$\Delta_{\vec{k}} = \sum_{\delta} e^{i\vec{k}\cdot\vec{\delta}} = \sum_{\delta}[\cos(\vec{k}\cdot\vec{\delta}) + i\sin(\vec{k}\cdot\vec{\delta})]$

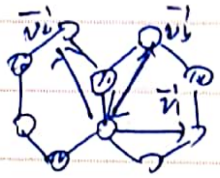
$h(\vec{k}) = -t\sum_{\delta}[\cos(\vec{k}\cdot\vec{\delta})\sigma_x + -\sin(\vec{k}\cdot\vec{\delta})\sigma_y]$

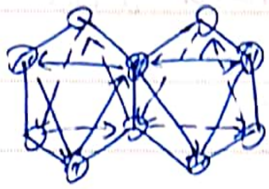
若考虑 inversion-symmetry-breaking on-site energy $+M$ on A and $-M$ on B，之前的 VN ~~处~~ 不能直接忽略，最终结果为

$$h_M(\vec{k}, M) = -t\sum_{\delta}[\cos(\vec{k}\cdot\vec{\delta})\sigma_x - \sin(\vec{k}\cdot\vec{\delta})\sigma_y + M\sigma_z]$$

$$= -t\sum_{\delta}\begin{pmatrix} M & \cos(\vec{k}\cdot\vec{\delta}) + i\sin(\vec{k}\cdot\vec{\delta}) \\ \cos(\vec{k}\cdot\vec{\delta}) - i\sin(\vec{k}\cdot\vec{\delta}) & -M \end{pmatrix}$$

H_{NNN} hopping:





$$H_{NNN} = -t'e^{i\phi}\sum_i a_{r_i}^{\dagger} a_{r_i+v_1} - t'\sum_i a_{r_i+v_1}^{\dagger} a_{r_i-v_3} - t'e^{i\phi}\sum_i a_{r_i-v_3}^{\dagger} a_{r_i} + h.c. + (a \to b \text{ and } \phi \to -\phi)$$

$\vec{v}_1 = (\sqrt{3}a, 0)$ $\quad \vec{v}_2 = (-\sqrt{3}/2\,a, 3/2\,a)$ $\quad \vec{v}_3 = (-\sqrt{3}/2\,a, -3/2\,a)$

换成 k 空间

$$H = -t'e^{i\phi}\sum_k a_k^{\dagger} a_k (e^{-ik\cdot v_1} + e^{-ik\cdot v_1} + e^{-ik\cdot v_3}) + h.c. + (a \to b \text{ and } \phi \to -\phi)$$

$$= \sum_k (a_k^{\dagger}\ b_k^{\dagger})\begin{pmatrix} -2t'(\cos(k\cdot v_1 - \phi) + \cos(k\cdot v_1 - \phi) + \cos(k\cdot v_3 - \phi)) & 0 \\ 0 & -2t'(\cos(k\cdot v_1 + \phi) + \cos(k\cdot v_1 + \phi) + \cos(k\cdot v_3 + \phi)) \end{pmatrix}\begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

或写成 Pauli 矩阵形式

$$H_{NNN} = 2t_2\cos\phi\left(\sum_i \cos(\vec{k}\cdot\vec{v}_i)\right) I + (-2t_2)\sin\phi\left(\sum_i \sin(\vec{k}\cdot\vec{v}_i)\right)\sigma_z$$

回顾：

① 几何相数值计算

$e^{i\gamma} = \prod_i \langle \psi_i | \psi_{i+1} \rangle$ ← 投影矩阵

好处： 1) Gauge Invariant

2) 便于实验研究

扫描全能王 创建

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3) 推广到 2d   Stokes theorem

转移矩阵

$$\langle\psi_1|\underbrace{\psi_2\rangle\langle\psi_2}_{P_2}|\underbrace{\psi_3\rangle\langle\psi_3}_{P_3}|\underbrace{\psi_4\rangle\langle\psi_4}_{P_4}|\underbrace{\psi_5\rangle\langle\psi_5}_{P_5}|\psi_1\rangle = Tr(P_1P_2P_3P_4P_5)$$

