如何数值计算几何相？

$$\gamma = i\int \langle \psi_R | \nabla_R | \psi_R \rangle \cdot d\vec{R}$$

$$= i\sum_i \langle \psi_i | \frac{(|\psi_j\rangle - |\psi_i\rangle)}{\delta R} \cdot \delta R$$

$$= i\sum_i \left(\langle \psi_i | \psi_j \rangle - 1 \right)$$

一个错误的做法.

(不是 Gauge Invariant)

① 作业.

量子化热导的推导

② 阅读材料：

1) Laughlin Nobel Lecture 1998

2) Laughlin PRB, 23, 5632 (1981)

3) Haldane 1988 PRL.

正确的做法.

$$e^{i\gamma} = e^{\int \langle \psi_R | \nabla_R | \psi_R \rangle \cdot d\vec{R}}$$

$$= e^{\sum_i \left(\langle \psi_i | \psi_{i+1} \rangle - 1 \right)}$$

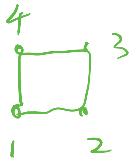
※ 成立条件 Gauge smooth.

$$= \prod_i e^{\langle \psi_i | \psi_{i+1} \rangle - 1}$$

$$= \prod_i \langle \psi_i | \psi_{i+1} \rangle$$

结论：

$$e^{i\gamma} = \prod_i \langle \psi_i | \psi_{i+1} \rangle \quad \text{Gauge Invariant.}$$

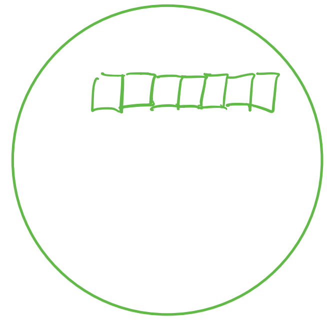


$$e^{i\gamma} = \langle \psi_1 | \psi_2 \rangle \langle \psi_2 | \psi_3 \rangle \langle \psi_3 | \psi_4 \rangle \langle \psi_4 | \psi_1 \rangle$$

Ref. Fukui (日本).

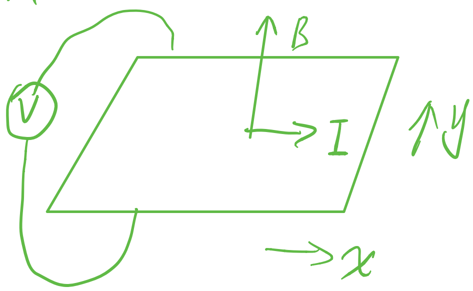
几何相、陈数

$$\int_\Sigma \vec{B}\cdot d\vec{s} = \sum_i \int \vec{B}\cdot d\vec{s}$$



Hall conductance

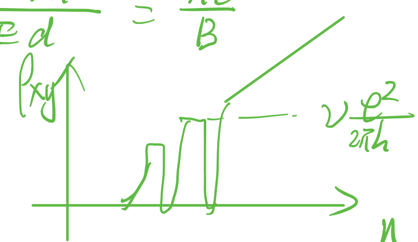
$G = \frac{1}{R}$, $R = \frac{U}{I}$



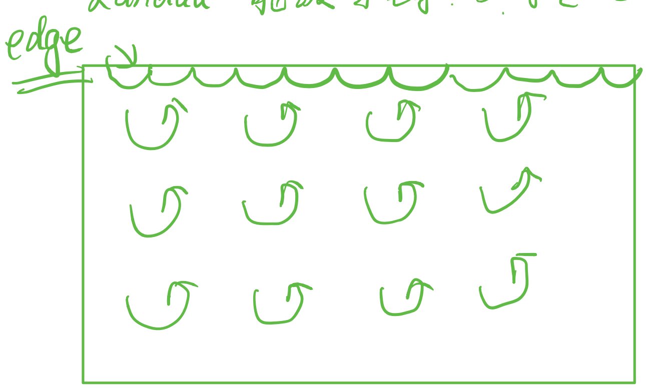
测载流子密度.

$$v = \frac{E}{B}$$

$$I = nevd \quad \Rightarrow G = \frac{nevd}{Ed} = \frac{ne}{B}$$

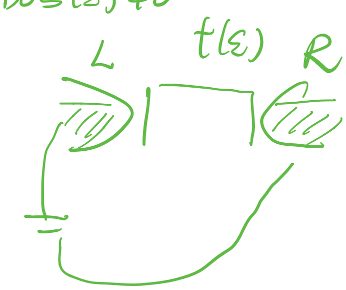


Landau 能级 与电子 ⇒ 导电 ⇒ $G=0$. for insulator.



