PHYSICAL REVIEW B VOLUME 50, NUMBER 15 15 OCTOBER 1994-I

Electron band structure in a two-dimensional periodic magnetic field

M. C. Chang and Q. Niu
Department of Physics, University of Texas, Austin, Texas 78712
(Received 24 March 1994; revised manuscript received 24 June 1994)

In this paper we study the energy spectrum of a two-dimensional electron gas (2DEG) in a two-dimensional periodic magnetic field. Both a square magnetic lattice and a triangular one are considered. We consider the general case where the magnetic field in a cell can be of any shape. A general feature of the band structure is bandwidth oscillation as a function of the Landau index. A triangular magnetic lattice on a 2DEG can be realized by the vortex lattice of a superconductor film coated on top of a heterojunction. Our calculation indicates a way of relating the energy spectrum of the 2DEG to the vortex structure. We have also derived conditions under which the effects of a weak magnetic modulation, periodic or not, may be reproduced by an electric potential modulation, and vice versa.

I. INTRODUCTION

The behavior of a two-dimensional electron gas (2DEG) in a homogeneous magnetic field modulated by a periodic electric potential or magnetic field is a subject of some recent investigations.[1–7] In the case of a one-dimensional electric modulation (1DEM), resonance between the cyclotron radius R_c and the period of modulation a results in an unusual magnetoresistance oscillation.[1,2] It was found that the electrons do not move along the trench formed by the modulation potential when $2R_c=(\kappa-\frac{1}{4})a$, and move with maximum velocity when $2R_c=(\kappa+\frac{1}{4})a$, where κ is an integer. Because R_c changes with the magnetic field, the magnetoresistance oscillates when the strength of the magnetic field increases. This result has been explained quantum mechanically by Gerhardts, Weiss, and von Klitzing,[1] and classically (when $R_c \gg a$) by Beenakker.[2] A variant system that uses a one-dimensional magnetic modulation (1DMM) instead of a 1DEM shows similar behaviors.[3] In this case the electron drift is absent when $2R_c=(\kappa+\frac{1}{4})a$, and a maximum drift velocity along the trench is obtained when $2R_c=(\kappa-\frac{1}{4})a$.

A delicate structure in the electron energy spectrum emerges when the modulation is two dimensional. It is well known that the spectrum in a magnetic field with a two-dimensional electric modulation (2DEM) exhibits fractal behavior—the so-called Hofstadter spectrum.[4] The fragmentation of bands will suppress the part of the magnetoresistance that comes from the variation of band conductivity, and only a weaker, diffusion-related magnetoresistance oscillation can be observed.[5] Fractal spectral structure also exists for a two-dimensional magnetic modulation (2DMM), which was studied by Wu and Ulloa recently. They also studied the collective excitations of the electron gas, and found that it does not map out the Hofstadter spectrum exactly.[7] Although it would be interesting to observe the excitation spectrum of a 2DMM system, there is still no attempt at an experimental realization that we know of.

In this paper we study the energy spectrum of the 2DEG as a function of quasimomentum, and show that there is a one-to-one correspondence between it and the spatial distribution of the magnetic field.[8] Both a rectangular magnetic lattice and a triangular one are considered. The latter can be generated using a superconductor in a vortex state. Therefore we can have a periodic modulation on a 2DEG by coating a superconductor film on top of a semiconductor heterojunction.[8] Using the connection between the energy spectrum and the shape of the magnetic modulation, we can use the 2DEG as a probe for getting some basic parameters of the superconductor film.

We also explore the possible connection between electric and magnetic modulations. There is a mapping at the operator level between the Hamiltonians of a 1DMM system and a 1DEM system.[9] No such simple mapping exists between 2D modulation systems. However, the matrix elements of the Hamiltonian of a 2DMM are similar in structure to those of a 2DEM. Hence, it is possible to have a formal connection between these two, albeit in a nontrivial way. The cases with one-dimensional and two-dimensional periodic modulations, and nonperiodic field distributions are discussed.

This paper is organized as follows: In Sec. II we study systems with weak perturbations. In Sec. III the exact energy spectrum is obtained by diagonalizing the Hamiltonian matrix. In Sec. IV a way of relating the vortex structure to the behavior of a 2DEG is discussed. In Sec. V we discuss the correspondence between the electric modulations and magnetic field modulations. The last section is a conclusion.

II. WEAK MODULATION

A. Rectangular lattice

Consider a 2DEG in a homogeneous magnetic field B_0 perturbed by a weak rectangular magnetic lattice $\tilde{B}(x,y)$. In order to simplify the notation, we choose the magnetic length $\lambda=(\hbar/eB_0)^{1/2}$ to be the unit of length, B_0 to be the unit of magnetic field, and $\hbar\omega_c$ to be the unit of ener-

0163-1829/94/50(15)/10843(8)/$06.00

gy, where $\omega_c = eB_0/m$ is the cyclotron frequency. This is the most natural choice of units in the sense that all physical quantities are around the order of unity in the regime of interest. The Hamiltonian can be written as

$$H = \frac{1}{2}\left[-i\frac{\partial}{\partial x} - y + A_x(x,y)\right]^2 + \frac{1}{2}\left[-i\frac{\partial}{\partial y} + A_y(x,y)\right]^2 . \quad (1)$$

We have chosen the vector potential of the homogeneous magnetic field to be $\mathbf{A}_0(x,y) = -y\hat{\mathbf{i}}$.

$$\mathbf{A} = (x,y) = A_x(x,y)\hat{\mathbf{i}} + A_y(x,y)\hat{\mathbf{j}}$$

is the vector potential of the perturbing field.