投影子 $P_i=|\psi_i\rangle\langle\psi_i|$

② Hall Conductance

key point: Landau level $\Rightarrow \varepsilon_g$

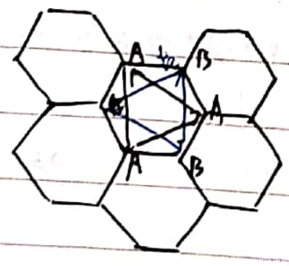
$$G=\frac{ne^2}{h}$$

Laughlin argument

基础: Landau level

Haldane 1988 没有 Landau level   根据: 规范势

2015 在冷原子中实现



Graphene Model

$$H=v\begin{pmatrix}0 & k_x+ik_y\\ k_x-ik_y & 0\end{pmatrix}$$

$$\varepsilon_k=\pm v|k|$$

需有山谷相互作用

关键: 1.为什么在非对角部分?

$$\sum_{ij} t_{ij}c_{iA}^\dagger c_{jB}\longrightarrow\sum_k(\cdots)\,c_{kA}^\dagger c_{kB}$$

一定没有 $c_{kA}^\dagger c_{kA}$. $c_{kB}^\dagger c_{kB}$

$$(c_{kA}^\dagger\quad c_{kB}^\dagger)\begin{pmatrix}0 & /// \\ /// & 0\end{pmatrix}\begin{pmatrix}c_{kA}\\ c_{kB}\end{pmatrix}$$

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2. 如何产生对角项？

引入对~~角项~~ 次近邻相互作用

$(\Sigma_A(t_2))\, C_{kA}^{\dagger} C_{kA} + (\Sigma_{kB}(t_2))\, C_{kB}^{\dagger} C_{kB}$

1) $t_2 \in \mathbb{R}$ C_{6v} 对称性

$\Sigma_{kA} = \Sigma_{kB}$，无影响

2) $t_2 e^{\pm i\phi}$

$\Sigma_{kA} \propto t_2 (A\cos\phi + B\sin\phi)$

$\Sigma_{kB} \propto t_2 (A\cos\phi - B\sin\phi)$

$$\begin{pmatrix} B_k \sin\phi & k_x + ik_y \\ k_x - ik_y & -B_k \sin\phi \end{pmatrix}$$

① Effective Zeeman Splitting

② Zeeman field $\propto B_k \sin\phi$ 与 $\vec{k}$ 有关

Shen S. Q. Extended Dirac eq.

$H = (M + D k^2)\sigma_z + v(k_x\sigma_x + k_y\sigma_y)$

无穷远处，自旋相同，可以用 one-point compactification

Haldane paper

1. 观点： $G = ne^2/h$ 来源 $\vec{A}$，而非 $\vec{B}$ [ref. AB effect]

2. 破坏 A、B 的等价性 引入 $B_k \sin\phi\, \sigma_z$

3. $H = v\begin{pmatrix} 0 & k_x + ik_y \\ k_x - ik_y & 0 \end{pmatrix}$ 磁场中 $H(\vec{k} + e\vec{A})$

eq2. – eq4. Dirac 点附近的 LL

Hilbert space of Dirac eq.

$H|\psi_A\rangle \to |\psi_B\rangle$

$H|\psi_B\rangle \to |\psi_A\rangle$

$|\psi_A\rangle \xrightarrow{H} |\psi_B\rangle \xrightarrow{H} |\psi_A\rangle$

$$H^2 = \begin{pmatrix} 0 & \pi_x + i\pi_y \\ \pi_x - i\pi_y & 0 \end{pmatrix}\begin{pmatrix} 0 & \pi_x + i\pi_y \\ \pi_x - i\pi_y & 0 \end{pmatrix}$$

