

拓扑定义:

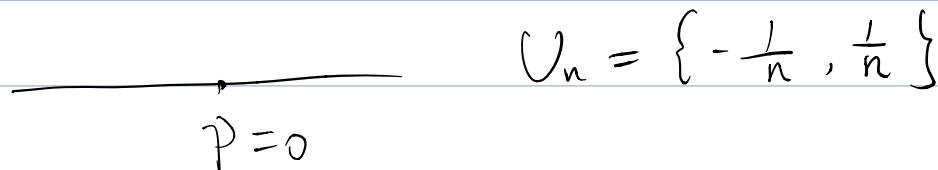
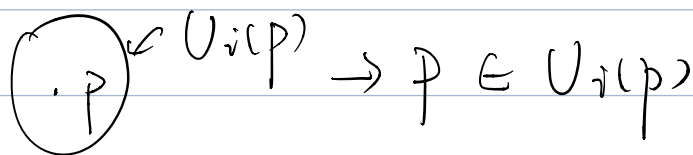
X is any set. $\mathcal{T} = \{U_i\}$, where $U_i \subset X$ is subset of

满足 $\begin{cases} \textcircled{1} \phi, X \in \mathcal{T} \\ \textcircled{2} \text{任意并} \in \mathcal{T} \\ \textcircled{3} \text{有限交} \in \mathcal{T} \Rightarrow \text{finite} \end{cases}$

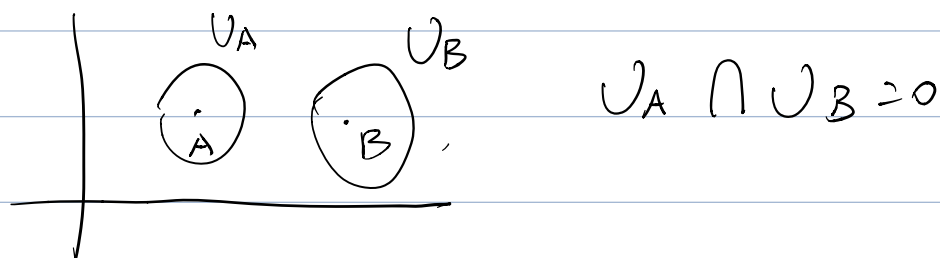
The pair (X, \mathcal{T}) is a topology.

Set $\Rightarrow X$ is a topology.

$\textcircled{1}$ Neighborhood (邻域/邻域)



$\textcircled{2}$ Hausdorff space (空间可分)



③ 覆盖: covering

开覆盖: open covering.

