Index calculations for the fermion-vortex system

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A method is developed for calculating the index of first-order operators involving as background fields the scalar and vector fields of the two-dimensional Abelian Higgs model. The method is applied to the Dirac operator and shown to relate the number of fermion zero modes to the topology of the Higgs field.

I. INTRODUCTION

Recently Jackiw and Rossi[1] investigated the Dirac equation describing charged fermions interacting with the scalar and vector fields of the two-dimensional Abelian Higgs model. They studied in particular the case where the background was a rotationally invariant multivortex configuration and found that there were $|n|$ zero-eigenvalue modes, where

$$n=\frac{1}{2\pi}\int_C dl_i \frac{\epsilon_{ab}\phi_a(\partial_i\phi)_b}{|\phi|^2}. \qquad (1.1)$$

Here ϕ_1 and ϕ_2 are the real and imaginary parts of the Higgs field and the integration is along a circle at spatial infinity. They conjectured that their results could be generalized beyond this particular case, and that it might be possible to obtain an index theorem for the Dirac operator which they studied. In this paper I verify their conjecture by deriving such a theorem.

The treatment in this paper differs somewhat from that in an earlier paper,[2] where an index theorem was derived for a rather similar operator which arose in the study of multivortex solutions. In that paper the index was found to be given in terms of the Pontrjagin number

$$n_A=\frac{q}{2\pi}\int d^2x\, F_{12}. \qquad (1.2)$$

(Here q is the charge of the scalar particle.) This may appear to conflict with the results of Jackiw and Rossi, who find the number of zero modes to be determined solely by the scalar field configuration. There is of course no real contradiction since n and n_A are equal if the background Higgs and gauge fields are solutions of the field equations, and the methods of Ref. 2 assume that this is the case. However, motivated by the fact that the existence of the zero modes found in Ref. 1 does not depend on this assumption, the conditions imposed on the background fields in this paper will be much weaker.

It may be useful to review some methods used in calculating indices of operators, noting in particular how these depend on the behavior at spatial infinity. Recall that the index of an elliptic differential operator is defined by

$$\mathcal{I}=\dim(\text{kernel } \mathfrak{D})-\dim(\text{kernel } \mathfrak{D}^\dagger). \qquad (1.3)$$

Now consider the quantity

$$\mathcal{I}(M^2)=\mathrm{Tr}\left(\frac{M^2}{\mathfrak{D}^\dagger\mathfrak{D}+M^2}\right)-\mathrm{Tr}\left(\frac{M^2}{\mathfrak{D}\mathfrak{D}^\dagger+M^2}\right). \qquad (1.4)$$

As M^2 tends to zero, only the contribution of the zero eigenvalues to $\mathcal{I}(M^2)$ survives. The normalizable zero modes of $\mathfrak{D}^\dagger\mathfrak{D}$, which are the same as those of $\mathfrak{D}$, each contribute 1; similarly, each of the normalizable zero modes of $\mathfrak{D}\mathfrak{D}^\dagger$ contributes -1. Therefore,

$$\mathcal{I}=\lim_{M^2\to 0}\mathcal{I}(M^2). \qquad (1.5)$$

(If the continuum portions of the spectra extend to zero, one must also consider the possibility of a contribution from this source; for the operators considered in this paper there will always be a gap between the discrete zero modes and the continuum.[3])

For such operators on a compact space the spectrum is entirely discrete. Since, as it is easy to show, the nonzero eigenvalues of $\mathfrak{D}^\dagger\mathfrak{D}$ and $\mathfrak{D}\mathfrak{D}^\dagger$ are the same, their contributions to Eq. (1.4) cancel, and $\mathcal{I}(M^2)$ is equal to the index for all values of M^2. $\mathcal{I}$ may therefore be obtained by evaluating $\mathcal{I}(M^2)$ in the limit $M^2\to\infty$, which turns out to be rather easy to do. As might be expected, a similar procedure may be used on an open Euclidean space if the background fields are well enough behaved at spatial infinity to allow compactification without the introduction of any but gauge singularities. Thus, for four-dimensional examples using instantons $\partial\mathcal{I}(M^2)/\partial M^2$ can be shown to vanish.[4] Similarly, for solutions of the Abelian Higgs model the field strengths and the covariant derivatives of the scalar field decrease exponentially at large distance, and the indices of operators involving such fields can be obtained by this procedure.