An eigenfunction of the unperturbed Hamiltonian for the nth level is

$$\psi_{nk_1}(x,y) = \frac{N_n}{\sqrt{L}} e^{-ik_1 x} e^{-(1/2)(y+k_1)^2} H_n(y+k_1) , \quad (2)$$

where N_n is a normalization constant, L is the size of the system, and k_1 is a constant. Since the perturbed system has rectangular symmetry, we would like to construct from (2) an unperturbed eigenstate with the same symmetry. Assume a_1 and a_2 to be the lattice constants. Discrete translation operators that commute with the Hamiltonian (1) are the magnetic translation operators $\tilde{T}_1, \tilde{T}_2$.

$$\tilde{T}_1 = T_1 , \qquad \tilde{T}_2 = e^{ia_2 x} T_2 , \quad (3)$$

where T_1 and T_2 are the usual translation operators. An unperturbed eigenstate that satisfies

$$\tilde{T}_1 \Psi_{nk_1k_2} = e^{ik_1 a_1} \Psi_{nk_1k_2} , \qquad \tilde{T}_2 \Psi_{nk_1k_2} = e^{ik_2 a_2} \Psi_{nk_1k_2} \quad (4)$$

is found to be

$$\Psi_{nk_1k_2} = \sum_{l=-\infty}^{\infty} e^{-ilk_2 a_2} \psi_{n,k_1 - 2\pi l/a_1} . \quad (5)$$

Equation (4) defines the quasimomenta k_1 and k_2, which are good quantum numbers. We assumed there is only one flux quantum $\Phi_0 = h/e$ per unit cell in getting Eq. (5). That is, we used $a_1 a_2 = 2\pi$ in the new unit system. Then, a state in a Landau level is only coupled to states in the other Landau levels with the same (k_1, k_2), relieving the necessity to consider degenerate perturbation. The case with arbitrary magnetic flux per unit cell is more complicated, and we will comment on it later.

The first-order energy perturbation can be found by calculating the diagonal matrix elements of the Hamiltonian on the unperturbed basis:

$$\mathcal{E}_n^{(1)}(k_1,k_2) = \frac{1}{2} \sum_{l,m=-\infty}^{\infty} \tilde{B}(l,m)[L_n^1(z_{lm}) + L_{n-1}^1(z_{lm})] \times e^{-z_{lm}/2}(-1)^{lm} e^{imk_1 a_1} \times e^{-ikl_2 a_2} ,$$
$$z_{lm} = 2\pi^2 \left[\left[\frac{l}{a_1}\right]^2 + \left[\frac{m}{a_2}\right]^2\right] , \quad (6)$$

where $\tilde{B}(l,m)$ are the Fourier components of $\tilde{B}(x,y)$,

$$\tilde{B}(x,y) = \sum_{lm} \tilde{B}(l,m) e^{2\pi i l x/a_1} e^{2\pi i m y/a_2} , \quad (7)$$

and L_n^1 is the associated Laguerre polynomial ($L_{-1}^1 \equiv 0$). Notice that $\tilde{B}(x,y)$ is in units of B_0. Equation (6) can be readily reduced to the result of a system with a one-dimensional sinusoidal perturbation in Ref. 3.

Using the asymptotic form of the associated Laguerre polynomials in the large-n limit, it can be shown that, in the four-point approximation [where all the Fourier components are zero except $\tilde{B}(\pm 1,0) = \tilde{B}(0,\pm 1) \neq 0$], the bandwidth is zero when $2\sqrt{n\pi} = (\kappa + \frac{1}{4})\pi$, and reaches maximum when $2\sqrt{n\pi} = (\kappa - \frac{1}{4})\pi$. Therefore the widths of the energy bands in Eq. (6) oscillate with the Landau index. Physically, it is clearer to write these two conditions as $2R_c = (\kappa \pm \frac{1}{4})a$, where the radius of cyclotron motion $R_c \simeq \sqrt{2n}$ when n is large, and $a = \sqrt{2\pi}$. These are the same as the resonant conditions for a 1DMM. The energy bands will be further split into p subbands when there are a rational number (say p/q) of flux quanta in a unit cell. A proper unperturbed wave function for such a system is

$$\Psi_{nk_1k_2} = \sum_{j=1}^{p} d_j \sum_{l=-\infty}^{\infty} e^{-i(l+j/p)k_2 q a_2} \psi_{nk_1 - (l+j/p)2\pi/a_1} . \quad (8)$$

The coefficients d_j are to be determined from a Harper-like equation after we turn on the perturbation;[10] more details can be found in Ref. 7.

B. Triangular lattice

An easy way to realize a two-dimensional periodic magnetic modulation would be using a superconductor vortex array with the symmetry of a triangular lattice. We choose $A_0(x,y) = -y\hat{\mathbf{i}}$ to make our Hamiltonian explicitly translationally invariant in x. The magnetic translation operators that commute with the Hamiltonian of this system are

$$\tilde{T}_1 = g_1 T_1 , \qquad \tilde{T}_2 = e^{i\sqrt{3}ax/2} g_2 T_2 , \quad (9)$$

where $g_1 = \exp(-i\pi)$ and $g_2 = \exp(-i\pi/2)$ are gauge factors for shifting the center of a hexagonal unit in $\mathcal{E}_n(k_1,k_2)$ to the origin. This will become clearer in a later discussion. Instead of putting g_1 and g_2 in the magnetic translation operators, we can put them on the right-hand side of Eq. (4). This amounts to a redefinition of the quasimomenta.

An unperturbed eigenstate with triangular symmetry is given by

$$\Psi_{nk_1k_2}=\sum_{r=-\infty}^{\infty}(-1)^{r(r-1)/2}e^{ri(k_1/2-k_2)a}\psi_{n,k_1-2\pi r/a}\ ; \tag{10}$$

again we assumed one flux quantum per unit cell ($\sqrt{3}a^2/2=2\pi$). The case of one-half flux quantum per unit cell will be discussed at the end of this section.

