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Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

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When a conducting layer is placed in a strong perpendicular magnetic field, there exist current-carrying electron states which are localized within approximately a cyclotron radius of the sample boundary but are extended around the perimeter of the sample. It is shown that these quasi-one-dimensional states remain extended and carry a current even in the presence of a moderate amount of disorder. The role of the edge states in the quantized Hall conductance is discussed in the context of the general explanation of Laughlin. An extension of Laughlin's analysis is also used to investigate the existence of extended states in a weakly disordered two-dimensional system, when a strong magnetic field is present.

I. INTRODUCTION

In a recent paper Laughlin has given a very elegant and general explanation of the phenomenon that under appropriate conditions, for a two-dimensional sample in a strong magnetic field, at $T=0$, the Hall conductance is quantized in *exact* multiples of the unit e^2/h.[1-4] The purpose of the present paper is to discuss some curious properties of electronic states in a magnetic field that are implied by Laughlin's analysis, and, incidentally, to clarify some details of Laughlin's argument. In particular, it is shown in Secs. II and III below that states at the perimeter of the sample are quasi-one-dimensional states which carry a current, and which do not become localized in the presence of a disordered potential of moderate strength. The perimeter states play an important role in the Hall measurement, if the Fermi levels are different at two edges of the sample.

Following the method of Laughlin,[1] we consider a film of annular geometry, in a magnetic field perpendicular to the plane of the film. In this case, the currents at the inner and outer edge are in opposite directions, and they contribute no net current around the annulus if the Fermi levels are the same at the two edges. If the two Fermi levels differ by an amount $e\Delta$, however, we find that the edge states contribute a net current δI around the ring given by

$$\delta I = ne^2\Delta/h \ , \tag{1}$$

where n is an integer. This contribution is consistent with the quantized Hall conductance, as the chemical potential difference Δ is included, along with any electrostatic potential present, in the potential difference that would be measured by a voltmeter connected between the inner and outer edges of the ring. Of course, the edge current and the quantity Δ are taken into account automatically in the general analysis of Laughlin.[1]

In Sec. IV below, we use an extension of Laughlin's analysis to investigate the question of whether extended states can exist in principle in the interior of a two-dimensional disordered system. We conclude that there must exist a band of extended states in the vicinity of the Landau energy, or at least an energy at which the localization length diverges, if the random potential is weak compared to the cyclotron energy $\hbar\omega_c$.

II. IDEAL SAMPLE

Let us first consider a collection of noninteracting electrons, confined in an ideal uniform film of annular geometry, with a uniform magnetic field $\vec{B}_0$ perpendicular to the plane of the sample. (See Fig. 1.) We assume in addition that there is a magnetic flux Φ, confined to the interior of a solenoid magnet threading the hole in the annulus, and we shall be able to vary the flux Φ without changing the magnetic field in the region where the electrons are confined. (This is a slight modification of the cylinder geometry considered by Laughlin.) We shall assume that no electric field is present so that the electrostatic potential seen by the electrons is constant in the interior of the film, and we assume that the dimensions of the annulus are very large compared to the cyclotron radius r_c for electrons in the magnetic field. We adopt the gauge where the vector potential $\vec{A}$ points in the

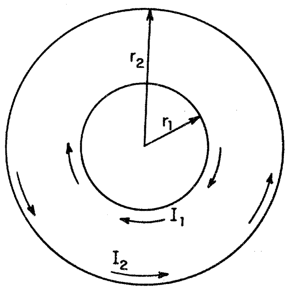

FIG. 1. Geometry of sample. Annular film, in region $r_1 < r < r_2$ is placed in uniform magnetic field B_0, pointing out of the page. Additional magnetic flux Φ is confined to region $r < r_1$. Curved arrows show direction of currents I_1 and I_2 at the boundaries of film.