$\Downarrow$

$Dos(\varepsilon) \neq 0$



$$\int \rho_L(\varepsilon+V)(1-\rho_R(\varepsilon))\, t(\varepsilon)\, d\varepsilon$$

[illegible]

① Landau Level ⟹ Insulator

但是 $I \neq 0$, $G \neq 0$.

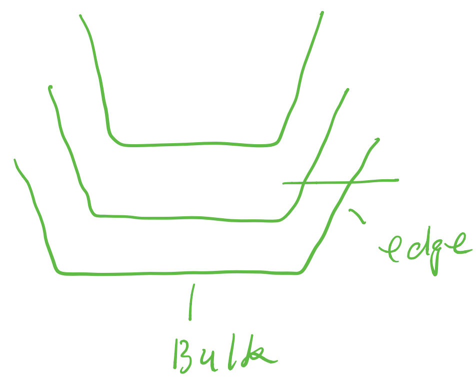
②. $G = \frac{ne}{B} \Rightarrow \nu\, e^2/2\pi\hbar$

③ ν 也可以是分数.

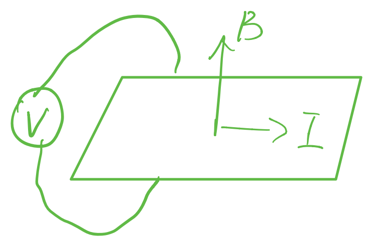
启示:

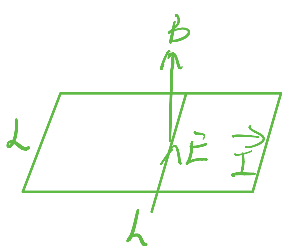
Bulk → Insulator

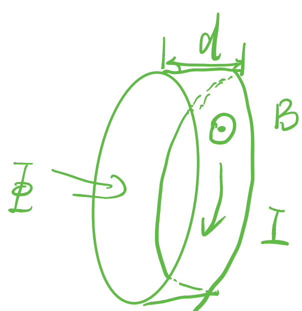
edge state



Laughlin's Argument







$\phi = \oint \vec{A} \cdot d\vec{l}$

$= AL$

第一个公式.

$$I = c\left(\frac{\partial U}{\partial \phi}\right) = c\,\frac{\delta U}{\delta \phi}$$

$E \propto \nu B$

量子化磁通 $\phi_0 = \frac{2\pi\hbar}{e}$

$$H(\phi)|\psi(\phi)\rangle = |\psi(\phi)\rangle E(\phi)$$

Feyman-Hellman 定理.

$$\langle \psi(\phi) | \left(\frac{\partial H}{\partial \phi}\right) | \psi(\phi) \rangle = \left(\frac{\partial u}{\partial \phi}\right)$$

另一方面，

$$\frac{\partial H}{\partial \phi} = \frac{1}{L}\left(\frac{\partial H}{\partial A}\right) \sim \underline{\underline{j}}$$

电磁场中 (minimal coupling)

$$L = L_0 + \vec{j} \cdot \vec{A}$$

$$I \sim c\frac{\delta u}{\delta \phi} \qquad \delta u = neV$$

$$\delta \phi = \frac{hc}{e}$$

$$= c\frac{ne\,V}{hc/e}$$

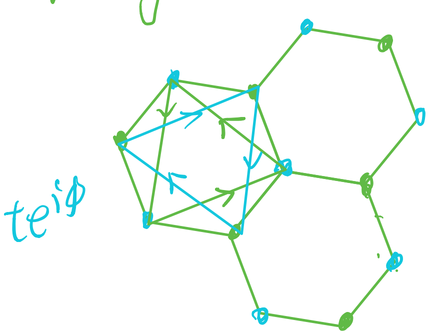
$$= n\frac{e^2}{h}V \quad \Rightarrow \quad \underline{G = n\frac{e^2}{h}}$$

$$\boxed{\text{QHE Without Landau Level}}$$

$$\boxed{\begin{array}{l}\text{作业.} \\ \underline{\text{推导 Laughlin's Argument}} \\ \text{中的 Landau level 和 Hall Resistance}\end{array}}$$

Haldane Model. (1988)

Honeycomb lattice.



