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拓扑相变：奇

拓扑场论：偶

目的：概念性和计算（求解 Hamiltonian）

☆ 复习 $\frac{1}{3}$，基本概念.

☆ 数学概念.

（线性代数，群论，同伦，同调）

教材《微积分初步》

外微分

$dxdy = J\,dx'dy'$

内容：EM（阅读，C.N. Yang，Phys Today 2004）

科学传记.

Monopole. $\oint \vec{B}\cdot d\vec{S} \neq 0$，Real space.

① Topological Insulator $\Leftarrow$ can be attached in Momentum space.

② Topological defect（Magnetic Material，$\vec{B}$

电极化，/液晶，$\vec{E}$

晶错 $\Rightarrow$ Thouless

BKT Phase Transition

③ Hall effect. & Models

几何相/绝热不变量. $I = \oint p\,dq$

（旧量子论中的量子化条件）

$\Rightarrow$ 分类，$\oint \vec{B}\cdot d\vec{S} = n$ $\Leftarrow$ 群论：分类的学问。

↓

什么群（元素

非阿贝尔拓扑物理.

Goldstone 定理.

Ref：1、Gauge Field, Knot and Gravity,

2、Topological Insulator, (HKU, shunqing, shen)

3、Topological Insulator and Topological superconductor

Bernevig (student of S.C. Zhang)

BHZ model

最早预言 Topological Insulator.

4、Geometry, topology and physic (1989)

(黄同信，交叉中心)

5、RMP, Altland & Simons CMFT, chapter 9.

6、数学书.

7、Wiki / 知乎

考核：$\frac{1}{3}$ 作业，$\frac{1}{3}$ 考试，$\frac{1}{3}$ project：读一篇文章，latex.

(开卷)

8 选 4.

要求：有一定计算能力.

开始：图像

$$\left\{\begin{aligned} H_k &= \\ H_{TB} &= \end{aligned}\right. \quad FT$$

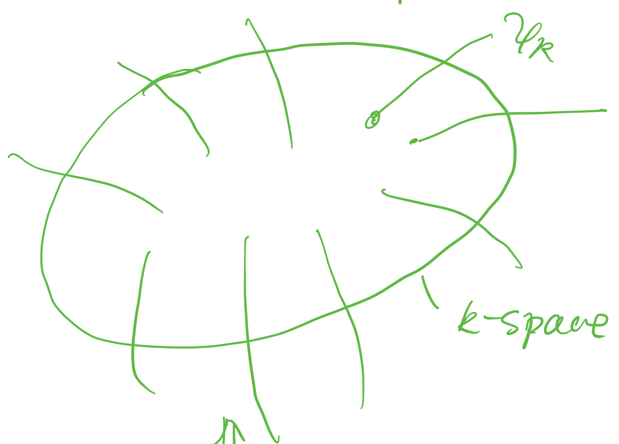
k 是什么？ 动量 ⇒ 参数. H_k 和 H_x 无本质区别

$H: K \to$ Hilbert Space. or $f: X \to Y$

$\psi_k \Rightarrow \psi$: $k \to$ vector space.

set: mapping

- 同态
- 同构
- 同伦
- 同调



⇑ fibre bundle.

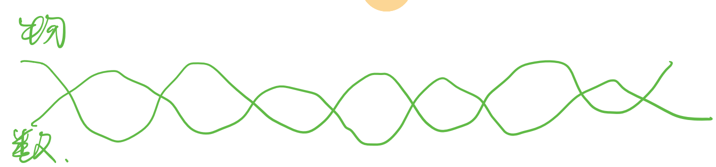
$\vec{A}$: connection;

↓

$\vec{B}$: curvature;

Wu-Yang solution.

陈省身传记



从几个概念开始:

1. dxdy 是什么
2. stokes 定理
3. 复变函数积分.
4. 什么是拓扑.

$$\int_{\partial M} \omega = \int_M d\omega \Rightarrow \langle \partial M, \omega \rangle = \langle M, d\omega \rangle$$

like $\langle \psi | A | \psi \rangle = \langle A^{+}\psi | \psi \rangle$

1. dxdy 微元面积.