By contrast, if there are long-range fields present $\partial\mathcal{I}(M^2)/\partial M^2$ need not vanish and the above method fails[5]; for the two-dimensional examples considered in this paper this occurs if the covariant derivatives of the Higgs field fall no faster than $1/|\vec{x}|$. It then becomes necessary to evaluate $\mathscr{I}(M^2)$ explicitly and then obtain the index from Eq. (1.5). Such a procedure was developed by Callias,[6] who used it to study operators involving 't Hooft-Polyakov monopoles, another case where long-range fields lead to a nonvanishing $\partial\mathscr{I}(M^2)/\partial M^2$.

The remainder of this paper is organized as follows. In Sec. II a general expression for the index of a large class of first-order differential operators is derived. It is used in Sec. III to evaluate the index of the Dirac operator of Ref. 1. Section IV contains some concluding remarks.

II. CALCULATING INDICES

In this section an expression will be obtained for the index of first-order operators $\mathfrak{D}$ of the form

$$\mathfrak{D} = P_i \partial_i + Q(x)\,. \tag{2.1}$$

The constant matrices P_i will be assumed to obey

$$\begin{aligned} P_i^\dagger P_j + P_j^\dagger P_i &= 2\delta_{ij} I\,,\\ P_i P_j^\dagger + P_j P_i^\dagger &= 2\delta_{ij} I \end{aligned} \tag{2.2}$$

so that

$$\begin{aligned} \mathfrak{D}^\dagger \mathfrak{D} &= -I(\partial_1{}^2 + \partial_2{}^2) - L_1\,,\\ \mathfrak{D}\mathfrak{D}^\dagger &= -I(\partial_1{}^2 + \partial_2{}^2) - L_2 \end{aligned} \tag{2.3}$$

with the L_i at most first-order differential operators.

It is convenient to combine $\mathfrak{D}$ and $\mathfrak{D}^\dagger$ into a single operator

$$\mathcal{O} = \begin{pmatrix} 0 & -\mathfrak{D}^\dagger \\ \mathfrak{D} & 0 \end{pmatrix} \tag{2.4}$$

and to write

$$\mathcal{O} = \Gamma_i \partial_i + K(x) \tag{2.5}$$

with

$$\Gamma_i = \begin{pmatrix} 0 & P_i^\dagger \\ P_i & 0 \end{pmatrix} \tag{2.6}$$

and

$$K = \begin{pmatrix} 0 & -Q^\dagger \\ Q & 0 \end{pmatrix}. \tag{2.7}$$

It follows from Eq. (2.2) that

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} I\,. \tag{2.8}$$

It is also useful to define a matrix

$$\Gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{2.9}$$

obeying

$$\{\Gamma_5, \mathcal{O}\} = 0\,. \tag{2.10}$$

$\mathscr{I}(M^2)$ may then be expressed in terms of $\mathcal{O}$ as

$$\begin{aligned} \mathscr{I}(M^2) &= \mathrm{Tr}\,\Gamma_5 \frac{M^2}{-\mathcal{O}^2 + M^2}\\ &= \int d^2x\, \mathrm{tr}\left\langle x \left| \Gamma_5 \frac{M^2}{-\mathcal{O}^2 + M^2} \right| x \right\rangle, \end{aligned} \tag{2.11}$$

where tr indicates a trace only over matrix indices.

Now consider a nonlocal current

$$J_i(x, y, M, \mu) = \mathrm{tr}\left(\left\langle x \left| \Gamma_5 \Gamma_i \frac{1}{\mathcal{O} + M} \right| y \right\rangle - \left\langle x \left| \Gamma_5 \Gamma_i \frac{1}{\mathcal{O} + \mu} \right| y \right\rangle \right). \tag{2.12}$$

