Magnon Hall Effect

I. TASK LIST

Anomalous Hall effect is well-know in electronic system. Moreover, there is also Anomalous Hall effect in magnon system. Unlike electrons, magnon does not carry charge so it will not feel electric field. But when we apply a temperature gradient, the statistical force will drive the magnon and cause the magnon Hall effect. In an anti-ferromagnetic honeycomb lattice in $x - y$ plane with an external magnetic field in $\hat{z}$ direction, the nearest DM interaction is 0 and the next nearest DM interaction is in the direction.

Calculate the Berry curvature and thermal conductivity. Check whether magnon Hall effect exists in this model. If not, tell why and what kind of transport phenomenon should exist.

II. GUIDANCE

1. Hamiltonian [1]

Firstly, you should write down the Hamiltonian of this system

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\langle\langle i,j \rangle\rangle} \nu_{ij} \boldsymbol{D}_{ij} \cdot (\boldsymbol{S}_i \times \boldsymbol{S}_j) + \mathcal{K} \sum_i S_{iz}^2, \tag{1}$$

where $\nu_{ij} = \pm 1$ is direction choice of DM interaction which is the same with that in [1]. Make sure you know the meaning of every term in the Hamiltonian above. By means of the Holstein-Primakoff transformation

$$S_i^+ = \sqrt{2S} a_i, \quad S_i^- = \sqrt{2S} a_i^\dagger, \quad S_i^z = S - a_i^\dagger a_i, \tag{2}$$

you can turn the Hamiltonian into second quantization form

$$H = \sum_{\boldsymbol{k}} \psi_{\boldsymbol{k}}^\dagger \mathcal{H}_{\boldsymbol{k}} \psi_{\boldsymbol{k}}, \tag{3}$$

where $\psi_{\boldsymbol{k}} = (a_{\boldsymbol{k}}, b_{\boldsymbol{k}}^\dagger)^T$ is Nambu basis in momentum space which can be derived by Fourier transform

$$\begin{pmatrix} a_i \\ b_i^\dagger \end{pmatrix} = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{R}_i} \begin{pmatrix} a_{\boldsymbol{k}} \\ b_{\boldsymbol{k}}^\dagger \end{pmatrix}. \tag{4}$$

2. Diagonalization [2]

Secondly, you should use Bogoliubov transform [3]

$$\begin{pmatrix} \alpha_{\boldsymbol{k}} \\ \beta_{\boldsymbol{k}}^\dagger \end{pmatrix} = \begin{pmatrix} u_{\boldsymbol{k}} & -v_{\boldsymbol{k}} \\ -v_{\boldsymbol{k}} & u_{\boldsymbol{k}} \end{pmatrix} \begin{pmatrix} a_{\boldsymbol{k}} \\ b_{\boldsymbol{k}}^\dagger \end{pmatrix} \tag{5}$$

to get equation of motion of new quasiparticle operators

$$i\hbar\dot{\alpha}_{\boldsymbol{k}} = [\alpha_{\boldsymbol{k}}, \mathcal{H}_{\boldsymbol{k}}]. \tag{6}$$

After obtaining the eigenequation, you can diagonalize it and get energy spectrum and wavefunctions.(Why not just diagonalize $\mathcal{H}_{\boldsymbol{k}}$? About this you can read [2, 3].)

3. Thermal Conductivity [4]

Next, you should use the energy spectrum and eigenvectors you have already got to calculate the Berry curvature of magnon. Take care of the σ_z when you deal with the magnon dynamics. So the Lagrangian will be like

$$\begin{aligned} \mathcal{L} &= \langle W \left| i\hbar\sigma_z(d/dt) - \mathcal{H}' \right| W \rangle, \\ \mathcal{H}' &= \mathcal{H}_k + U(r), \end{aligned} \tag{7}$$

where $U(\boldsymbol{r})$ is confining potential felt by magnon.

Finally, consider the potential-driven magnon Hall effect and get the thermal conductivity. The dynamics of the wave packet is described by the semiclassical equation of motion

$$\begin{aligned} \dot{\boldsymbol{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{n\boldsymbol{k}}}{\partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}_n(\boldsymbol{k}), \\ \hbar\dot{\boldsymbol{k}} &= -\nabla U(\boldsymbol{r}). \end{aligned} \tag{8}$$

You may derive the thermal conductivity through

$$\begin{aligned} j_Q &= \int d\varepsilon f(\varepsilon)\varepsilon\dot{r}, \\ j_{Qx} &= \kappa_{xy}\partial_y U(r), \end{aligned} \tag{9}$$

where $f(\varepsilon)$ is Bose-Einstein distribution function, ε is energy of magnonm, and κ_{xy} is thermal conductivity driven by potential.

This model is the same with that in [1]. For each part, you can go to corresponding reference for detailed process.

[1] R. Cheng, S. Okamoto, and D. Xiao, Spin nernst effect of magnons in collinear antiferromagnets, Phys. Rev. Lett. **117**, 217202 (2016).

[2] S. Toth and B. Lake, Linear spin wave theory for single-q incommensurate magnetic structures, Journal of Physics: Condensed Matter **27**, 166002 (2015).

[3] 固体理论 (高等教育出版社, 2002).

[4] R. Matsumoto and S. Murakami, Rotational motion of magnons and the thermal hall effect, Phys. Rev. B **84**, 184406 (2011).