azimuthal (θ) direction, and the magnitude of $\vec{A}$ depends only on the distance from the center of the annulus:

$$A = \tfrac{1}{2} B_0 r + \Phi / 2\pi r \ . \tag{2}$$

Away from the edges of the film, the electronic states in this geometry have the form

$$\psi_{m,\nu}(\vec{r}) \simeq \text{const} \times e^{im\theta} f_\nu (r - r_m) \ , \tag{3}$$

where m and ν are integers, with $\nu \geq 0$, f_ν is the $\nu+1$ eigenstate of a one-dimensional harmonic oscillator, and the radius r_m is determined by

$$B_0 \pi r_m^2 = m \Phi_0 - \Phi \ . \tag{4}$$

Here Φ_0 is the flux quantum, hc/e. The width of f is of order r_c, where r_c is the cyclotron radius. Of course, Eq. (3) is only applicable if r_m is in the range $r_1 < r_m < r_2$, with $r_m - r_1$ and $r_2 - r_m$ large compared to r_c. We shall assume throughout that r_c is small compared to r_1 and $r_2 - r_1$. The energies of the states (3) are given by the Landau formula

$$E_{m,\nu} = \hbar\omega_c (\nu + \tfrac{1}{2}) \ , \tag{5}$$

where ω_c is the cyclotron frequency determined by the field B_0 and the carrier effective mass m^*:

$$\omega_c = |eB_0| / m^* c \ . \tag{6}$$

The electron density $|\psi_{m,\nu}(r)|^2$ associated with Eq. (3) is symmetric about the radius r_m, and decays rapidly for $|r - r_m| / r_0 >> 1$. The current carried by the state is given by

$$I_{m,\nu} = \frac{e}{m^*} \int_0^\infty dr \, |\psi_{m,\nu}(\vec{r})|^2 \left[\frac{m\hbar}{r} - \frac{eA(r)}{c} \right]$$
$$\simeq \frac{e^2 B_0}{m^* c} \int_0^\infty dr \, |\psi_{m,\nu}|^2 (r_m - r) \ . \tag{7}$$

The integral may be taken over the radial coordinate r, at any fixed value of θ. The net current vanishes for states in the interior of the annulus, since the probability densities of the harmonic oscillator states are symmetric about the point $r = r_m$.

The situation is very different when r_m is closer than a few times r_c to an edge of the sample. Then the condition that the wave function vanish at the edges of the sample will shift the energies of the eigenstates away from the Landau energies (5).

Let us focus our attention on the behavior near the outer edge of the annulus, and let us continue to use the index ν to label the number of nodes in the radial wave function. We may then write the electronic wave functions as

$$\psi_{m,\nu}(\vec{r}) = \text{const} \times e^{im\theta} g_\nu (r - r_m, r_2 - r_m) \ , \tag{8}$$

where $g_\nu(x,s)$ is a wave function which is defined in the region $-\infty < x < s$ and has ν nodes, which vanishes for $x \to s$ and $x \to -\infty$, and which obeys the eigenvalue equation

$$\left[-\frac{\hbar^2}{2m^*} \frac{d^2}{dx^2} + \frac{B_0^2 e^2 x^2}{2m^* c^2} \right] g_\nu = E g_\nu \ . \tag{9}$$

Now it is clear that the eigenvalue $E_{m,\nu}$ will approach the value $E_\nu = \hbar\omega_c(\nu + \frac{1}{2})$, for $r_2 - r_m >> r_c$. The energy $E_{m,\nu}$ will increase monotonically as r_m increases, passing through the value $E_{m,\nu} = \hbar\omega_c (2\nu + \frac{3}{2})$, when $r_m = r_2$, and increasing eventually as $(r_m - r_2)^2 e^2 B_0^2 / 2m^* c^2$ for $r_m - r_2 > r_c$. The energy curve is sketched in Fig. 2.

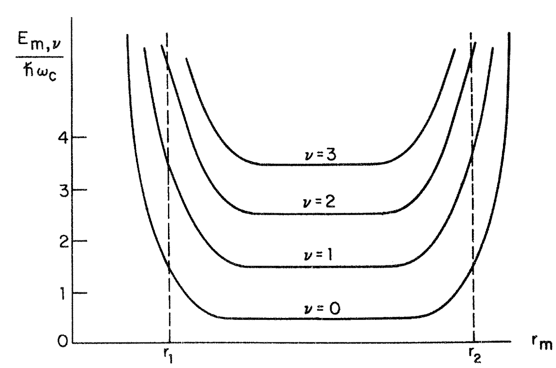

FIG. 2. Energy levels of nonrandom system, in units of $\hbar\omega_c$, as a function of the parameter r_m. The latter quantity is determined by the azimuthal quantum number m, according to Eq. (4), and it is the radius at which the azimuthal current density vanishes for quantum number m. The radius r_m is the center of the wave function $\psi_{m\nu}$ provided that r_m is not too close to the boundary r_1 or r_2.

