True-MCSA: A Framework for Truthful Double Multi-Channel Spectrum Auctions
Zhili Chen, He Huang, Yu-e Sun, and Liusheng Huang

Abstract—Spectrum auctions motivate existing spectrum owners (as sellers) to lease their selected idle channels to new spectrum users (as buyers) who need the spectrum desperately. The most significant requirement is how to make the auctions economic-robust (truthful in particular) while enabling spectrum reuse. Furthermore, in practice, both sellers and buyers would require to trade multiple channels at one time, while guaranteeing their individual profitability. Unfortunately, existing designs can not meet all these requirements simultaneously. We address these requirements by proposing True-MCSA, a framework for truthful double multi-channel spectrum auctions. True-MCSA introduces novel virtual buyer group (VBG) splitting and bidding algorithms, and applies a proper winner determination and pricing mechanism to achieve truthfulness and other economic properties, meanwhile successfully dealing with multi-channel requests from both buyers and sellers and improving spectrum utilization. Our experiments show that the auction efficiency is impacted by the economic factors with efficiency degradations within 30%, under different settings. Furthermore, the experimental results indicate that we can improve the auction efficiency by choosing a proper bidding algorithm and using a positive base bid. True-MCSA makes an important contribution on enabling spectrum reuse to improve auction efficiency in multi-channel cases.

Index Terms—Spectrum auction, truthfulness, double auction, multi-channel.

I. INTRODUCTION

THE demand for radio spectrum use has been growing rapidly with the dramatic development of the mobile telecommunication industry in the last decades. However, the growth of wireless networks has been hampered by the previous inefficient spectrum distribution. In the past decade, the FCC (Federal Communications Commission) and its counterparts across the world have been using single-sided auctions to assign spectrum to wireless service providers in terms of predetermined national/regional long-term leases. This static allocation scheme has led to an artificial shortage of spectrum: new wireless applications starve for spectrum, while large chunks of spectrum remain idle most of the time under their current owners. This allocation inefficiency has prompted a wide interest in an open, market-based approach for redistributing the spectrum where new users can gain access to the spectrum they desperately need and existing owners can gain financial incentives to “lease” their idle spectrum. Additionally, as the development of multi-radio wireless networks, which is regarded as an enabling technology of next generation wireless network communications, the multi-channel requirement from one single user in the spectrum distribution becomes more and more popular.

Spectrum auctions are among the best-known market-based spectrum allocation mechanisms due to their perceived fairness and allocation efficiency: everyone has an equal opportunity to win and the spectrum channels are sold to bidders who value them the most. Unlike conventional FCC-style auctions that target long-term national/regional leases to service providers, the spectrum auctions we addressed in this paper allow sellers to lease their idle spectrum channels to buyers that can be small wireless networks, individual infrastructure networks or home networks, and provide a promising solution for efficient dynamic spectrum redistributions. There have been some researches [1][2][3][4] for the spectrum auctions targeting spectrum redistribution. However, none of them provides a truthful double multi-channel spectrum auction solution.

In this paper, we propose a framework for TRUthful douBlE Multi-Channel Spectrum Auctions (True-MCSA) where each seller or buyer requests arbitrary number of spectrum channels to sell or buy based on their individual needs. With True-MCSA, we coherently design the steps of buyer group formation, virtual buyer group (VBG) splitting and bidding, winner determination, and pricing to achieve truthfulness and other economic properties, to improve spectrum utilization and to successfully deal with multi-channel requests. True-MCSA provides a simple framework to address truthful double multi-channel spectrum auctions with spectrum reuse.

The paper makes the following key contributions.

1) We propose a framework, True-MCSA, for truthful double multi-channel spectrum auctions. In True-MCSA, we successfully deal with multi-channel requests from both sellers and buyers by introducing virtual buyer group (VBG) splitting and bidding together with applying a novel winner determination and pricing mechanism. True-MCSA provides an efficient and trust-worthy environment for spectrum sellers and buyers to trade arbitrary number of spectrum channels. Through the auctions, each seller sells all the channels it bid if winning, sells none if losing; each buyer buys at most the number of channels it bid if winning, buys nothing if losing.

2) We formally prove that True-MCSA is of three key...
economic-properties, namely individual rationality, ex-post budget balance and truthfulness for both sellers and buyers. These properties ensure that the auctions are economic-robust, the auctioneer have incentives to setup an auction, and the bidders have incentives to bid in the auction.

(3) We do extensive experiments to show the auction efficiency of our framework compared to that of the pure allocation (PA) algorithm, and the impacts of allocation algorithms, bidding algorithms, bidding patterns and buyer distributions on the auction efficiency.