(The regulator mass μ will eventually be taken to infinity.) A straightforward calculation using the identities

$$\begin{aligned} \delta(x-y) &= \left[\Gamma_i \frac{\partial}{\partial x_i} + K(x) + M \right] \left\langle x \left| \frac{1}{\mathcal{O} + M} \right| y \right\rangle\\ &= \left\langle x \left| \frac{1}{\mathcal{O} + M} \right| y \right\rangle \left[-\frac{\overleftarrow{\partial}}{\partial y_i} \Gamma_i + K(y) + M \right] \end{aligned} \tag{2.13}$$

yields

$$\begin{aligned} \left(\frac{\partial}{\partial x_i} + \frac{\partial}{\partial y_i} \right) J_i(x, y, M, \mu) = &-2\,\mathrm{tr}\left\langle x \left| \Gamma_5 \frac{M}{\mathcal{O} + M} \right| y \right\rangle + 2\,\mathrm{tr}\left\langle x \left| \Gamma_5 \frac{\mu}{\mathcal{O} + \mu} \right| y \right\rangle\\ &+ \mathrm{tr}\left([K(x) - K(y)]\, \Gamma_5 \left\langle x \left| \frac{1}{\mathcal{O} + M} - \frac{1}{\mathcal{O} + \mu} \right| y \right\rangle \right). \end{aligned} \tag{2.14}$$

Now let y approach x. Because of the regulator, this does not introduce any singularity in J_i, while each of the matrix elements on the right-hand side of Eq. (2.14) remains finite. In particular, the last term on the right-hand side vanishes. With the aid of Eq. (2.10) one obtains

$$\begin{aligned} \partial_i J_i(x, x, M, \mu) = &-2\,\mathrm{tr}\left\langle x \left| \Gamma_5 \frac{M^2}{-\mathcal{O}^2 + M^2} \right| y \right\rangle\\ &+ 2\,\mathrm{tr}\left\langle x \left| \Gamma_5 \frac{\mu^2}{-\mathcal{O}^2 + \mu^2} \right| y \right\rangle. \end{aligned} \tag{2.15}$$

From this it follows that

$$\mathcal{I}(M^2) = \mathcal{I}(\mu^2) - \tfrac{1}{2}\int d^2x\, \partial_i J_i(x,x,M,\mu)$$

$$= \mathcal{I}(\mu^2) + \tfrac{1}{2}\int_C dl_i\, \epsilon_{ij} J_j(x,x,M,\mu)\,, \qquad (2.16)$$

where the integration in the last term is along a circle at spatial infinity. [Note that $\mathcal{I}(M^2)$ is independent of M^2 if the asymptotic behavior is such that J_i decreases faster than $1/|x|$.] Finally, taking M to zero and μ to infinity gives

$$\mathcal{I} = \mathcal{I}(\infty) + \tfrac{1}{2}\int_C dl_i\, \epsilon_{ij} J_j(x,x,0,\infty)\,. \qquad (2.17)$$

To evaluate the first term on the right-hand side, note that Eqs. (2.5) and (2.8) lead to

$$-\mathcal{O}^2 = \Delta - L \qquad (2.18)$$

with

$$\Delta = -I(\partial_1{}^2 + \partial_2{}^2) \qquad (2.19)$$

and

$$L = (\Gamma_i K + K\Gamma_i)\partial_i + \Gamma_i(\partial_i K) + K^2\,. \qquad (2.20)$$

We may then write

$$\frac{\mu^2}{-\mathcal{O}^2+\mu^2} = \mu^2[(\Delta+\mu^2)^{-1} + (\Delta+\mu^2)^{-1}L(\Delta+\mu^2)^{-1} + \cdots]\,. \qquad (2.21)$$

If this expression is substituted into Eq. (2.11), all terms beyond that linear in L vanish in the limit $\mu^2 \to \infty$.[7] L then enters only through the quantity $\mathrm{tr}(\Gamma_5 L)$; with the aid of Eq. (2.10) this is seen to be

$$\mathrm{tr}\Gamma_5 L = \mathrm{tr}\Gamma_5\Gamma_i(\partial_i K)\,. \qquad (2.22)$$

Using

$$\left\langle x \left| \frac{1}{(\Delta+\mu^2)^2} \right| x \right\rangle = \int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2+\mu^2)^2} = \frac{1}{4\pi\mu^2}\,, \qquad (2.23)$$

we obtain

$$\mathcal{I}(\infty) = \lim_{\mu^2\to\infty}\int d^2x\, \mathrm{tr}(\Gamma_5 L)\mu^2 \left\langle x \left| \frac{1}{(\Delta+\mu^2)^2} \right| x \right\rangle = \frac{1}{4\pi}\int d^2x\, \mathrm{tr}\Gamma_5\Gamma_i(\partial_i K)\,. \qquad (2.24)$$