Since the density $|\psi_{m,\nu}(\vec{r})|^2$ is no longer symmetric about $r=r_m$, we no longer expect that $I_{m,\nu}=0$. In fact, it is readily established that

$$I_{m,\nu}=-c\frac{\partial E_{m,\nu}}{\partial \Phi}=\frac{e}{h}\frac{\partial E_{m,\nu}}{\partial m} . \qquad (10)$$

For $B_0>0$, we find that $I_{m,\nu}>0$, for $r_m \simeq r_2$, while $I_{m,\nu}<0$, near the inner edge $r_m \simeq r_1$.

Note that the quantity $|\partial E_{m,\nu}/\partial m|$ is just the energy separation between adjacent energy levels for a given quantum number ν. Thus the total current carried by states of a given ν in a small-energy interval δE is equal to $(e/h)\delta E$ at the outer edge of the sample, and $-(e/h)\delta E$ at the inner edge. (We neglect here any spin or valley degeneracy of the carriers.)

Let us suppose that the Fermi level lies in between the energies E_ν of two Landau levels $\nu=n-1$ and $\nu=n$, in the interior of the sample. Suppose also that near r_2 and r_1 there are Fermi levels $E_F^{(2)}$ and $E_F^{(1)}$, respectively, which differ from each other, but still lie in the interval between E_{n-1} and E_n. Then the total current carried by the edge states between $E_F^{(2)}$ and $E_F^{(1)}$ is clearly given by $neh^{-1}(E_F^{(2)}-E_F^{(1)})$, in agreement with Eq. (1).

In a real experiment, the measured Hall potential eV is the sum of an electrostatic potential eV_0 and the difference in Fermi levels $E_F^{(2)}-E_F^{(1)}$. The edge current is then only a *fraction* of the total Hall current, given by $(E_F^{(2)}-E_F^{(1)})/eV \approx \alpha n r_c \hbar\omega_c C/e^2$ where C is the capacitance per unit length of the edge states, and α is a number of order unity.

III. DISORDERED SAMPLE

Now we must show that the edge currents are not destroyed by a moderate amount of disorder in the sample. Let us consider the effect of a weak random potential $V(\vec{r})$, with $|V(\vec{r})| << \hbar\omega_c$. Let us consider for simplicity a situation where the Fermi level E_F lies midway between the unperturbed Landau energies E_0 and E_1. It is then clear that there will be no energy eigenstates with E near E_F in the interior of the sample, but there will remain two bands of states with E near E_F which are radially localized near r_2 and r_1, respectively.

Consider an energy eigenstate ψ from the band at r_2, and write the state as superposition of the eigenstates $\psi_{m\nu}$ of the nonrandom system:

$$\psi(\vec{r})=\sum_{m\nu} c_{m\nu}\psi_{m\nu}(\vec{r}) . \qquad (11)$$

The expansion coefficient $c_{m\nu}$ will be relatively large for $\nu=0$ and r_m near to, but slightly smaller than r_2. The coefficient $c_{m\nu}$ will be smaller by a factor of order $V(\vec{r})/\hbar\omega_c$ for $\nu\geq 1$, and $c_{m\nu}$ will be "exponentially small" for $|r_2-r_m|>>r_c$.

The azimuthal current carried by the state ψ is given by

$$\langle I\rangle=\sum_{m\nu\nu'} c^*_{m\nu'}c_{m\nu}I_{m\nu\nu'} , \qquad (12)$$

where

$$I_{m\nu\nu'}\equiv\frac{e}{2\pi m^*}\int\int dr\,d\theta\,\psi^*_{m\nu'}(\vec{r})\psi_{m\nu}(\vec{r}) \times\left[\frac{m\hbar}{r}-\frac{eA(r)}{c}\right] . \qquad (13)$$

Note that azimuthal current must be independent of θ, since current conservation requires $\vec{\nabla}\cdot\langle\vec{j}(\vec{r})\rangle=0$, where $\langle\vec{j}(\vec{r})\rangle$ is the current density carried by any exact eigenstate of the Hamiltonian. We see that $I_{m\nu\nu'}$ is identical to $I_{m\nu}$ when $\nu=\nu'$. Furthermore, for r_m near r_2, I_{m01} is of the same order as I_{m0}, namely of order $e\omega_c r_c/r_2$. It follows then that the off-diagonal contribution $(\nu\neq\nu')$ to Eq. (12) cannot cancel the positive diagonal contribution $(\nu=\nu'=0)$, when $V(\vec{r})/\hbar\omega_c$ is small; hence the current $\langle I\rangle$ is nonzero. It follows also from current conservation that the eigenstate ψ is not localized azimuthally in any region of θ, but must be spread more or less uniformly around the annulus.

