

2021. 3. 18.

外微分.

$$1\text{-form}, W = \sum_i f_i dx_i \leftarrow dH = \dot{q} dp - \dot{p} dq.$$

$$\leftarrow dU = p dV - T dS.$$

$$2\text{-form}, W = \sum_{ij} f_{ij} dx_i \wedge dx_j \leftarrow \oint \vec{B} \cdot d\vec{S} = \oint B_x dy dz + \oint B_y dz dx + \oint B_z dx dy.$$

$$3\text{-form}, W = \sum_{ijk} f_{ijk} dx_i \wedge dx_j \wedge dx_k.$$

$$\uparrow$$

$$\oint \vec{P} \cdot d\vec{V}$$

$$W = P dx + Q dy + R dz$$

$$dW = dP \wedge dx + dQ \wedge dy + dR \wedge dz$$

$$dP = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz.$$

$$\Rightarrow dW = \frac{\partial P}{\partial y} dy \wedge dx + \frac{\partial P}{\partial z} dz \wedge dx.$$

+ ...

+ ...

$$= \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx \wedge dy + \dots + \dots$$

$\therefore \int W = \int dW.$

$$\int \underbrace{P dx dy + Q dy dz + R dz dx}_{w} \rightarrow \int w$$

$$= \int (\partial P / \partial x + \partial Q / \partial y + \partial R / \partial z) dx dy dz \rightarrow \int dw$$

$$\therefore \int_{D-1 \text{ 维}} w = \int_{D \text{ 维}} dw. \quad (\text{可能会发散})$$

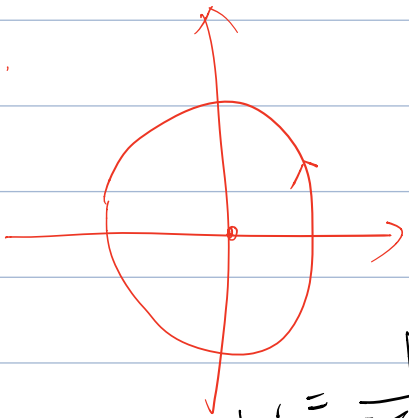
要求 Smooth.

① 磁单极子 (Topo. charge).

$$Q = \oint \vec{B} \cdot d\vec{S} = \int (\nabla \times \vec{A}) \cdot d\vec{S} = \int (\nabla \cdot (\nabla \times \vec{A})) dV$$

= 0 ? \leftarrow 定理失效,
实际上不为 0. \rightarrow

②.



$$\frac{1}{2\pi i} \oint \frac{dz}{z} = 0 \quad ?$$

$$w = \frac{1}{2\pi i} \frac{dz}{z} = d \frac{\ln z}{2\pi i}, \quad dw = d^2 \frac{\ln z}{2\pi i} = 0.$$

$$d(dx) = 0$$