$$X = \bigcup_{i=1}^N A_i \rightarrow \text{open set}$$

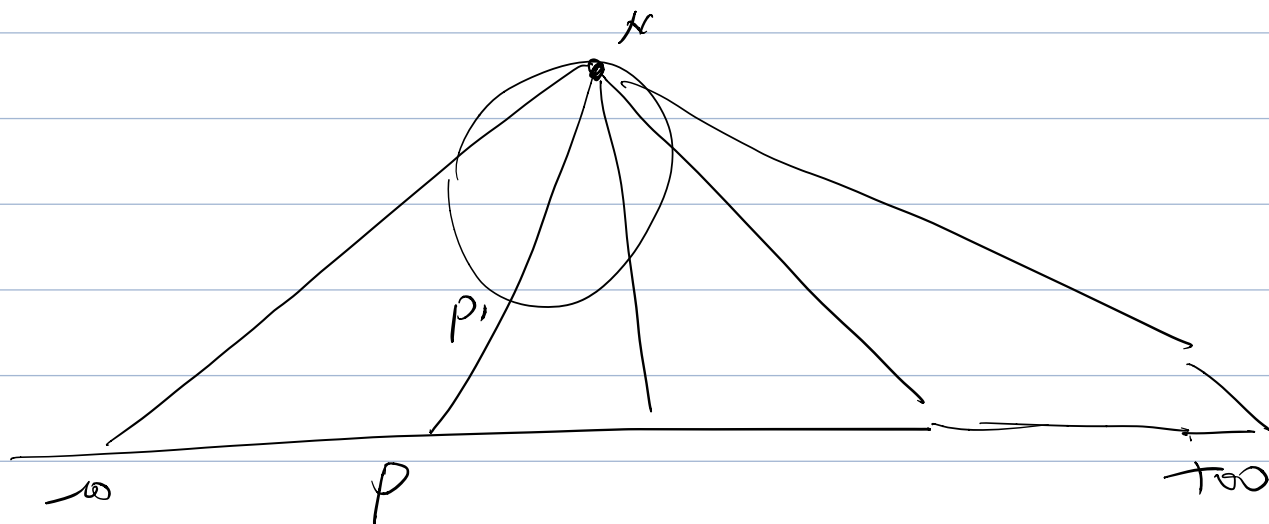
$$A_i \in X \quad [0, \infty)$$

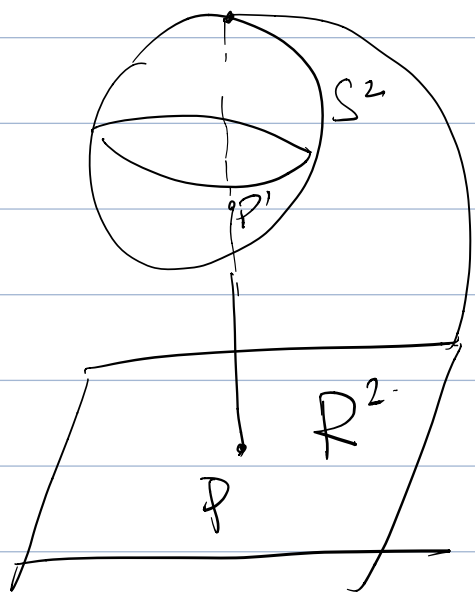
特殊: $\begin{cases} \textcircled{1} N \rightarrow \infty \\ \textcircled{2} N \rightarrow \text{有限} \end{cases}$

$$\left. \begin{aligned} S' - \{N\} &= R' \\ S' &\cong R' \cup \{\infty\} \end{aligned} \right\}$$

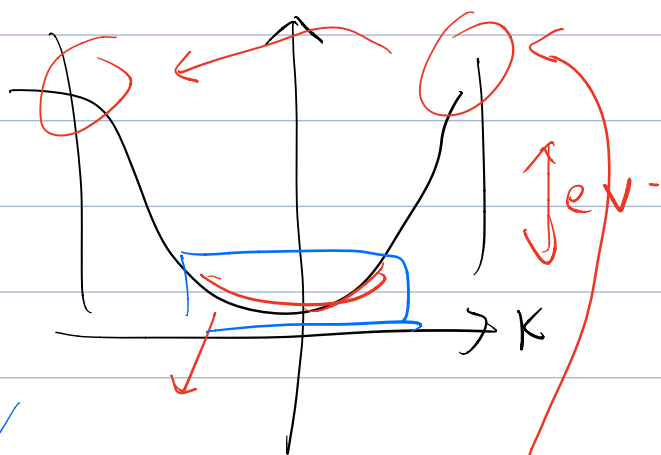
one-point compactification: $\Leftrightarrow S^n \cong R^n$

R . S' 是否一样? 不是



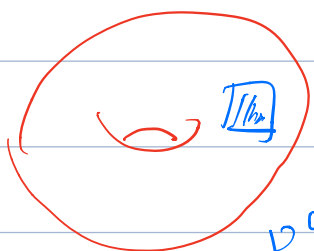


物理应用



只考虑这个, 不考虑!

因为 $k_{BT} / 300k \sim 2? \text{ meV.}$



R^d

等价性???

$$R^d \cup \{\infty\} = S^d.$$

$$Z = \sum_{k \in \Theta} e^{-\beta \epsilon_k} \sim \int_{-\infty}^{\infty} e^{-\beta (1c^2/2m^*)}$$

④ Connection.