It is clear, physically, that the situation is unaltered if there are a few isolated regions with $V(\vec{r})>>\hbar\omega_c$. Although there may be localized bound states or resonances in the regions of strong potential, the current-carrying edge states will simply be displaced, locally, to go around these regions. Of course, if the random potential becomes sufficiently strong that electron scattering rate is large compared to ω_c, it is no longer useful to employ the Landau levels as starting points and the arguments given here breakdown.

The arguments given above can be extended, with little difficulty, to the case where E is midway between the $\nu=1$ and $\nu=2$ Landau levels, etc. In this case there will be several values of ν for which the expansion coefficients $c_{m\nu}$ can be large. The contributions of the off-diagonal terms $(\nu\neq\nu')$ in Eq. (12) to the current carried by the state ψ are nevertheless small for $V(\vec{r})/\hbar\omega_c<<1$, because the matrix element $I_{m\nu\nu'}$ is diagonal in m, while the largest values of $c_{m\nu}$ occur at different values of m

for different oscillator levels ν. The reasoning clearly breaks down, on the other hand, if E_F is too close to an unperturbed energy E_ν.

Our argument that $\langle I \rangle \neq 0$ for an edge state in a weakly disordered system did not show that the current carried satisfies Eq. (1) exactly in this case. The validity of this equation may be most easily established by considering what happens as one adiabatically increases the threading flux Φ by one flux quantum, in the manner described by Laughlin, in Ref. 1. We shall not repeat that analysis here in detail, but we shall mention some essential features in the following section.

IV. DO EXTENDED STATES EXIST IN TWO DIMENTIONS?

A starting point of Laughlin's analysis of the quantized Hall conductance is the assumption that for a collection of noninteracting electrons in an infinite two-dimensional sample with weak disorder, in a strong perpendicular magnetic field, there exist energy bands of extended states ("Landau levels") separated by energy regions of localized states and/or energy gaps where there are no states at all. Laughlin shows that if the Fermi energy occurs at a position outside the bands of extended Landau states, and if the flux Φ threading the hole of an annular sample is increased adiabatically by one flux quantum Φ_0, then the net effect will be to transfer an integral number n electrons from the Fermi level at the outer edge to states at the Fermi level of the inner edge of the sample. Since the net change in the energy of the sample is $-neV$, where V is the voltage difference between the outer and inner edge of the sample, and since the work done in the flux change is equal to $-c^{-1}I\Phi_0$, where I is the current around the loop, Laughlin has established that $I/V = nec/\Phi_0$.

It is natural to identify the integer n with the number of bands of extended states below the Fermi energy (multiplited by the spin and valley degeneracy of the carriers), and it is natural to suppose that for weak disorder this number will be the same as the number of Landau levels that would occur below E_F in the absence of disorder. If the disorder is sufficiently strong, however, so that all states below E_F are localized, then we would obtain the integer $n=0$, and the quantized Hall conductance would not be observed.

It is now generally believed that in the absence of a magnetic field or other mechanisms to break the time-reversal invariance of the Schrödinger equation, the electronic states in a two-dimensional random potential are *always* localized, in principle.[5] When time-reversal symmetry is broken, the leading term responsible for localization in the renormalization-group equations is known to be absent; nevertheless, it has remained an open question whether extended states can exist in a two-dimensional system under these conditions.[5,6]

If two-dimensional states were actually always localized there would seem to be a serious problem, in principle, with the starting point of Laughlin's theory. One could take the point of view that the experimental existence of a nonzero, quantized Hall conductance is sufficient evidence for the existence of extended states, and that further discussion of this point is unnecessary.[7] For the sake of intellectual completeness, however, it seems worthwhile to note that the existence of extended states and of nonzero Hall conductance can actually be demonstrated theoretically, at least in the case of a weakly disordered sample in a strong magnetic field, by an extension of Laughlin's arguments, which will be given below. (Actually, we cannot rule out the possibility that the energy regions of extended states have vanishing width in the limit of an infinite sample, but this possibility would still be compatible with a nonzero quantized Hall conductance.) In addition, the theoretical argument can be applied directly to the theoretically important case of noninteracting electrons, whereas the electron-electron interactions could *a priori* be important in the experimental systems.[8] In the discussion below, we shall in fact confine ourselves to the noninteracting case, although a small modification of the arguments also confirms that the electron-electron interaction, if it is not too strong, will not destroy a nonzero, quantized Hall conductance.[9]