The first-order energy perturbation is

$$\mathcal{E}_n^{(1)}(k_1,k_2)=\frac{1}{2}\sum_{r,s=-\infty}^{\infty}\tilde{B}(r,s)[L_n^1(w_{rs})+L_{n-1}^1(w_{rs})]\times e^{-w_{rs}/2}(-1)^{rs}e^{is(k_1a+\pi)}\times e^{-ir(k_2a+\pi)}\ ,$$

$$w_{rs}=\frac{2\pi}{\sqrt{3}}(s^2-sr+r^2)\ . \tag{11}$$

$\tilde{B}(r,s)$ are the Fourier components of $\tilde{B}(x,y)$,

$$\tilde{B}(x,y)=\sum_{r,s}\tilde{B}(r,s)e^{i(x\hat{\mathbf{i}}+y\hat{\mathbf{j}})\cdot(r\hat{\mathbf{b}}_1+s\hat{\mathbf{b}}_2)}\ , \tag{12}$$

where $\hat{\mathbf{b}}_1=(2\pi/a)(\hat{\mathbf{i}}-\hat{\mathbf{j}}/\sqrt{3})$ and $\hat{\mathbf{b}}_2=(2\pi/a)(2\hat{\mathbf{j}}/\sqrt{3})$ are unit vectors of the reciprocal lattice.

We let all of the Fourier components be zero except six of them (we will call this a six-point approximation): $\tilde{B}(\pm1,0),\tilde{B}(0,\pm1)$, and $\tilde{B}(\pm1,\pm1)$, which are all equivalent to each other. Then we have

$$\tilde{B}(x,y)=2\tilde{B}(1,0)\left[2\cos\left[\frac{2\pi}{a}x\right]\cos\left[\frac{2\pi}{\sqrt{3}a}y\right]+\cos\left[\frac{2\pi}{\sqrt{3}a}2y\right]\right]\ . \tag{13}$$

The corresponding energy perturbation is

$$\mathcal{E}_n(k_1,k_2)=-e^{-\pi/\sqrt{3}}\tilde{B}(1,0)(L_n^1+L_{n-1}^1)\times\{\cos(ak_1)+\cos(ak_2)+\cos[a(k_1-k_2)]\}\ . \tag{14}$$

Notice that k_1 and k_2 are along the nonorthogonal directions $\tilde{T}_1$ and $\tilde{T}_2$; we need to make a transformation

$$k_1=k_x\ ,\qquad k_2=\frac{k_x}{2}+\frac{\sqrt{3}}{2}k_y\ , \tag{15}$$

to put $\mathcal{E}_n$ on an orthogonal basis k_x and k_y. See Fig. 1 for plots of $\tilde{B}(x,y)$ and $\mathcal{E}_0(k_x,k_y)$, where $\tilde{B}(1,0)=0.05$. Notice that the magnitude of variation of $\tilde{B}(x,y)$ is $9\tilde{B}(1,0)$ [from $-3\tilde{B}(1,0)$ to $6\tilde{B}(1,0)$], while that of $\mathcal{E}_0(k_x,k_y)$ is $4.5e^{-\pi/\sqrt{3}}\tilde{B}(1,0)$, which is a much smaller variation. It appears that the electrons sense the average magnetic field more than they sense the fluctuation. The plot of $\mathcal{E}_0(k_x,k_y)$ will be shifted to the right by π if both factors g_1 and g_2 in Eq. (9) are unity.

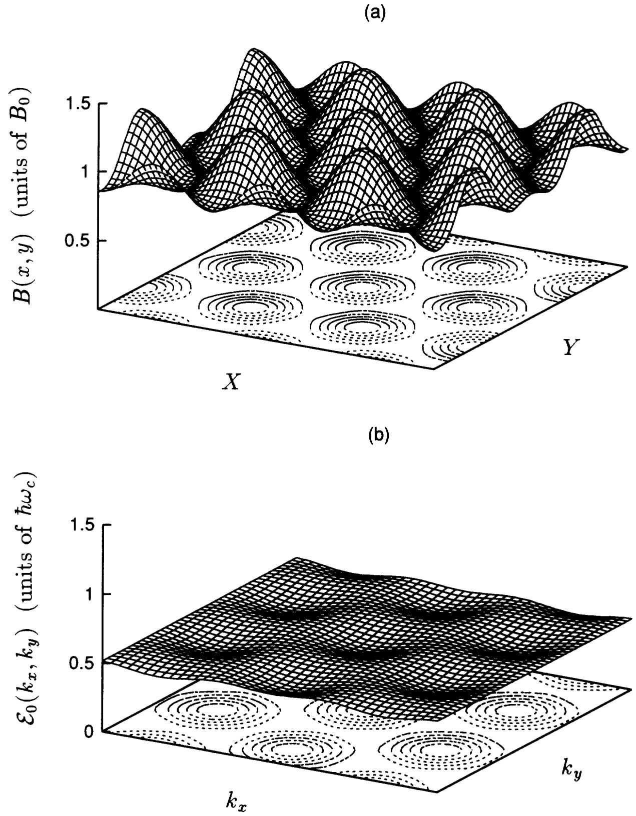

FIG. 1. (a) Plot of the magnetic field $B(x,y)$ with triangular lattice symmetry in the six-point approximation. (b) Ground-state energy of an electron in a homogeneous magnetic field modulated by the magnetic field in (a).