$(\vec{\pi} = \vec{p} + e\vec{A})$

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$$= \begin{pmatrix} (\Pi_x+i\Pi_y)(\Pi_x-i\Pi_y) & 0 \\ 0 & (\Pi_x-i\Pi_y)(\Pi_x+i\Pi_y) \end{pmatrix}$$

$$(\Pi_x+i\Pi_y)(\Pi_x-i\Pi_y) = \Pi_x^2+\Pi_y^2 - i[\Pi_x,\Pi_y]$$

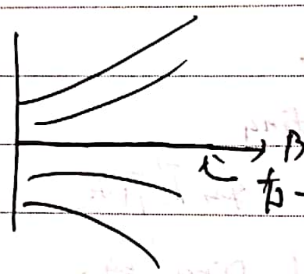
$$(\Pi_x-i\Pi_y)(\Pi_x+i\Pi_y) = \Pi_x^2+\Pi_y^2 + i[\Pi_x,\Pi_y]$$

$$[\Pi_x,\Pi_y] = [P_x+eA_x, P_y+eA_y] = e(P_xA_y - P_yA_x) = e(\nabla\times A)_i =$$

$$\begin{cases} \Pi_x = \sqrt{eB}\,a \\ \Pi_y = \sqrt{eB}\,a^\dagger \end{cases} \qquad [a, a^\dagger] = 1$$

$$\varepsilon_{\alpha n\pm} = \pm\sqrt{(mc^2)^2 + n\hbar eBc^2} \qquad n \geq 1$$

$$n=0 \qquad \varepsilon_{\alpha 0} = 0$$



有一个态的LL平均为0

4) 能谱

$$H = \varepsilon_0(k) + d_x(k)\sigma_x + d_y(k)\sigma_x + d_z(k)\sigma_z$$

$$= \varepsilon_0(k) + \begin{pmatrix} d_z & d_x - id_y \\ d_x+id_y & -d_z \end{pmatrix}$$

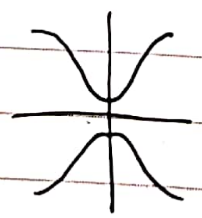
(Berry phase $H = \vec{B}\cdot\vec{\sigma}$)

$B_x = d_x \quad B_y = d_y \quad B_z = d_z$

$$d = (d_x, d_y, d_z)$$

$$|d| = \sqrt{d_x^2+d_y^2+d_z^2}$$

$$\varepsilon = \varepsilon_0 \pm |d|$$



Qi–Wu–Zhang PRB, 74, 085308 (2006)

$d_x = \sin k_y$

$d_y = \cancel{\sin k_x} - \sin k_x$

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$d_z = c(2 - \cos k_x - \cos k_y - e_s)$

$|\vec{d}| = 0 \Leftrightarrow \begin{cases} d_x = 0 \\ d_y = 0 \\ d_z = 0 \end{cases} \Rightarrow k_x, k_y = 0, \pi \quad e_s = 0, 2, 4$

gap=0 时 发生相变

$$\begin{array}{ccc} & |d|=0 & \\ 0 & 2 & 4 \quad \to e_s \\ n & & -n \end{array}$$

$e_s \to \pm\infty$ 时，而导能带 ...平凡的

若 $(0,2)$ 中间 $G = ne^2/h$，则 $(2,4)$ 中间 $G = -ne^2/h$

计算：

$H = \begin{pmatrix} d_z & d_x - id_y \\ d_x + id_y & -d_z \end{pmatrix}$

$H|\psi\rangle = -|d||\psi\rangle \qquad |\psi\rangle = \begin{pmatrix} x \\ y \end{pmatrix} \qquad |x|^2 + |y|^2 = 1$

$\begin{cases} d_z x + (d_x - id_y) y = -|d| x \\ (d_z + |d|) x = (id_y - d_x) y \end{cases}$

$|\psi\rangle \propto \begin{pmatrix} id_y - d_x \\ d_z + |d| \end{pmatrix} = \frac{1}{\sqrt{d_x^2 + d_y^2 + (d_z + |d|)^2}} \begin{pmatrix} id_y - d_x \\ d_z + |d| \end{pmatrix}$