空间的方块.

$$A = \textcircled{x} \dots \dots \textcircled{y}$$

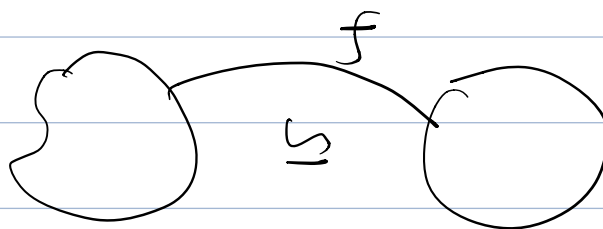
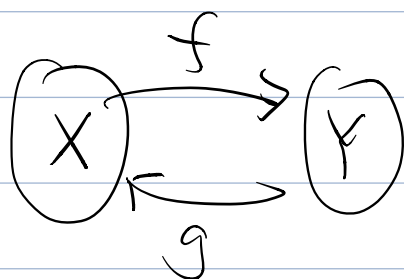
⑤. 同胚 \Rightarrow 空间同胚. 等价

同态: 状态不同的 } 群同态
同构: 结构一样. } 同构

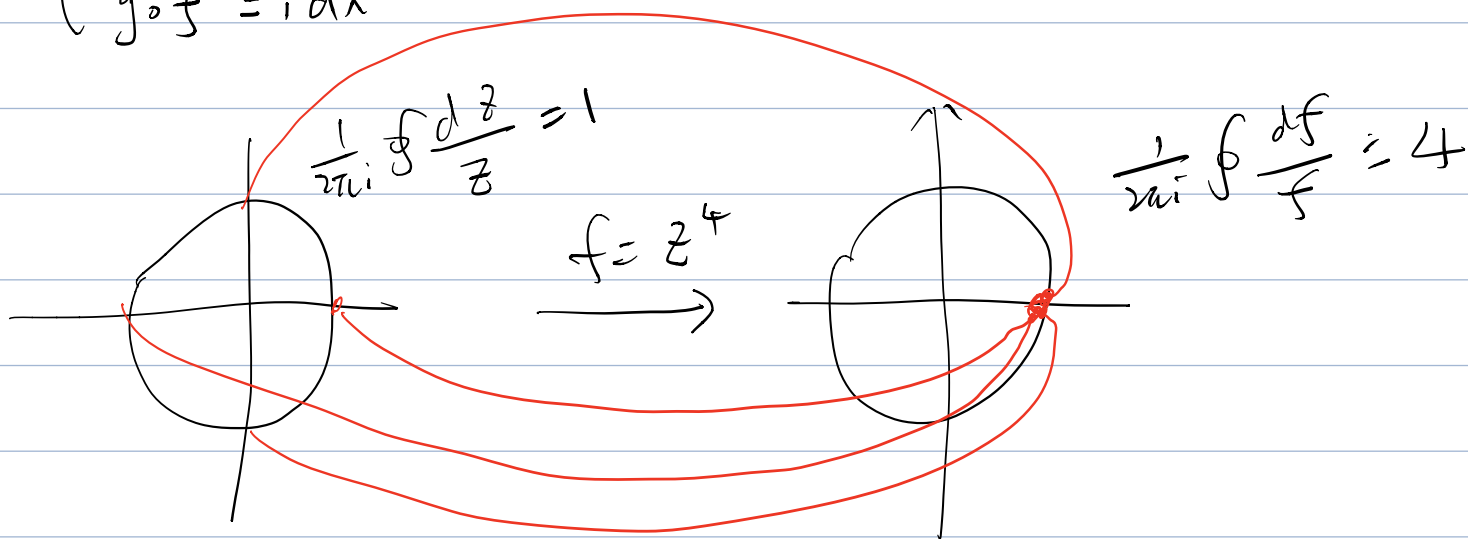
$f: X \rightarrow Y$ 是连续.