In a real physical situation, the triangular array is formed by a superconductor vortex state, where the magnetic flux per plaquette is $\Phi_0/2$ instead of Φ_0. In this case the magnetic translation operators defined in Eq. (3) do not commute ($T_1T_2=-T_2T_1$), and the quasimomenta in Eq. (4) are no longer good quantum numbers. We have to choose a unit cell that consists of two magnetic plaquettes to have a mutually commuting set of H, T_1, and T_2. Assume a is the lattice constant of the magnetic plaquette; then there is one flux quantum Φ_0 within the area $(\sqrt{3}/2)a_1a_2$ of the unit cell, where $a_1=2a,a_2=a$. The energy perturbation for this system is similar to Eq. (11), but with two differences. First, $(-1)^{rs}$ is replaced by $(-1)^{2rs}$, and thus can be dropped. Second, k_1a in the exponent is replaced by k_1a_1, and k_2a by $2k_2a_2$. Hence there are two identical regions in the first Brillouin zone. This is related to the following fact. Instead of choosing $a_1=2a$, we can choose $a_2=2a$, and $a_1=a$. The overlapping region of these two different Brillouin zones has the same energy spectrum because we are free to enlarge our unit cell either way. The dispersion surfaces in the other half regions of either Brillouin zone just replicate the energy spectrum in the overlapping region. The existence of two identical regions in the Brillouin zone is related to the twofold degeneracy of such a system.

III. EXACT ENERGY SPECTRUM

The exact energy spectrum for an electron in a magnetic field with strong modulation can be found by diagonalizing the Hamiltonian (1) numerically. For a square magnetic lattice, the result of diagonalization in the four-point approximation is shown in Fig. 2. The energies from the perturbation calculation are lower than the exact energies since a positive-definite contribution from the quadratic terms of the vector potential is neglected in calculating Eq. (6). Notice that the magnetic modulation is of the same strength as the homogeneous component, but the energy shift is just a fraction of $\hbar\omega_c$.

Matrix elements of the Hamiltonian (1) with triangular modulation are listed below for reference. Write the Hamiltonian as $H=H_0+H_1+H_2$, where H_1 is linear in A, and H_2 is quadratic in A. The matrix elements of H_1 on the unperturbed basis (10) are found to be

$$\langle n'|H_1(k_1,k_2)|n\rangle_{(n'\geq n)}=\frac{1}{2}\sum_{r,s}\tilde{B}(r,s)\left[L_n^{n'-n}(w_{rs})-\left[\frac{n'+n}{w_{rs}}L_n^{n'-n}(w_{rs})-\frac{2n'}{w_{rs}}L_{n-1}^{n'-n}(w_{rs})\right]\right]G_{n'n}(r,s)\ . \tag{16}$$

The matrix elements of H_2 are

$$\langle n'|H_2(k_1,k_2)|n\rangle_{(n'\geq n)}$$
$$=\sum_{\substack{r,s\\ r',s'}}[A_x(r,s)A_x(r',s')+A_y(r,s)A_y(r',s')]$$
$$\times L_n^{n'-n}(w_{r+r',s+s'})G_{n'n}(r+r',s+s')\ , \tag{17}$$

where the function $G_{n'n}$ is defined to be

$$G_{n'n}(r,s)=\left[\frac{n!}{n'!}\right]^{1/2}(\sqrt{2}\pi)^{n'-n}\left[\frac{ir}{a}+\frac{2s-r}{\sqrt{3}a}\right]^{n'-n}$$
$$\times e^{-w_{rs}/2}(-1)^{rs}e^{is(k_1a+\pi)}$$
$$\times e^{-ir(k_2a+\pi)}\ . \tag{18}$$

The vector potential $\mathbf{A}(r,s)$ is related to $\tilde{\mathbf{B}}(r,s)$ by $\tilde{\mathbf{B}}(r,s)=i\mathbf{q}\times\mathbf{A}(r,s)$, where $\mathbf{q}=r\hat{\mathbf{b}}_1+s\hat{\mathbf{b}}_2\neq 0$. Using the Coulomb gauge condition, we have

$$\mathbf{A}(r,s)=i\frac{\mathbf{q}}{|\mathbf{q}|^2}\times\tilde{\mathbf{B}}(r,s)\ . \tag{19}$$

Under the six-point approximation, there is only one independent component, say $A_x(1,0)$, of the vector potential, which is related to $\tilde{B}(1,0)$ by

$$A_x(1,0)=(\sqrt{3}a/8\pi i)\tilde{B}(1,0)\ .$$

The energy spectrum for a system with a six-point approximation is shown in Fig. 3; the modulation strength $\tilde{B}(1,0)$ is chosen such that the minimum of the total magnetic field $B(x,y)=B_0+\tilde{B}(x,y)$ is zero. Numerical calculation shows that the transition amplitudes between different Landau levels oscillate with respect to the level separation Δn when Δn is small, and drop quickly to zero beyond $\Delta n\simeq 10$. The asymptotic behavior goes roughly like $1/(\Delta n)!$ We were careful to choose the rank of the matrix being diagonalized large enough to ensure the convergence of our result.