• a

• b

a, b sublattice

只有最近邻 Graphene ⇒ Dirac 锥.

$$H(k) = \left(\begin{array}{c|c} 0 & f(p) \\ \hline f^*(p) & 0 \end{array}\right) \rightarrow E(k).$$

加入质量项 $\left(\begin{array}{c|c} m & f(p) \\ \hline f^*(p) & -m \end{array}\right)$

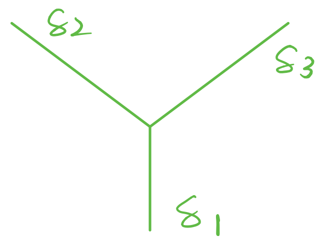
加入一个相位项. $t'\varphi$

系统的 Hamiltonian , nearest-neighboring

$$H_{NN} = -t\sum_i \left(a_i^\dagger b_{i+s_1} + a_i^\dagger b_{i+s_2} + a_i^\dagger b_{i+s_3}\right) + h.c$$

Fourier Transformation

$$\begin{cases} a_i = \frac{1}{\sqrt{N}}\sum_k e^{i\vec{k}\cdot\vec{R}_i^a} a_k \\ b_i = \frac{1}{\sqrt{N}}\sum_k e^{i\vec{k}\cdot\vec{R}_i^b} b_k \end{cases}$$



$s_1 = (0, -1)$

$s_2 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$s_3 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

催补量是好量子数.

$$\sum_{ij} C(i-j)\, a_i^\dagger a_j$$

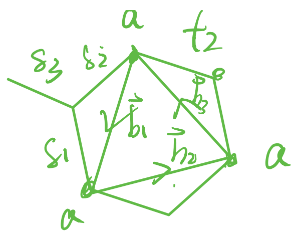
$$= \sum_{ij}\sum_{kq} C(i-j)\, a_k^\dagger a_q\, e^{-i\vec{k}\cdot\vec{R}_i + i\vec{q}\cdot\vec{R}_j}$$

$$= \sum_{kq}\left(\sum_{ij} C(i-j)\, e^{-i\vec{k}\cdot\vec{R}_i + i\vec{q}\cdot\vec{R}_j}\right) a_k^\dagger a_q$$

$$H_{NN} = -t\sum_k a_k^\dagger b_k \left(\sum_{s_i} e^{i\vec{k}\cdot\vec{s}_i}\right) + h.c$$

$$= -t\sum_k (a_k^\dagger, b_k^\dagger)\begin{pmatrix} & \Sigma \\ \Sigma^* & \end{pmatrix}\begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

次近邻相互作用.



$$\begin{cases} \vec{b}_1 = \vec{s}_1 - \vec{s}_2 \\ \vec{b}_2 = \vec{s}_2 - \vec{s}_3 \\ \vec{b}_3 = \vec{s}_3 - \vec{s}_1 \end{cases}$$

$$\begin{cases} H^{a}_{NNN} = -t_2 e^{i\phi} \sum_i \left[\left(a_i^\dagger a_{i+b_1} + a^\dagger_{i+b_1} a_{i-b_3} + a^\dagger_{i-b_3} a_i \right) + h.c \right] \\ H^{b}_{NNN} = -t_2 e^{-i\phi} \sum_i \left[b_i^\dagger b_{i-b_3} + b^\dagger_{i-b_3} b_{i+s_2} + b^\dagger_{i+s_2} b_i \right) + h.c \end{cases}$$

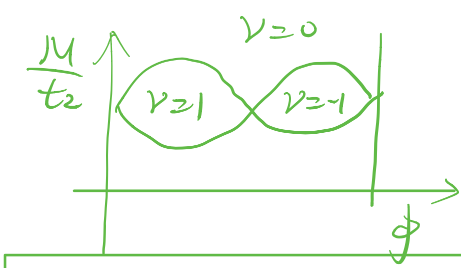
$$\xrightarrow{FT} \quad H^{a}_{NNN} = \sum_k E^{a}_k a_k^\dagger a_k \qquad E^{a}_k = E_k(\phi)$$

$$H^{b}_{NNN} = \sum_k E^{b}_k b_k^\dagger b_k \qquad E^{b}_k = E_k(-\phi)$$

$$E^{a}_k = \cos(\phi) + \underline{\sin(\phi)}$$

$$\begin{cases} a_k^\dagger a_k \left(\sum_i e^{i\vec{k}\cdot\vec{b}_i + i\phi} \right) + h.c \\ b_k^\dagger b_k \left(\sum_i e^{i\vec{k}\cdot\vec{b}_i - i\phi} \right) + h.c \end{cases}$$

$$= 2t_2 \cos(\phi) \left[\sum_i \cos(\vec{k}\cdot\vec{b}_i) \right] I_{2\times2} + \left[M - 2t \sin(\phi) \sum_i \sin(\vec{k}\cdot\vec{b}_i) \right] \sigma^z$$

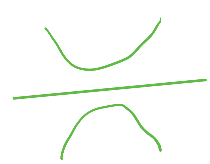


Q~W-Z Model

PRB, 74, 085308 (2006)

$H = \vec{a}\cdot\vec{\sigma}$

$E = \pm\sqrt{a_k^2 + b_k^2 + c_k^2}$



Gap