同胚双射.

$g: Y \rightarrow X$ 连续.



$$\begin{cases} f \circ g = id_Y \\ g \circ f = id_X \end{cases} \quad \text{identity map}$$



f^{-1} 不存在

- \star Euler #, Chern #.
- \star algebra structure (Group, Ring).
- \star Connectness, Compactness
- Hausdorff property

如何判断 X 与 Y 同胚? 不知道!

- \star 如果 $X \cong Y \Rightarrow$ 上述性质相同.
- \star 反之不对 / 不一定.

例子:

① $[-1, 1]$ 与 $(-1, 1)$ 不同胚。
但 $(-1, 1) \subseteq \mathbb{R}$.

②. $A \subseteq B, B \subseteq C, \Rightarrow A \subseteq C$
 $S^1 \subseteq (-1, 1)$

③. $D^2 = \{x^2 + y^2 < 1 \mid x, y \in \mathbb{R}\}$
 $\subseteq \mathbb{R}^2$

④. $O \subseteq \square \Leftarrow f(x, y) = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}} \right)$

Euler 示性数.

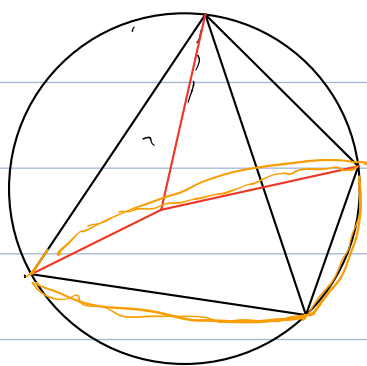
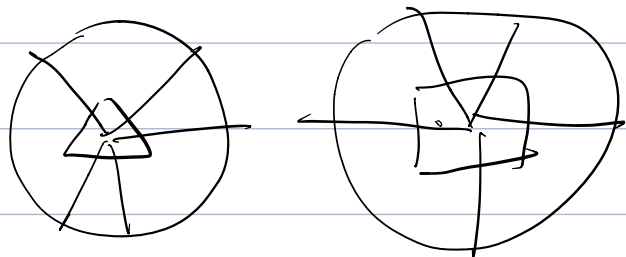
(Euler characteristic)

- 1. 空间可以用 \mathbb{Z} 区分.
- 2. 第一个例子为 Euler 示性数
- 3. 推广 \Rightarrow Gauss-Bonnet number
Chern # / TKNN #

K . 多面体

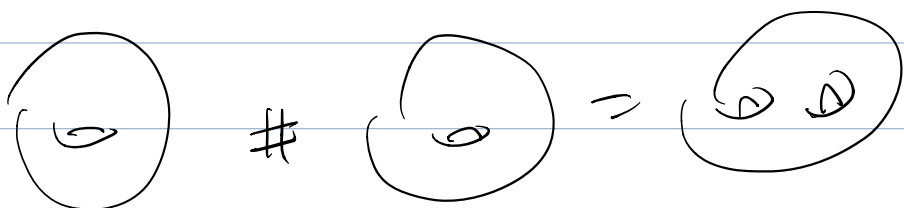
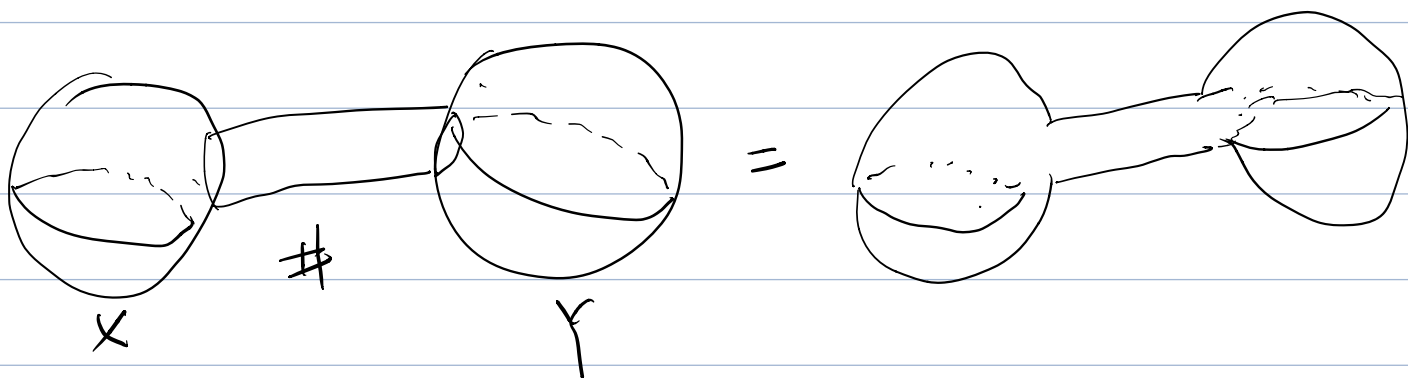
$$\chi(K) = \text{点} - \text{边} + \text{面} = V - E + F$$