To evaluate the second term in Eq. (2.17), note first that Eq. (2.10) allows J_j to be rewritten as

$$J_j(x,x,M,\mu) = \mathrm{tr}\left\langle x \left| \Gamma_5\Gamma_j\mathcal{O}\left(\frac{1}{-\mathcal{O}^2+M^2} - \frac{1}{-\mathcal{O}^2+\mu^2}\right)\right| x \right\rangle. \qquad (2.25)$$

To proceed further, it is necessary to make some assumptions concerning the behavior of $\mathcal{O}^2$ at large $|x|$. Let us suppose[8] that it is of the form

$$-\mathcal{O}^2 = \Delta + a^2 I - B_i(x)\partial_i - C(x) \qquad (2.26)$$

with a^2 a positive constant and $B_i(x)$ and $C(x)$ falling at least as fast as $1/|x|$. From Eq. (2.20) it follows that

$$B_i = \Gamma_i K + K\Gamma_i\,. \qquad (2.27)$$

Expanding $(-\mathcal{O}^2+M^2)^{-1}$ about $(\Delta+a^2+M^2)^{-1}$ and substituting into Eq. (2.25) gives

$$\begin{aligned} J_j(x,x,M,\mu) &= \mathrm{tr}\left\langle x \left| \Gamma_5\Gamma_j\mathcal{O}\left[\frac{1}{\Delta+a^2+M^2} - \frac{1}{\Delta+a^2+\mu^2}\right]\right| x \right\rangle \\ &\quad + \mathrm{tr}\left\langle x \left| \Gamma_5\Gamma_j\mathcal{O}(B_k\partial_k + C)\left[\frac{1}{(\Delta+a^2+M^2)^2} - \frac{1}{(\Delta+a^2+\mu^2)^2}\right]\right| x \right\rangle + O\left(\frac{1}{x^2}\right) \\ &= \frac{1}{4\pi}\ln\left(\frac{\mu^2+a^2}{M^2+a^2}\right)\mathrm{tr}(\Gamma_5\Gamma_j K) - \frac{1}{8\pi}\ln\left(\frac{\mu^2+a^2}{M^2+a^2}\right)\mathrm{tr}(\Gamma_5\Gamma_j\Gamma_k B_k) \\ &\quad + \frac{1}{4\pi}\left(\frac{1}{a^2+M^2} - \frac{1}{a^2+\mu^2}\right)\mathrm{tr}(\Gamma_5\Gamma_j KC) + O\left(\frac{1}{x^2}\right)\,. \end{aligned} \qquad (2.28)$$

Using Eqs. (2.8), (2.10), and (2.27) it is easy to show that the first two terms on the right-hand side cancel. Taking μ to infinity and $M=0$ then gives

$$J_j(x,x,0,\infty) = \frac{1}{4\pi a^2}\mathrm{tr}(\Gamma_5\Gamma_j KC) + O\left(\frac{1}{x^2}\right)\,. \qquad (2.29)$$

Substituting this, together with Eq. (2.24), into Eq. (2.17) gives the desired index.

III. THE DIRAC OPERATOR

In this section the results of Sec. II will be used to obtain the index of the Dirac operator studied by Jackiw and Rossi.[1] They considered a two-component fermion field interacting with gauge and Higgs background fields according to the Lagrangian

$$\mathcal{L} = \bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \frac{ig}{2}\phi\bar{\psi}\psi^c + \frac{ig^*}{2}\phi^*\bar{\psi}^c\psi\,. \qquad (3.1)$$

Here

$$\psi_i^c = C_{ij}\bar{\psi}_j \qquad (3.2)$$

with C the charge-conjugation matrix. Note that charge conservation requires the electric charge e of the fermion to be half that of the scalar. The Dirac equation following from this Lagrangian may be written as

$$i\frac{\partial\psi}{\partial t} = \vec{\alpha}\cdot(-i\vec{\nabla} - e\vec{A})\psi - g\phi\sigma_2\psi^* \qquad (3.3)$$

with $\vec{\alpha}$ being the pair of Pauli matrices (σ_1, σ_2). Because both ψ and ψ^* appear in Eq. (3.3), separation of the time variable requires writing ψ as a sum of two terms, one with a factor of e^{-iEt}, the other with e^{iEt}. For $E=0$ these two collapse to a single term obeying

$$0 = \vec{\alpha}\cdot(-i\vec{\nabla} - e\vec{A})\psi - g\phi\sigma_2\psi^* . \qquad (3.4)$$

