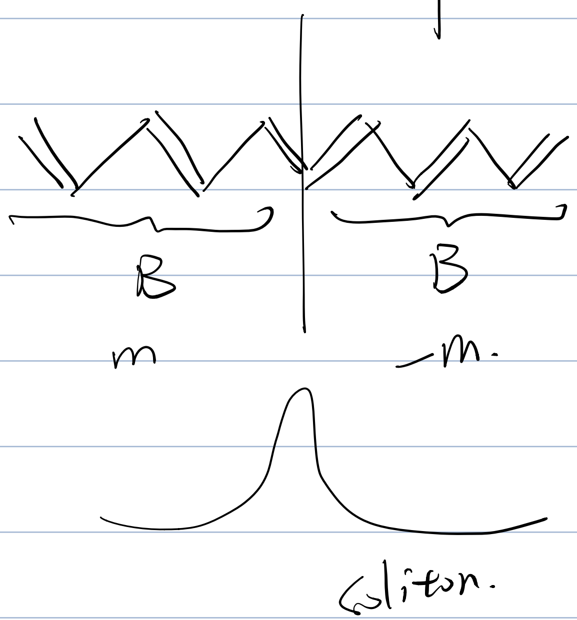
$\begin{cases} \varepsilon_k = -2t_0 \cos(ka) \\ \sigma_k = 4\alpha u \sin(ka). \end{cases}$

由 $T(a) \longrightarrow T(2a)$

$\Downarrow$

与 Haldane 类似 (由 Su 中 提供)





以上三篇文章 ☆☆☆ Su Wu pei (PRB)

☆ HW: PRL. 46. 738 (1981) 分数化. $u_n = u\cos(\frac{2}{3}\pi n - \theta)$