$dxdy = J dx'dy'$ $\quad [dxdy] = m^2 \Rightarrow$ wedge ($\wedge$)

是乘积 (点乘, 叉乘) $\vec{a} \times \vec{b}$.

高维积分 $\wedge$

假设 $dxdy$ 是普通乘法.

$$\begin{cases} x = a_{11}x' + a_{12}y' \\ y = a_{21}x' + a_{22}y' \end{cases}$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} dx' \\ dy' \end{pmatrix}$$

$$\Rightarrow dxdy = a_{11}a_{21}dx'dx' + a_{11}a_{22}dx'dy' + a_{12}a_{21}dy'dx' + a_{12}a_{22}dx'dy'$$

$$= (a_{11}a_{22} - a_{12}a_{21})dx'dy'$$

$dxdy$(乘)满足 $dx'dx' = dy'dy' = 0$ (积都不等于零)

$dx'dy' = -dy'dx'$ $\Rightarrow$ 不是点乘.

叉乘有类似的结论.

$[dxdy] = m^2$, 量纲不一样

$[\vec{a} \times \vec{b}] \neq [\vec{c}]$

定义 $dxdy = \underline{dx \wedge dy}$

特点: $\begin{cases} dx \wedge dx = 0. \\ dx \wedge dy = -dy \wedge dx \end{cases}$

$d^2 = 0$

$d(dx) = 0$

$(dx \wedge dy) \wedge dz = dx \wedge (dy \wedge dz)$ 结合律.

同调群 / 上同调群 对空间分类.

2、引入 form (形式)

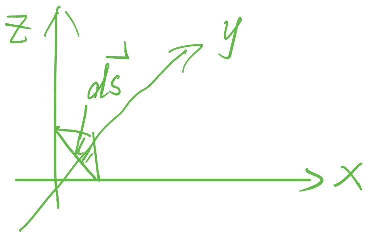
线 面 体

$\begin{cases} \text{1-form} & \omega = \sum f_i dx^i & \in \Lambda^1 \text{ 是.} \\ \text{2-form} & \omega = \sum f_{ij} dx^i \wedge dx^j & \in \Lambda^2 \\ \text{3-form} & & \Lambda^3 \end{cases}$

$\omega = \sum_i f_{ijk}\, dx^i \wedge dx^j \wedge dx^k$

对应 1-form {经典力学. $dH = \sum_i f_i dx^i$, $H = H(p, q)$
统计力学 $du = p\,dv - T\,ds + \mu\, dN$ $dH = \frac{\partial H}{\partial p} \cdot dp - \frac{\partial H}{\partial q} dq$

2-form $\oint \vec{B} \cdot d\vec{s}$



$$d\vec{s} = (dy \wedge dz,\ dz \wedge dx,\ dx \wedge dy)$$

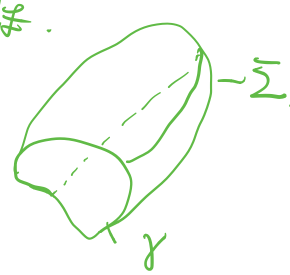
3-form. $\oint \phi\, dx dy dz = \oint \phi\, dV$

$$\int \vec{E} \cdot d\vec{s} = \int (\nabla \vec{\phi}) dV$$

$$\int_M d\omega = \Omega.$$

3、 Stocks 定理. 是微积分的高峰. [可能收敛, smooth]

$$I = \int P\,dx + Q\,dy + R\,dz.$$



$$\gamma = \partial \Sigma$$

$$= \int \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy + \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) dy\,dz$$
$$+ \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) dz\,dx$$

{与微观结构无关,
只和整体相关.