When the variation of magnetic field is large, adjacent energy levels begin to mix with each other. This can be seen from Fig. 4. The modulation part is nine times larger than the homogeneous part; therefore in some re-

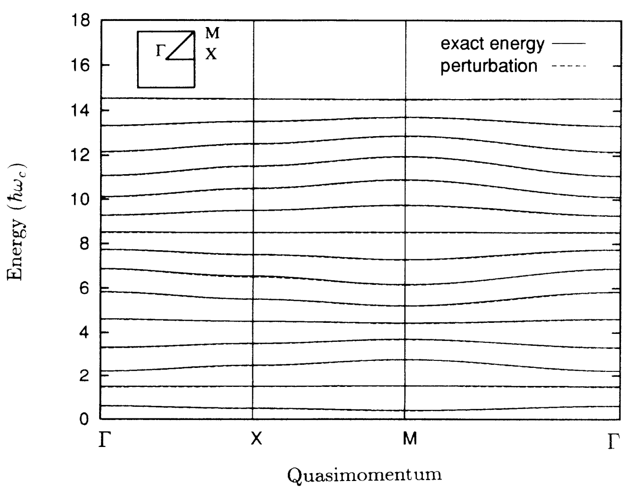

FIG. 2. Broadened Landau levels for a 2DEG in a homogeneous magnetic field modulated by a square magnetic lattice. The perturbing field is $\tilde{B}(x,y)=0.5(\cos x+\cos y)$. Dashed lines are for the first-order energy perturbation, and solid lines are for the exact energy spectrum. Only the lowest 15 levels are shown here.

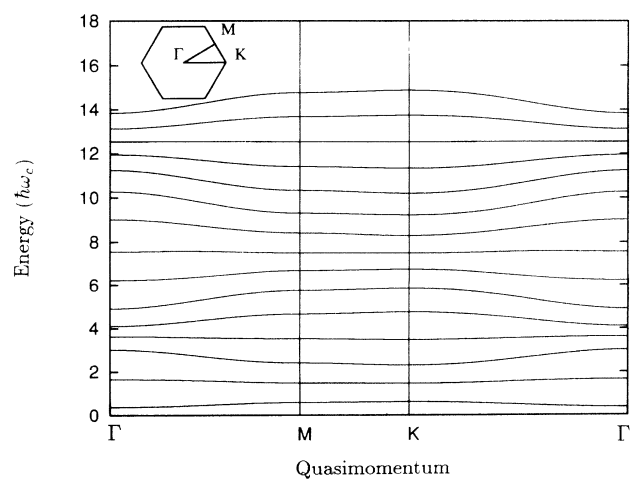

FIG. 3. Exact energy spectrum for a 2DEG in a homogeneous magnetic field modulated by a triangular magnetic lattice in the six-point approximation, $\tilde{B}(1,0)=\frac{1}{3}$.

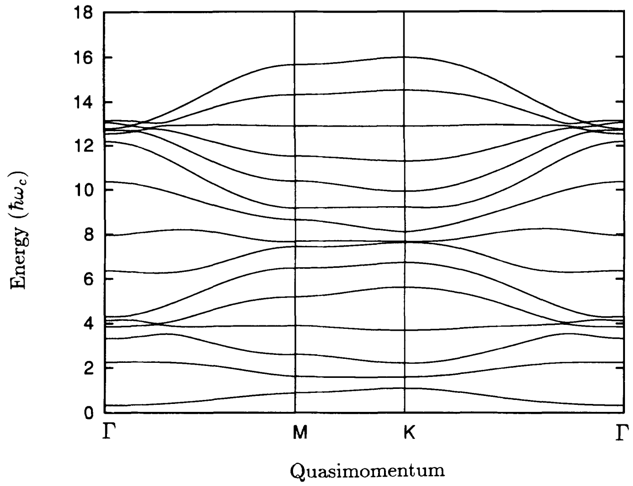

FIG. 4. Exact energy spectrum for a 2DEG in a homogeneous magnetic field modulated by a triangular magnetic lattice in the six-point approximation, $\tilde{B}(1,0)=1$. Many of the curves at the upper left and right are actually separated with higher resolution.

gion of the unit cell the total magnetic field reverses the direction. Notice that these levels in general do not touch. This conforms to a theorem by Von Neumann and Wigner, which states that at least three parameters need to vary for a generic Hamiltonian to have an accidental degeneracy.[11]

Aharonov and Casher once showed that the ground-state energy of a spin-$\frac{1}{2}$ particle in a nonuniform magnetic field is zero.[12] This result is checked numerically in this periodic case by adding a spin-interaction term $(\sigma_z/2)B(x,y)$ to the Hamiltonian (1). Since (10) is not a very good basis for this new Hamiltonian, it is necessary to diagonalize a larger Hamiltonian matrix to get a correct result. The lowest dispersion curve we get for this Hamiltonian with a spin term is found to be flat and zero with negligible deviation, which agrees very well with Aharonov and Casher's result.

IV. ENERGY SPECTRUM AND VORTEX STRUCTURE

It is not hard to see that, when the energy perturbation is small, we can determine $B(x,y)$ with the help of Eqs. (11) and (12) *if the energy spectrum* $\mathcal{E}_n(k_1,k_2)$ *over the entire first Brillouin zone can be mapped out precisely.* Therefore, in principle, the 2DEG beneath the superconductor film can be a probe for measuring the vortex structure. However, this is a difficult experimental task compared to other methods like neutron diffraction, scanning tunneling microscopy,[13] and electron-wave holography.[14] Angle-resolved photoemission spectroscopy (ARPES) can be a tool for measuring the band structure, but it can barely probe beyond 10 Å below the surface, while our 2DEG is buried down in the heterojunction. Besides, the electron density of a conventional 2DEG ($\simeq 10^{11}$ cm^{-2}) is too low for ARPES to have the precision we need. In spite of these difficulties, at least the bandwidth can be measured with confidence, and this can give us some information about the vortex state. In this section we will take a simple model of vortex structure as our starting point, and demonstrate how the bandwidth of the 2DEG may help us determine the coherence length of the superconductor.