The solutions of this equation may be chosen to be eigenmodes of σ_3. Doing so and multiplying Eq. (3.4) on the left by σ_2, one obtains

$$\begin{aligned} 0 &= [\sigma_3(-\partial_1 + ieA_1) + (-i\partial_2 - eA_2)]\psi - g\phi\psi^* \\ &= [\lambda(-\partial_1 + ieA_1) + (-i\partial_2 - eA_2)]\psi - g\phi\psi^* . \end{aligned} \qquad (3.5)$$

In the second equality $\lambda = \pm 1$ and ψ has been reduced to a single complex function of $\vec{x}$. Equation (3.5) is nonlinear in that it contains both ψ and its complex conjugate; it may be transformed to a pair of linear equations by writing

$$\psi(\vec{x}) = u(\vec{x}) + v(\vec{x}) \qquad (3.6)$$

with u and v being real functions. Taking $\lambda = 1$ leads to

$$0 = \mathfrak{D}\begin{pmatrix} u \\ v \end{pmatrix} \qquad (3.7)$$

with $\mathfrak{D}$ a 2×2 matrix of the form

$$\begin{aligned} \mathfrak{D} = (-\partial_1 + i\tau_2\partial_2) + e(-A_2 - i\tau_2 A_1) \\ + g(-\tau_3\phi_1 - \tau_1\phi_2) , \end{aligned} \qquad (3.8)$$

while setting $\lambda = -1$ leads to

$$0 = \mathfrak{D}^\dagger\begin{pmatrix} u \\ v \end{pmatrix} . \qquad (3.9)$$

The index of $\mathfrak{D}$ therefore satisfies

$$\mathfrak{I} = N_+ - N_- , \qquad (3.10)$$

where $N_\pm$ is the number of fermion zero modes with $\sigma_3\psi = \pm\psi$. Since $\mathfrak{D}$ is of the form (2.1) with the P_i satisfying Eq. (2.2), its index may be evaluated using the results of Sec. II.

A straightforward calculation gives

$$\begin{aligned} \mathfrak{D}^\dagger\mathfrak{D} = I(-\partial_1{}^2 - \partial_2{}^2 + e^2\vec{A} + g^2|\phi|^2 - eF_{12}) \\ - ie\tau_2(2A_i\partial_i + \partial_i A_i) \\ + g\{-\tau_3[(D_1\phi)_1 + (D_2\phi)_2] \\ + \tau_1[-(D_1\phi)_2 + (D_2\phi)_1]\} , \end{aligned} \qquad (3.11)$$

where

$$(D_i\phi)_a = \partial_i\phi_a + 2eA_i\epsilon_{ab}\phi_b \qquad (3.12)$$

is the covariant derivative of the Higgs field. (Recall that the charge of the scalar is twice that of the fermion.) $\mathfrak{D}\mathfrak{D}^\dagger$ may be obtained from Eq. (3.11) by making the substitutions $F_{12} \to -F_{12}$ and $(D_1\phi)_a \to -(D_1\phi)_a$. $\mathcal{O}^2$ will be of the desired form (2.26) (with $a^2 = g^2|\phi|^2$) provided that $|\phi|$ tends to a constant at spatial infinity while A_i and $(D_i\phi)_a$ fall at least as fast as $1/|x|$. The traces occurring in Eqs. (2.24) and (2.29) are

$$\mathrm{tr}\,\Gamma_5\Gamma_i(\partial_i K) = 4eF_{12} \qquad (3.13)$$

and

$$\mathrm{tr}\,\Gamma_5\Gamma_j KC = 4g^2\epsilon_{jk}\epsilon_{ab}\phi_a(D_k\phi)_b + O\left(\frac{1}{x^2}\right) . \qquad (3.14)$$

Substituting these into Eqs. (2.24) and (2.29) and using Eq. (2.17) gives

$$\mathfrak{I} = \frac{e}{\pi}\int d^2x\, F_{12} + \frac{1}{2\pi}\int_C dl_i \frac{\epsilon_{ab}\phi_a(D_i\phi)_b}{|\phi|^2} . \qquad (3.15)$$

These two terms may be combined by noting that

$$\frac{e}{\pi}\int d^2x\, F_{12} = \frac{e}{\pi}\int_C dl_i\, A_i . \qquad (3.16)$$

Finally, substituting this into Eq. (3.15) and using Eq. (3.12), we obtain

$$\begin{aligned} \mathfrak{I} &= \frac{1}{2\pi}\int_C dl_i \frac{\epsilon_{ab}\phi_a(\partial_i\phi)_b}{|\phi|^2} \\ &= n . \end{aligned} \qquad (3.17)$$

This agrees with the explicit calculations of Ref. 1.