$|\psi\rangle = R\begin{pmatrix} u \\ v \end{pmatrix} \qquad R$：归一化因子 $\quad u = id_y - d_x \quad v = d_z + |d|$

求 $\gamma = i\oint \langle\psi|\nabla_k|\psi\rangle \cdot d\vec{k}$

注意 1) well-defined $\quad e^{i\gamma} = \prod \langle\psi_i|\psi_{i+1}\rangle \quad \gamma \in [0, 2\pi)$

2) Berry 联系 $G = ne^2/h$

$|\psi\rangle = R\begin{pmatrix} u \\ v \end{pmatrix} \Leftrightarrow R^2(|u|^2 + |v|^2) = 1 \qquad R \in \mathbb{R}$

$\vec{A} = \langle\psi|\nabla|\psi\rangle$

$= (u^* \; v^*) R \nabla R \begin{pmatrix} u \\ v \end{pmatrix}$

$= (u^* \; v^*)(R\nabla R)\begin{pmatrix} u \\ v \end{pmatrix} + R^2 (u^* \; v^*) \begin{pmatrix} \nabla u \\ \nabla v \end{pmatrix}$

$= (|u|^2 + |v|^2) R\nabla R + R^2 (u^* \nabla u + v^* \nabla v)$

$= \nabla(\ln R) + R^2(u^* \nabla u + v^* \nabla v)$

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$$= \nabla \ln R + \frac{u^* \nabla u + v^* \nabla v}{|u|^2 + |v|^2}$$

$\oint (\nabla \ln R) \cdot d\vec{R}$ 系统有 gap, $R \neq 0$

$\Rightarrow \oint (\nabla \ln R) \cdot d\vec{R} = \oint d \ln R = 0$

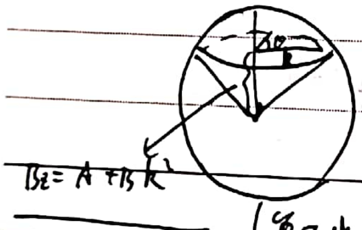
考虑 $\begin{cases} d_x = k_x \\ d_y = k_y \\ d_z = A + Bk^2 \end{cases}$

$u = i d_y - d_x = i k_y - k_x$

$v = A + Bk^2 + \sqrt{k_x^2 + k_y^2 + (A + Bk^2)^2}$

考虑 $|k| \to \infty$ 让 $k_x = k\cos\theta$ $k_y = k\sin\theta$

$H = \begin{pmatrix} A + Bk^2 & k e^{i\theta} \\ k e^{-i\theta} & A - Bk^2 \end{pmatrix} \Leftrightarrow |\psi\rangle = \begin{pmatrix} e^{i\theta} \tilde{u} \\ \tilde{v} \end{pmatrix}$



(算不出来)

$$A_x = \frac{u^* \partial_{k_x} u + v^* \partial_{k_x} v}{|u|^2 + |v|^2}$$

$$A_y = \frac{u^* \partial_{k_y} u + v^* \partial_{k_y} v}{|u|^2 + |v|^2}$$

作业: 1) A_x, A_y 计算出来

D[f, x]

2) $\oint A_x dk_x + A_y dk_y$ Stokes 定理, $\oint B \, dk_x dk_y$

$B = (\partial_x A_y - \partial_y A_x)$

FullSimplify

3) 书结合 $k_x, k_y \to k, \theta$

Models

① $d_x = k_x$ $d_y = k_y$ $d_z = A + Bk^2$ $(-\infty, +\infty)$

②. QWZ . model

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$\begin{cases} \text{SSH Model 1d} \quad 1976 \quad \text{Soliton} \Rightarrow \text{分数化} \quad \text{聚乙炔} \\ \text{Jackiw-Raebbi model : 场论} \end{cases}$

ref. PRB, 22, 2099 (1980)

PRL, 42, 1698 (1979)

PRL, 46, 738 (1981)