A convenient choice for the structure of a vortex was proposed by Clem.[15] The magnetic field distribution of a single vortex in the Clem model is

$$b(\rho)=\frac{\Phi_0}{4\pi\lambda\xi_v}\frac{K_0((\rho^2+\xi_v^2)^{1/2}/\lambda)}{K_1(\xi_v/\lambda)}\ , \tag{20}$$

where K_0 and K_1 are the modified Bessel functions, ρ is a radial coordinate, λ is the penetration depth, and ξ_v is a variation parameter that is of the same order as the coherence length. ξ_v and λ are the only two parameters in this model. The magnetic field of a vortex array is $B_s(\mathbf{r})=\sum_{\mathbf{R}} b(\mathbf{r}-\mathbf{R})$, where the summation is over all sites of the vortices. Therefore the Fourier components

$$B_s(r,s)=\frac{1}{L^2}\int B_s(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d\mathbf{r}$$
$$=\frac{1}{A_{\text{cell}}}\frac{\Phi_0}{2\lambda}\frac{1}{Q}\frac{K_1(Q\xi_v)}{K_1(\xi_v/\lambda)}\ , \tag{21}$$

where A_{cell} is the area of a unit cell, $\mathbf{q}=r\hat{\mathbf{b}}_1+\hat{\mathbf{s}}_2$, and $Q\equiv(|\mathbf{q}|^2+\lambda^{-2})^{1/2}$.

The magnitude of oscillation of the magnetic field $B_s(x,y)$ at the surface of the superconductor is attenuated through the distance d between the superconductor film and the 2DEG. The Fourier component of wave vector $\mathbf{q}$ is damped out by a factor $\exp(-|\mathbf{q}|d)$.[1] Therefore the Fourier components of the magnitude field that appear in Eq. (11) are actually $\tilde{B}(r,s)=B_s(r,s)e^{-|\mathbf{q}|d}$, where

$$|\mathbf{q}|=4\pi/(\sqrt{3}a)\sqrt{r^2-rs+s^2}\ .$$

For example, if $a=1500$ Å (for $B_0=0.1$ T), and $d=400$ Å, then $\tilde{B}(1,0)=0.16B_s(1,0)$, $\tilde{B}(1,-1)=0.04B_s(1,-1)$. The modulation on the 2DEG is small unless B_0 is much less than 0.1 T, or d is much less than 400 Å. Therefore in most cases the perturbation formula works well.

Assuming that the vortex array is dense enough so that the six-point approximation is valid, then the width of the nth band is

$$\Delta\mathcal{E}_n=4.5e^{-\pi/\sqrt{3}}\tilde{B}(1,0)[L_n^1(2\pi/\sqrt{3})+L_{n-1}^1(2\pi/\sqrt{3})]\ . \tag{22}$$

One way of measuring the bandwidth is by measuring the differential Hall conductivity of the 2DEG, which in the collisionless limit is

$$\sigma_H'(E)=\frac{e}{B_0}D(E)f(E)\ , \tag{23}$$

where $D(E)=\sum_n\int d^2\mathbf{k}/4\pi^2\delta(E-\mathcal{E}_n(\mathbf{k}))$ is the density of states, and $f(E)$ is the Fermi distribution. Notice that we have put back the real units in the above formula. This expression can be derived from the Kubo formula, under the assumption that the modulation is small compared to the average field. The effect of modulation appears through the $\mathcal{E}(\mathbf{k})$'s in the density of states and in the Fermi distribution. Equation (23) is the same as the classical formula $\sigma_H = \rho_e e/B_0$, where ρ_e is the density of mobile electrons.

In the presence of disorder, $D(E)$ has to be replaced by the density of extended states $D_e(E)$, or, more precisely, the density of Chern numbers.[16] For a homogeneous magnetic field, it has been shown that $D_e(E)$ is a δ function for each Landau level. When the magnetic field is modulated, we expect that the width of $D_e(E)$ will be approximately equal to the bandwidth in the weak-disorder limit. A tricky point in measuring the bandwidth is that we need a large B to have a good resolution in a quantum Hall system, but that tends to pack the vortex array tightly and shrink the bandwidths. Therefore it is necessary to find a compromise between these two constraints. A theoretical calculation of the Hall conductivity according to (23) at zero temperature is shown in Fig. 5. It varies continuously when the Fermi energy is inside the band, and takes quantized values when the Fermi energy is in the gaps. The slopes are discontinuous at the band edges since the densities of states are discontinuous for a 2D system. How a finite disorder will change the extended density of states remains an open question.

From the bandwidths we can get one Fourier component $\tilde{B}(1,0)$ and hence an algebraic relation between ξ_v and λ through Eqs. (21) and (22). This is useful because the coherence length of a type-II superconductor with large κ is usually more difficult to measure than the penetration depth. Therefore, if λ is known beforehand, measurement of the bandwidth can give us the coherence length. The measured values of $\tilde{B}(1,0)$ from different bands can be used as a consistency check of this approach. In principle it is also possible to determine the temperature dependence of ξ_v since the bandwidth will be different when ξ_v is changed by temperature.

The Hall conductivity gives more information about the band structure than just the bandwidths. One may compare the density of states measured according to Eq. (23) and that calculated from the theoretical model. If the sample is clean enough, such a comparison should yield information about the higher Fourier components of the magnetic field, and therefore more information about the magnetic flux lattice.

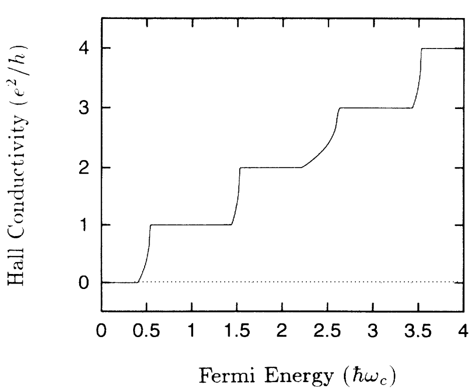

FIG. 5. Hall conductivity at zero temperature. Due to the broadened Landau levels, the transition from one plateau to another is gradual. $\tilde{B}(1,0)$ here is 0.2.