$$\omega = P\,dx + Q\,dy + R\,dz$$

$$d\omega = dP\wedge dx + dQ\wedge dy + dR\wedge dz$$

$$= \left(\frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz\right)\wedge dx$$

$$+\left(\frac{\partial Q}{\partial x}dx + \frac{\partial Q}{\partial y}dy + \frac{\partial Q}{\partial z}dz\right)\wedge dy$$

$$+\left(\frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy + \frac{\partial R}{\partial z}dz\right)\wedge dz$$

$$= \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\wedge dy$$

$$+\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)dy\wedge dz$$

$$+\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)dz\wedge dx$$

$$\int_S P\,dx\,dy + Q\,dy\,dz + R\,dz\,dx$$

$$\underbrace{= \int_{\partial\Sigma}\omega \;\overset{\text{书}}{=}\; \int_\Sigma\left(\frac{\partial P}{\partial z} + \frac{\partial Q}{\partial x} + \frac{\partial R}{\partial z}\right)dx\,dy\,dz.}$$

物理上的粒子.

$$\oint \vec{E}\cdot d\vec{S} = Q_+ - Q_-$$

$$\vec{E} = \nabla\phi = (\phi_x, \phi_y, \phi_z)$$

$$\oint \vec{E}\,d\vec{S} = \int \phi_z\,dx\,dy + \phi_y\,dz\,dx + \phi_x\,dy\,dz$$

$$= \int_\Sigma \underbrace{(\phi_{xx} + \phi_{yy} + \phi_{zz})}_{\nabla\varphi = \rho}dx\,dy\,dz$$

fail down

复变函数上的积分. $\frac{1}{2\pi i}\oint\frac{dz}{z} = \int_\gamma \frac{1}{2\pi i}\frac{dz}{z} = \int_{\partial\Sigma}\omega = \int_\Sigma d\omega \overset{\text{wrong}}{=} 0$

$$\omega = \frac{1}{2\pi i}\frac{1}{z}dz = d\left(\frac{1}{2\pi i}\ln z\right) = d\eta$$

Monopole : Dirac paper

$$0 \neq \Phi_m = \int_S \vec{B}\cdot d\vec{S} = \int_S (\nabla\times\vec{A})\cdot d\vec{S}$$

$$= \int_V \nabla\cdot(\nabla\times\vec{A})\cdot dV = 0.$$

⇓

Topological charge.

奇点导致.

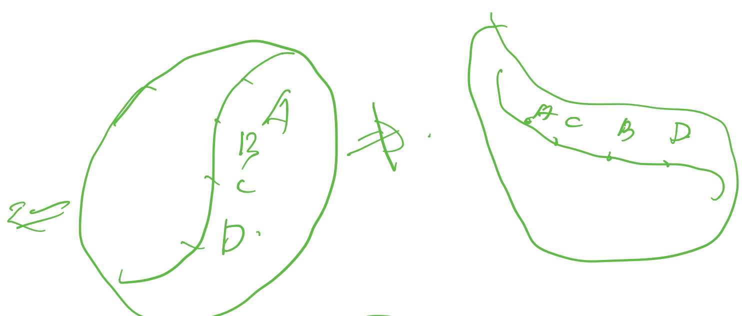
$$\langle \partial M, \omega \rangle = \langle M, d\omega \rangle$$

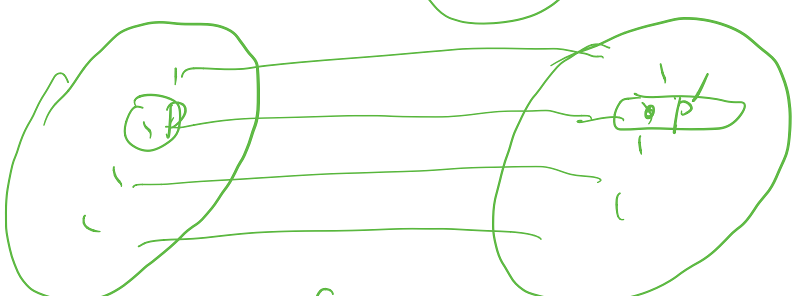
$\begin{cases} \text{Hermitian} \\ \text{adjoint} \end{cases}$

4、Topology 是什么？

$\begin{cases} \text{放弃. 连续性，距离} \\ \text{保持 相邻的性质.} \end{cases}$

开集





f: 映射.

$p' = f(p)$

$u' = f(u)$