Jackiw-Rebbi model

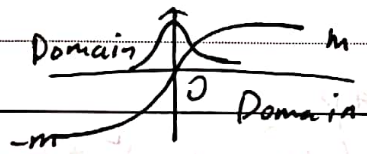
$H = p\sigma_x + m(x)\sigma_z \qquad p = -i\frac{\partial}{\partial x}$

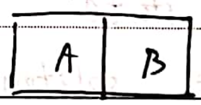
☆ $m(x) = m \qquad p \to k$

$H = k\sigma_x + m\sigma_z \Rightarrow \varepsilon = \pm\sqrt{k^2 + m^2}$

特点 $\varepsilon(m) = \varepsilon(-m)$

degeneracy





存在束缚态 $H|\psi\rangle \equiv 0$

解: $\begin{pmatrix} m(x) & -i\frac{\partial}{\partial x} \\ -i\frac{\partial}{\partial x} & -m(x) \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0$

$\begin{cases} mu - i\frac{\partial}{\partial x}v = 0 \\ -i\frac{\partial}{\partial x}u - mv = 0 \end{cases}$

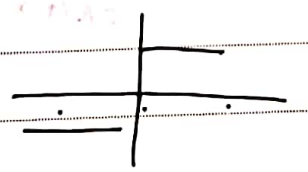
$|\psi\rangle \sim \begin{pmatrix} a \\ b \end{pmatrix} e^{\theta(x)} \qquad u = ae^{\theta} \qquad v = be^{\theta}$

$\begin{cases} mae^{\theta} - ibe^{\theta}\theta' = 0 \Rightarrow ma - ib\theta' = 0 \\ -iae^{\theta}\theta' - mbe^{\theta} = 0 \Rightarrow -ia\theta' - mb = 0 \end{cases}$

$\begin{cases} ma = ib\theta' \\ -ia\theta' = mb \end{cases}$

相乘, $a^2 = -b^2 \Rightarrow |\psi\rangle \sim \begin{pmatrix} a \\ b \end{pmatrix} e^{\theta(x)} = \begin{pmatrix} 1 \\ \pm i \end{pmatrix} e^{-\int_0^x m(x')dx'}$

例 $\Rightarrow$ $m(x) =$



$-i\frac{\partial}{\partial x}m + m\frac{\partial}{\partial x} = -i\left(\frac{\partial}{\partial x}m\right)$

$= -ig\,\delta(x)$

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$$H=\begin{pmatrix} m & -i\frac{\partial}{\partial x} \\ -i\frac{\partial}{\partial x} & -m \end{pmatrix}$$

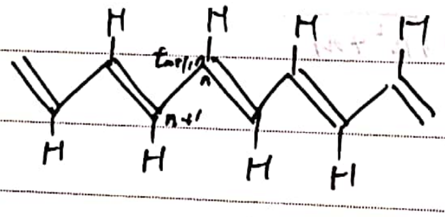
$$H^2=\begin{pmatrix} m^2-\frac{\partial^2}{\partial x^2} & \cdots \\ \cdots & m^2-\frac{\partial^2}{\partial x^2} \end{pmatrix}$$

$$H^2 = m^2-\frac{\partial^2}{\partial x^2} \pm g\delta(x)$$

Jackiw-Rebbi model $\Rightarrow$ $\sqrt{1d \quad \delta(x) \text{ potential}}$

Pereils 不稳定性

Model



$\begin{cases} 2\text{个 degenerate ground state} \\ \text{From Jackiw-Rebbi} \Rightarrow \text{soliton in gap} \end{cases}$

$$H=-\sum_n \big(t_{n+1,n}\underbrace{c_{n+1}^\dagger c_n}_{\text{电子}}+h.c.\big)+\underbrace{\sum_n \tfrac{1}{2}k(u_{n+1}-u_n)^2+\tfrac{1}{2}m\dot{u}_n^2}_{\text{phonon}}$$

$$t_{n+1,n}=t_0-\alpha(u_{n+1}-u_n)$$

怎么做:

1) 量子力学处理: 量子化 行不通

2) 经典处理 $u_n=(-1)^n u$ $\quad u$ 常数 $\quad \dot{u}=0$

$$u_n = u\cos(\tfrac{2}{3}\pi n+\theta)$$

$$\begin{aligned} t_{n+1,n} &= t_0-\alpha(u_{n+1}-u_n) \\ &= t_0+2\alpha u(-1)^n \\ &= \begin{cases} t_0+t_1 \\ t_0-t_1 \end{cases} \qquad t_1=2\alpha u \end{aligned}$$

$$H=-\sum_n t_{n+1,n}c_{n+1}^\dagger c_n+\underbrace{\sum_n \tfrac{1}{2}k(u_{n+1}-u_n)^2}_{\frac{1}{2}ku^2\times 4\cdot N=2Nku^2}$$

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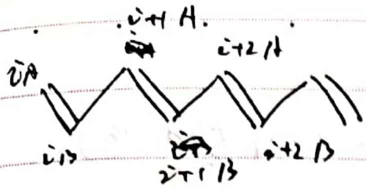


$(t_0+t_1) C_{iA}^\dagger C_{iB} + (t_0-t_1) C_{iB}^\dagger C_{i+1\,A}$

FT ~~(t_0+t_1)~~ $(t_0+t_1) e^{ik(R_B^i - R_A^i)} C_{kA}^\dagger C_{kB} + (t_0-t_1) e^{ik(R_A^{i+1} - R_B^i)} C_{kB}^\dagger C_{kA}$

$= (t_0+t_1) e^{ika} C_{kA}^\dagger C_{kB} + (t_0-t_1) e^{ika} C_{kB}^\dagger C_{kA}$

若晶格常数不同，

$= (t_0+t_1) e^{ik(a-2u)} C_{kA}^\dagger C_{kB} + (t_0-t_1) e^{ik(a+2u)} C_{kB}^\dagger C_{kA}$

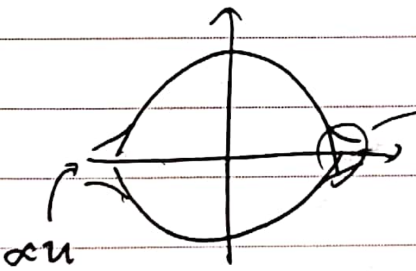
$H = \begin{pmatrix} 0 & \gamma \\ \gamma^* & 0 \end{pmatrix} \Rightarrow \pm|\gamma|$

结果： $E_k = \pm\sqrt{\varepsilon_k^2+\Delta_k^2}$

$$\begin{cases} \varepsilon_k = -2t_0\cos ka \\ \Delta_k = 4\alpha u \sin ka \end{cases}$$

平移 $T(a) \to T(2a)$

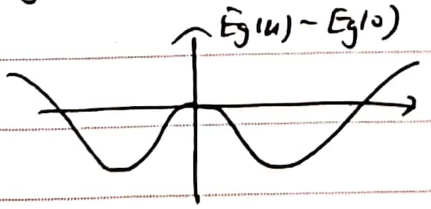
Haldane Model $C_{6v} \to \sin\phi\,\sigma_z$

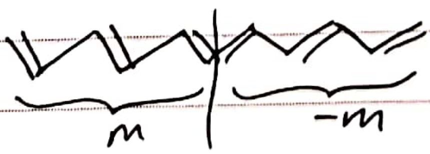


$H = p\sigma_x + m\sigma_z$

形成二聚的反而打开gap，由金属变成为绝缘体

$E_g = -\sum_k \sqrt{\varepsilon_k^2+\Delta_k^2} + 2Nku^2$





会出现 soliton.

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读 Su 三篇文章

作业：PRL 46, 738 (1981)　　公数化 $u_n = u\cos(\frac{2}{3}\pi n - \theta)$

☆推导

☆ $Q = 0, \pm\frac{1}{3}e, \pm\frac{2}{3}e$ 求解

$|\phi\rangle \to j \propto \phi^* \partial\phi - \partial\phi^* \phi \longrightarrow \rho \to Q = \int \rho\, dx$

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