We begin by generalizing the annular geometry of Fig. 1 as follows. We divide the sample into three concentric regions, bounded by radii $r_1 < r_1' < r_2' < r_2$. For $r_1 < r < r_1'$ and $r_2' < r < r_2$, we assume the potential $V(\vec{r})=0$. For $r_1' < r < r_2'$, we assume a weak random potential $V(\vec{r}) << \hbar\omega_c$. There is no macroscopic electrostatic field present, and we assume infinite reflecting walls at r_1 and r_2 as before. We shall assume the dimensions of the sample to be arbitrarily large compared to any microscopic length.

The electronic energy levels in this geometry are indicated in Fig. 3. In the border regions $r_1 < r < r_1'$ and $r_2' < r < r_2$, the analysis of Sec. II applies, and the electronic states are well under-

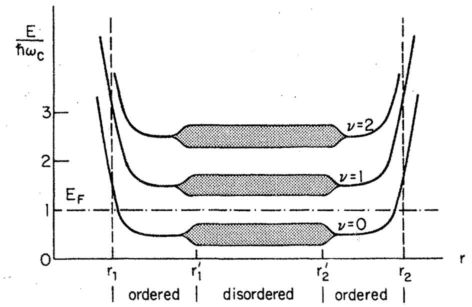

FIG. 3. Energy bands, as a function of position for the inhomogeneous geometry described in Sec. IV. Regions $r_1 < r < r'_1$ and $r'_2 < r < r_2$ contain ordered "ideal" conductor, while region $r'_1 < r < r'_2$ contains a weak random potential $V(\vec{r})$.

stood. The states have energies $E_{m\nu}$ which are given by the Landau formula $E_\nu = \hbar\omega_c(\nu + \frac{1}{2})$, except at the boundaries r_1 and r_2, where they are pushed upward in energy as in Fig. 2. Now, in the interior disordered region $r'_1 < r < r'_2$, we expect that the states will occur in a series of energy bands of finite width, centered about the energies E_ν. If the potential $V(\vec{r})$ is sufficiently weak there should be no states in the region midway between two Landau levels. (Alternatively, if there are a small number of strong impurities, there may be a small density of isolated impurity levels in the mid-gap region; these states will be localized on a scale of order r_c, however, and will not be important for our argument.)

We may now choose one of two hypotheses:

(a) The states in the disordered region are *localized at all energies* with a finite energy-dependent localization length $\lambda(E)$.

(b) The states near the center of each magnetic energy band are *delocalized,* or at least $\lambda(E) \rightarrow \infty$ for some energy E in the band.

We shall adopt hypothesis (a), and see that this leads to a contradiction.

Assume that, initially, all electron states in the sample are filled up to a Fermi level E_F, which we choose to lie at the energy $\hbar\omega_c$, midway between the $\nu=0$ and $\nu=1$ Landau levels. Let $\lambda_{\max}$ be the maximum value of $\lambda(E)$, for $E < E_F$, and choose $r'_2 - r'_1 >> \lambda_{\max}$. Let us now increase adiabatically the flux Φ through the hole in the annulus, by one flux quantum Φ_0. Since, initially, there was no net current flowing in the sample, there is no work done in this process, or, more accurately, the work $-c^{-1}\int I\,d\Phi$ is inversely proportional to the size of the system, as the induced current will be small for large r. We also know that the electronic wave functions in the ordered regions will contract slightly during the flux change so that at the end there is one state unoccupied just below E_F, at $r \approx r_2$, and one new state occupied just above E_F, at $r \approx r_1$.

