拓扑不变量 (Nakahara,Page86)

2021 年 4 月 8 日

1　原文

Now our main question is: *'How can we characterize the equivalence classes of homeomorphism?'* In fact, we do not know the complete answer to this question yet. Instead, we have a rather modest statement, that is, if two spaces have different **'topological invariants'**, they are not homeomorphic to each other. Here topological invariants are those quantities which are conserved under homeomorphisms. A topological invariant may be a number such as the number of connected components of the space, an algebraic structure such as a group or a ring which is constructed out of the space, or something like connectedness, compactness or the Hausdorff property. (Although it seems to be intuitively clear that these are topological invariants, we have to prove that they indeed are. We omit the proofs. An interested reader may consult any textbook on topology.) If we knew the complete set of topological invariants we could specify the equivalence class by giving these invariants. However, so far we know a partial set of topological invariants, which means that even if all the known topological invariants of two topological spaces coincide, they may not be homeomorphic to each other. Instead, what we can say at most is: *if two topological spaces have different topological invariants they cannot be homeomorphic to each other.*

2　翻译

现在我们的主要问题是:"*我们如何刻画同胚的等价类？*"。事实上，我们还不知道这个问题的完整答案。相反，我们有一个相当温和的声明，即，如果两个空间有不同

的“拓扑不变量”，它们彼此不是同胚的。；这里拓扑不变量指的是在同胚下守恒的量。**一个拓扑不变量可以是一个数字，比如空间中连通分支的数量，一个代数结构（比如由空间构造的一个群或一个环），或者一些东西，比如连通性、紧致性或 Hausdroff 性质（虽然直观上这些是拓扑不变量似乎很清晰，但我们必须证明它们确实是。我们省略了证明。感兴趣的读者可以查阅任何有关拓扑学的教科书。）**如果我们知道拓扑不变量的完整集合，我们可以通过给出这些不变量来指定等价类。然而，到目前为止，我们只知道了部分拓扑不变量，这意味着即使已知的两个拓扑空间的所有已知的拓扑不变量一致，它们也可能不是彼此同胚的。我们最多只能说: *如果两个拓扑空间具有不同的拓扑不变量，那么它们不可能互为同胚。*