IV. CONCLUDING REMARKS

The calculation of the index of the Dirac operator $\mathfrak{D}$ does not in itself determine the number of fermion zero modes. Rather, since $\mathfrak{I}$ is the difference between the positive quantities N_+ and N_-, it gives only a lower bound on the number of such modes. In some cases it is possible to obtain further information. An example of this is obtained by taking the scalar field potential to be that of the Ginzburg-Landau theory, with the parameters chosen to correspond to the borderline between a type-I and a type-II superconductor.[9] The theory then has a number of features which are similar to ones encountered in the study of self-dual Yang-Mills fields in four-dimensional Euclidean space. Thus, solutions of the second-order Euler-Lagrange equations can be obtained by solving the first-order equations[10]

$$(D_1\phi)_a = \pm\epsilon_{ab}(D_2\phi)_b\,,$$
$$eF_{12} = \pm\epsilon^2(v^2-\phi^2)\,. \quad (4.1)$$

Here the upper and lower signs correspond to positive and negative n, respectively, and v is the vacuum expectation value of ϕ. Furthermore, the energy of such solutions is proportional to $|n|$.

For a solution with $n>0$, Eq. (3.11) and the remarks following it, together with Eq. (4.1), give

$$\mathfrak{D}\mathfrak{D}^\dagger = (-\partial_j + e\tau_2 A_j)^2 + e^2(v^2 - |\phi|^2) + g^2|\phi|^2\,. \quad (4.2)$$

For such a solution one would expect $(v^2 - |\phi|^2)$ to be everywhere positive, so that $\mathfrak{D}\mathfrak{D}^\dagger$ would be manifestly positive. (In any case $\mathfrak{D}\mathfrak{D}^\dagger$ is positive if one chooses $g^2 \geq e^2$.) If $\mathfrak{D}\mathfrak{D}^\dagger$ is positive then neither it nor $\mathfrak{D}^\dagger$ has any zero eigenvalues and $N_-=0$. There are then precisely $N_+=n$ fermion zero modes. Similarly, for solutions of Eq. (4.1) with $n<0$, $\mathfrak{D}^\dagger\mathfrak{D}$ is given by the right-hand side of Eq. (4.2). The above arguments then lead to $N_+=0$ and $N_-=|n|$.

Finally, it should be noted that the methods of this paper can be applied to the operator encountered when seeking zero energy fluctuations about a solution of Eq. (4.1).[2] While it is most natural to consider this operator with ϕ and A_i obeying the field equations, one may of course study it for more general values of the fields. Using the expressions obtained in Sec. II to evaluate the index[11] leads to

$$\mathcal{I} = \frac{2e}{\pi}\int d^2x\, F_{12} + \frac{1}{\pi}\int_C dl_i\, \frac{\epsilon_{ab}\phi_a(D_i\phi)_b}{|\phi|^2}\,, \quad (4.3)$$

which is just twice the result found in Eq. (3.15) for the Dirac operator. For solutions of the field equations the second integral vanishes and one has the expression previously obtained.

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[5]E. Weinberg, Phys. Rev. D 20, 936 (1979).

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[7]There are terms quadratic in L containing two derivatives which are potentially nonvanishing in the limit $\mu^2 \to \infty$. However, these terms vanish after the traces are taken.

[8]It is possible, although a bit more cumbersome, to proceed with somewhat weaker assumptions.

[9]E. B. Bogomol'nyi, Yad. Fiz. 24, 861 (1976) [Sov. J. Nucl. Phys. 24, 449 (1976)].

[10]It should be kept in mind that the charge of the scalar is $2e$.

[11]In order to obtain an $\mathcal{O}^2$ of the form of Eq. (2.26), it is necessary to use the background gauge condition $0 = \partial_i \delta A_i - 2e\,\epsilon_{ab}\phi_a\delta\phi_b$, rather than the Coulomb gauge condition used in Ref. 2.