When the strength of the modulation is large, neighboring bands are mixed due to strong oscillations (see Fig. 4); also Eq. (23) for the Hall conductivity is no longer valid. However, each filled band still contributes an integer number of e^2/h to the Hall conductivity as long as it is not in touch with the other bands (usually adjacent bands are separated by avoided crossings). These avoided crossings (or local gaps) can be closed by tuning $\tilde{B}$ to a particular value $\tilde{B}^*$. It does not violate the Von Neumann–Wigner theorem since three parameters $(k_x, k_y, \tilde{B})$ are used to induce an accidental degeneracy. When a local gap collapses, the Hall plateau that corresponds to that gap disappears. A new plateau emerges when the gap is reopened by increasing $\tilde{B}$ away from $\tilde{B}^*$. This plateau may or may not stick to its original value. In case it does not, σ_H will jump up or down by an integer multiple of e^2/h.[17] This dramatic effect can also be achieved by using an electric modulation to control the band crossings.

V. CORRESPONDENCE BETWEEN GENERAL MAGNETIC AND ELECTRIC MODULATIONS

A. One-dimensional modulation

It is well known that the equation of motion of a 2D electron in a constant magnetic field can be reduced to that of a 1D electron in a parabolic electric potential. This analogy can be generalized when the magnetic field is modulated along one direction,[9] where the corresponding electric potential $V(x)$ is determined by

$$B(x) = \frac{d\sqrt{2V(x)}}{dx} . \tag{24}$$

For a weak 1DMM with $B(x) = 1 + \tilde{B}\cos(2\pi/ax)$, the corresponding electric potential is

$$V(x) \simeq \frac{(x-c)^2}{2} + \tilde{B}(x-c)\frac{a}{2\pi}\sin\frac{2\pi}{a}x . \tag{25}$$

Besides the "all-magnetic" and "all-electric" systems, it is also possible to transform the term $(x-c)^2/2$ in Eq. (25) back to a homogeneous magnetic field, and keep the second term intact. This is equivalent to a mixed system with a magnetic field plus a modulating electric field. Following Beenakker's analysis on the drift of the guiding center of an electron in a 1DEM,[2] the mean square drift velocity in the classical limit $R_c \gg a$ is found to be

$$\langle v_d^2 \rangle = \left[\tilde{B}\frac{a}{2\pi} \right]^2 \frac{R_c}{a} \cos^2 \left[\frac{2\pi R_c}{a} - \frac{3\pi}{4} \right] , \tag{26}$$

which is the same as the one in the paper by Xue and Xiao.[3] It has maxima under the resonant condition $2R_c = (\kappa - \frac{1}{4})a$.

B. Two-dimensional periodic modulation

There is no simple mapping as in Eq. (24) when the modulation is bidirectional. However, a connection still exists when the perturbation is small. Consider a 2DEG in a rectangular 2DEM. The first-order energy perturbation is similar to that in Eq. (6). The only difference is that

$$\tilde{B}(l,m)[L_n^1(z_{lm})+L_{n-1}^1(z_{lm})]$$

is replaced by $2V_1(l,m)L_n(z_{lm})$. Therefore, as far as the energy spectrum goes, a magnetic modulation can be simulated using an electric potential whose Fourier components satisfy

$$2V_1(l,m)L_n(z_{lm})=\tilde{B}(l,m)[L_n^1(z_{lm})+L_{n-1}^1(z_{lm})] . \quad (27)$$

It can be readily reduced to a one-dimensional relation if all the components with $l\neq 0$ are zero. Unlike the effective potential in Eq. (25), this one is valid only in the one-band approximation, and is different for different bands.

A curious feature of this formula is that if a z_{lm} happens to coincide with a zero of the Laguerre polynomial, the (l,m)th Fourier component of the effective potential does not exist, simply because there is no way a $V_1(l,m)$ can contribute to the broadening of the nth level, no matter how large it is. Therefore, for a given Landau level with $n\neq 0$, an electric modulation can only mimic a magnetic modulation whose Fourier components $\tilde{B}(l,m)$ vanish when z_{lm} are at zeros of L_n. Conversely, a magnetic modulation can only mimic an electric modulation whose Fourier components $V_1(l,m)$ vanish when z_{lm} are at zeros of $L_n^1+L_{n-1}^1$.

C. General two-dimensional field distribution

It is possible to extend the above connection to a random magnetic field. The only restriction is that the random component be weak compared to the homogeneous component. Consider the matrix elements of the Hamiltonian in the unperturbed basis (2) (no requirement of one flux quantum per unit cell). Express the magnetic field in a Fourier integral instead of Fourier series since there is no discrete translational symmetry here; then we have

$$\langle n,q-q'|H_1^B(x,y)|n,q\rangle=\frac{1}{2}\int dk_2\tilde{B}(q',k_2)(L_n^1+L_{n-1}^1) \times e^{-(1/4)(q'^2+k_2^2)} \times e^{(i/2)q'k_2}e^{ik_2q} , \quad (28)$$

where the argument in the associated Laguerre polynomial is $(q'^2+k_2^2)/2$. For a general electric potential $V(x,y)$, the matrix elements are

$$\langle n,q-q'|H_1^E(x,y)|n,q\rangle =\int dk_2V(q',k_2)L_ne^{-(1/4)(q'^2+k_2^2)}e^{(i/2)q'k_2}e^{ik_2q} . \quad (29)$$