This change in occupation costs no energy in the limit of a large sample. If, however, in the disordered region the states below the Fermi surface are all localized, there will be no way to transport an electron across this region, since, as discussed by Laughlin, localized states remain unchanged during the flux increase, except for an uninteresting phase factor $e^{i\theta(\vec{r})}$. Then the electron removed from $r \approx r_2$ must be "transferred" to a new occupied state at $r \approx r'_2$, and the new electron at $r \approx r_1$ must be associated with a hole near $r \approx r'_1$. However, by construction, there are no states in the interior of the sample with energy near E_F (except perhaps for some strongly localized impurity states, whose occupation cannot change during the flux increase). It follows that the required change of occupation must cost an energy of order $\hbar\omega_c$, which would be a violation of conservation of energy. Therefore, there must be at least some delocalized states below the Fermi level, even in the disordered region of the sample.

It is interesting to ask what happens to the above argument when the random potential is sufficiently strong that all states below the Fermi energy are localized in the disordered region. It seems that the bands of extended states do not disappear, but rather are pushed upwards in energy as the disorder is increased, and that the Hall conductance ceases when the lowest extended band rises above the Fermi energy.[10] In an inhomogeneous geometry such as that considered above, there will be current-carrying states at the Fermi level near the boundaries of the disordered region (radii r'_1 and r'_2) analogous to the current-carrying states at the edge of the sample. Under these circumstances, the addition of a flux quantum will transfer an electron from a state at r_2 to a state at r'_2 and another electron from a state at r'_1 to a state at r_1, so that the Laughlin argument cannot be applied to the sample as a whole. The Hall current will then be determined by the voltage drops across the nondisordered regions only.

As a final remark, we note that by using the geometry described above, we have put Laughlin's derivation of the exact quantization of the Hall conductance in a form which does not require any *a priori* assumption about the behavior of extended states in the disordered region, during the adiabatic change of Φ. We have only made use of the known behavior of the wave functions in the ordered boundary regions, and the relatively trivial behavior of any localized states at the Fermi level during the change of Φ. The transfer of charge through the disordered region, and the quantized relation for I/V, are then implied by conservation of energy and particle number.

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[1]R. B. Laughlin, Phys. Rev. B 23, 5632 (1981).

[2]The quantized Hall conductance was predicted originally, on the basis of an approximate calculation of a simple model, by T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. 39, 279 (1975). Subsequently, the quantization was observed to hold with great precision experimentally by K. V. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980), and by others. Recent experimental results from a number of laboratories are reported in the Proceedings of the Fourth International Conference on Electronic Properties of Two-Dimensional Systems, New London, New Hampshire, 1981 [Surf. Sci. (in press)].

[3]See also the discussion of R. E. Prange, Phys. Rev. B 23, 4802 (1981), and references therein, and H. Aoki and T. Ando, Solid State Commun. 38, 1079 (1981).

[4]D. J. Thouless [J. Phys. C 14, 3475 (1981)] has taken an approach quite different from Laughlin's, and has presented an argument, based on the convergence of perturbation theory, that the quantization of Hall conductance remains exact in the presence of a random potential smaller than $\frac{1}{2}\hbar\omega_c$.

[5]See P. A. Lee and D. S. Fisher, Phys. Rev. Lett. 47, 882 (1981), and references therein.

[6]P. A. Lee (private communication); S. Hikami (private communication).

[7]Ando and Aoki, Ref. 3, and also Thouless, Ref. 4, have also emphasized that a nonzero Hall conductance at $T=0$ implies the existence of some extended states below the Fermi level.

[8]It has been suggested by H. Fukuyama, P. M. Platzman, P. A. Lee, and P. W. Anderson that the electron-electron interaction must be taken into account explicitly if one wishes to understand experiments in which the carrier density is varied and the Fermi level passes through a region of extended states [H. Fukuyama (private communication); H. Fukuyama and P. M. Platzman (unpublished)].

[9]One way to consider electron-electron interactions is via a *Gedanken* experiment, where the electron-electron interaction applies only when the electrons are inside the disordered region $r_1' < r < r_2'$ of Fig. 3. If the interaction is not too strong, there must remain an energy gap between the first and second Landau levels. If necessary, we may add a constant background potential in the disordered region to keep the Fermi level in this gap. The requirements of conservation of energy and of particle number then imply that a nonzero integral number of electrons is transferred through the disordered region, when the flux Φ is increased by one flux quantum, just as in the noninteracting case, analyzed in Sec. IV. If there is a finite density of localized states at the Fermi level in the noninteracting case, then we must make the additional reasonable assumption that these states remain localized (i.e., nonconducting) in the presence of the electron-electron interaction.

[10]This suggestion was also made by R. B. Laughlin (private communication).