一点紧致化的重要性 (Nakahara,Page84)

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1 原文

The reader might not appreciate the significance of compactness from the definition and the few examples given here. It should be noted, however, that some mathematical analyses as well as physics become rather simple on a compact space. For example, let us consider a system of electrons in a solid. If the solid is non-compact with infinite volume, we have to deal with quantum statistical mechanics in an infinite volume. It is known that this is mathematically quite complicated and requires knowledge of the advanced theory of Hilbert spaces. What we usually do is to confine the system in a finite volume V surrounded by hard walls so that the electron wavefunction vanishes at the walls, or to impose periodic boundary conditions on the walls, which amounts to putting the system in a torus, see example 2.5(b). In any case, the system is now put in a compact space. Then we may construct the Fock space whose excitations are labelled by discrete indices. Another significance of compactness in physics will be found when we study extended objects such as instantons and Belavin–Polyakov monopoles, see section 4.8. In field theories, we usually assume that the field approaches some asymptotic form corresponding to the vacuum (or one of the vacua) at spatial infinities. Similarly, a class of order parameter distributions in which the spatial infinities have a common order parameter is an interesting class to study from various points of view as we shall see later. Since all points at infinity are mapped to a point, we have effectively compactified the non-compact space R^n to a compact space $S^n = R^n \cup \{\infty\}$. This procedure is called the one-point compactification.

2 翻译

读者也许尚不能从这里给出的定义和几个例子中理解到紧致的重要性。然而值得注意的是，一些数学和物理的分析在紧致空间中变得相当简单。例如，让我们考虑固体中的电子系统。如果固体在无限大的非紧致的空间上，我们就不得不处理无限大体积下的量子统计力学。而众所周知，这在数学上是相当复杂的，需要了解希尔伯特空间的高等理论。我们通常所做的则是将系统限制在一个硬壁包围的有限的体积 V 中，或者在边界上施加周期性边界条件，这相当于将系统置于一个轮胎面上，见例 2.5(b)。无论任何情况下，系统都被放在一个紧致的空间里。然后我们就可以构造 Fock 空间，而此空间中的激发态是可以用离散指标标记的。紧致性在物理学中的另一个重要意义则会体现在我们研究诸如瞬子和 Belavin-Polyakov 单极子等对象时（见第 4.8 节）。在场论中，我们通常假设场逼近于一个无穷远处的真空 (或真空之一) 的渐近形式。类似地，有一类在空间无穷大处都有相同值的序参量会是我们在后面从不同角度来研究有趣的例子。由于无穷远处的所有点都映射到一个点上，我们有效地将非紧致空间 R^n 紧致化成一个紧致空间 $S^n = R^n \cup \{\infty\}$。这个过程称为一点紧致化。