Since the basis $|n,q\rangle$ form a complete set for the nth level, it follows that the two operators H_1^B and H_1^E are equivalent in the one-band approximation if (28) and (29) are equivalent for any q and q'. Assuming the equivalence of these two matrices, we can take inverse Fourier transforms of the integrals with respect to q to get an identity between the integrands

$$\tilde{B}(L_n^1+L_{n-1}^1)\exp[-(q'^2+k_2^2)/4]\exp(iq'k_2/2)$$

and

$$VL_n\exp[-(q'^2+k_2^2)/4]\exp(iq'k_2/2) .$$

After dividing out the common exponential part, it follows that $H_1^B=H_1^E$ if and only if

$$2L_nV(k_1,k_2)=(L_n^1+L_{n-1}^1)\tilde{B}(k_1,k_2), \quad \forall k_1,k_2 , \quad (30)$$

which is a continuous version of Eq. (27). The discussion in the preceding subsection about the singularities in the Fourier components applies to this case as well. This relation might be helpful to researchers working on problems of random magnetic field.

VI. CONCLUSION

We have calculated the energy spectra of 2D electrons in two-dimensional periodic magnetic fields. The spectra are expressed in quasimomenta that are good quantum numbers of the system. It is found that the Landau levels are broadened in an osillatory manner with respect to the band index. When the modulation of the magnetic field is strong, dispersion curves are mixed but show avoided crossings. The magnetic lattice with triangular symmetry can be realized by using a superconductor film in a vortex state. A connection between the electric properties of the 2DEG and the magnetic properties of the vortex arrays is studied. It shows a possible way of getting information about the vortices through measurements of the bandwidth of the 2DEG. Finally, the connection between electric field modulation and magnetic field modulation is discussed. We find the effective electric potentials for weak 2D magnetic modulations. The effective potentials differ from band to band, and do not always exist.

ACKNOWLEDGMENTS

We would like to thank D. Gavenda, J. Markert, C. K. Shih, and T. Wang for helpful discussions. This work is supported by NIST and the Welch Foundation.

[1]R. R. Gerhardts, D. Weiss, and K. v. Klitzing, Phys. Rev. Lett. **62**, 1173 (1989); R. W. Winkler, J. P. Kotthaus, and K. Ploog, *ibid.* **62**, 1177 (1989).
[2]P. Vasilopoulos and F. M. Peeters, Phys. Rev. Lett. **63**, 2120 (1989); C. Zhang and R. R. Gerhardts, Phys. Rev. B **41**, 12 850 (1990); R. R. Gerhardts, *ibid.* **45**, 3449 (1991); C. W. J. Beenakker, Phys. Rev. Lett. **62**, 2020 (1989).
[3]D. P. Xue and G. Xiao, Phys. Rev. B **45**, 5986 (1992); X. Wu

and S. E. Ulloa, *ibid.* **47**, 7182 (1993); R. Yagi and Y. Iye, J. Phys. Soc. Jpn. **62**, 1279 (1993).

[4]D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976).

[5]H. Fang and P. J. Stiles, Phys. Rev. B **41**, 10 171 (1990); R. R. Gerhardts, D. Weiss, and U. Wulf, *ibid.* **43**, 5192 (1991); Y. Tan and D. J. Thouless, *ibid.* **49**, 1827 (1994).

[6]B. A. Dubrovin and S. P. Novikov, Zh. Eksp. Theor. Fiz. **79**, 1006 (1980) [Sov. Phys. JETP **52**, 511 (1980)]; Sov. Math. Dokl. **22**, 240 (1980) [Dokl. Akad. Nauk. SSSR **253**, 1293 (1980)]; S. P. Novikov, *ibid.* **23**, 298 (1981) [*ibid.* **257**, 538 (1981)].

[7]X. Wu and S. E. Ulloa, Phys. Rev. B **47**, 10 028 (1993). See also D. Yoshioka and Y. Iye, J. Phys. Soc. Jpn. **87**, 448 (1987); G. J. O. Schmidt, Phys. Rev. B **47**, 13 007 (1993).

[8]Experiments using similar setups have been done in the studies of weak localization of electrons in an inhomogeneous magnetic field. See S. J. Bending, K. v. Klitzing, and K. Ploog, Phys. Rev. Lett. **65**, 1060 (1990); A. K. Geim, S. J. Bending, and I. V. Grigorieva, *ibid.* **69**, 2252 (1992).

[9]F. M. Peeters and A. Matulis, Phys. Rev. B **48**, 15 166 (1993).

[10]D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. de Nijs, Phys. Rev. Lett. **49**, 405 (1982).

[11]J. Von Neumann and E. P. Wigner, Phys. Z **30**, 467 (1929).

[12]Y. Aharonov and A. Casher, Phys. Rev. A **19**, 2461 (1979). The degeneracy of the ground-state energy for two-dimensional electrons (with spin) in an inhomogeneous magnetic field has been generalized to the situation that the electrons are embedded in an arbitrary two-dimensional curved surface, with the magnetic field being everywhere orthogonal to the surface. See R. Alicki, J. R. Klauder, and J. Lewandowski (unpublished).

[13]H. F. Hess, R. B. Robinson, and J. V. Waszczak, Phys. Rev. Lett. **64**, 2711 (1990); F. Gygi and M. Schluter, Phys. Rev. B **43**, 7609 (1991).

[14]S. Hasegawa, T. Matsuda, J. Endo, N. Osakabe, M. Igarashi, T. Kobayashi, M. Naito, A. Tonomura, and R. Aoki, Phys. Rev. B **43**, 7631 (1991).

[15]J. R. Clem, J. Low Temp. Phys. **18**, 427 (1975).

[16]Y. Huo and R. N. Bhatt, Phys. Rev. Lett. **68**, 1375 (1992).

[17]A detailed study has been done by M. Nielson and P. Hedegard (unpublished).