$$X(\Delta) = X(\square) = X(O)$$



Connected sum 连通和 #

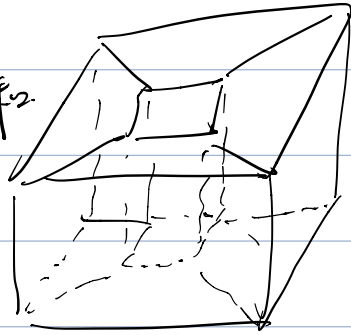
$$X(X \# Y) = X(X) + X(Y) - 2$$



亏格 $X(K) = 2(1-g)$. g 调子数.

$$\begin{cases} X(\textcircled{\circ}) = 0 \\ X(\textcircled{\circ\circ}) = -2 \end{cases}$$

连通和



$$V = 16, E = 32, F = 16$$

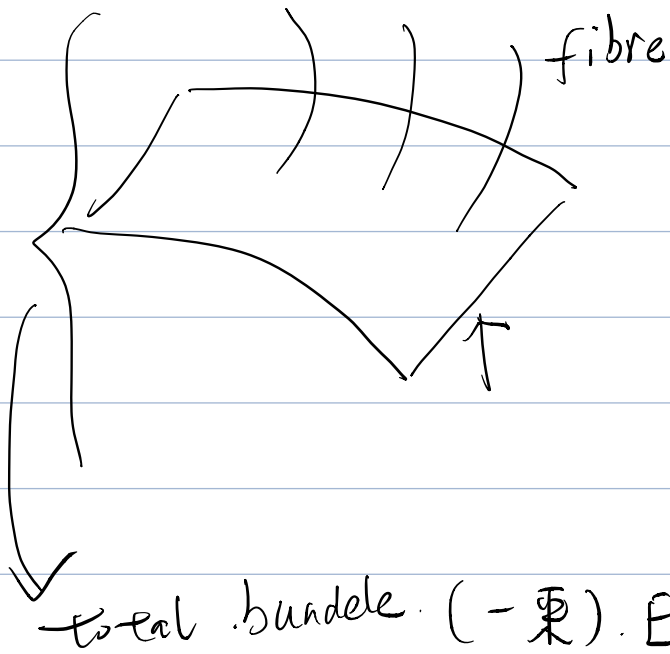
$$V - E + F = 0$$

纤维丛.

F : fibre. (\mathbb{C}^k or p ?)

E : total space

B : Base space.
CS' of K , g in $\pi_1(B)$



total bundle. (一栗). E .

E $\begin{cases} \text{locally } \simeq B \times F \\ \text{globally not- (不等于 } B \times F \text{)} \end{cases}$???



A series of horizontal blue lines for writing, spaced evenly down the page.