The rest of the paper is organized as follows: Section II is the preliminaries. Section III describes the concept and design of True-MCSA, and Section IV is the proof of auction properties. In section V, we carry out extensive simulation experiments to evaluate the performance of our design. Section VI introduces the related works. Finally, Section VII is the conclusions and future work.

II. Preliminaries

In this section, we provide the problem model of double multi-channel spectrum auctions, and discuss our design goals to implement the auctions.

A. Problem Model

We consider a single-round double multi-channel spectrum auction with one auctioneer, \(M\) sellers, and \(N\) buyers. We assume that each seller contributes multiple channels and each buyer requests multiple channels. The auction is sealed-bid and private. Bidders submit their bids privately to the auctioneer without any knowledge of others.

For a seller \(m\), its bid is denoted by \((s_m, c_m)\) (\(s_m > 0\) and \(c_m \geq 1\)), meaning that \(m\) requires the minimum per-channel payment \(s_m\) to sell \(c_m\) channels; \(v^w_m\) and \(c^w_m\) are its true valuation of each channel and true number of channels provided; \(p^w_m\) is the per-channel payment received if it wins the auction; and its utility is \(u^w_m = c^w_m \cdot (p^w_m - v^w_m)\) if it wins \(c^w_m\) (\(1 \leq c^w_m \leq c_m\)) channels in the auction, and 0 otherwise. For a buyer \(n\), its bid is denoted by \((b_n, d_n)\) (\(b_n > 0\) and \(d_n \geq 1\)), which represents that the buyer is willing to pay the maximum price \(b_n\) for each channel, and it requires \(d_n\) channels; \(v^p_n\) and \(d^p_n\) are its true valuation of each channel and true number of channels requested; \(p^p_n\) is the per-channel price it pays if it wins the auction; and its utility is \(u^p_n = d^p_n \cdot (p^p_n - v^p_n)\) if it wins \(d^p_n\) (\(1 \leq d^p_n \leq d_n\)) channels in the auction, and 0 otherwise.

In the auctions, we assume that both sellers and buyers can bid the per-channel price untruthfully, while the buyers can also bid the number of channels requested untruthfully and the sellers always bid the true number of channels he can provide. The last assumption is based on the practical scenario in which the auctioneer normally asks the sellers to provide the spectrum channels and check them before auctions, which prevents a lie about the number of selling channels. We also assume that when \(d^w_n > d^p_n\), the utility of each extra channel for buyers is not more than zero.

Note that our problem model implies that all spectrum channels are homogenous and are available to all buyers. This may occur in practice when all spectrum channels have similar frequencies and quality, in the same authorized zones. For some other practical scenarios, channels may have to be modeled as being heterogeneous. Although, this work focus on truthful double multi-channel auctions in homogenous case, it is probable to be extended to the heterogeneous case. Intuitively, the heterogeneous case can be split into several homogenous subcases according to types of channels auctioned, one subcase for each type, without considering the interdependence among the subcases. However, how to deal with this interdependence to achieve truthfulness is challenging and still needs further studies and careful designs. So we leave it a future work.

B. Design Goals

Our first design goal is to exploit the spatial reusability of radio spectrum. Unlike conventional goods, spectrum is reusable among bidders subjecting to the spatial interference constraints: bidders in close proximity cannot use the same spectrum frequency simultaneously but well-separated bidders can. In the case of multi-channel spectrum auctions, different buyer request quite different number of spectrum channels, how to exploit and maximize the spatial reusability is challenging.

Our second design goal is to ensure economic-robustness of the auctions. Truthfulness, individual rationality and budget balance are the three critical properties required to design economic-robust double auctions [1][5][6]. Although TRUST proposed by [1] has well achieved all the three properties in one-channel spectrum auctions, how to achieve these economic properties in multi-channel spectrum auctions has not been addressed. In the multi-channel spectrum auctions, both sellers and buyers request different numbers of channels, which makes it challenging to design the auction process (i.e. determination of winners and prices) and to achieve economic robustness.

We now define the three economic properties in double multi-channel spectrum auctions:

1. Truthfulness. A double multi-channel spectrum auction is truthful if no matter how other players bid, no seller \(m\) or buyer \(n\) can improve its own utility by biding untruthfully (\(s_m \neq v^w_m\) for sellers and \(b_n \neq v^p_n\) or \(d_n \neq d^p_n\) or both for buyers).

Truthfulness is essential to avoid market manipulation and ensure auction fairness and efficiency. In untruthful auctions, selfish bidders can manipulate their bids to game the system to increase their utilities but decrease others’. In truthful auctions, the dominate strategy for bidders is to bid truthfully, eliminating the fear of market manipulation and the overhead of strategizing over others. With the true valuations, the auctioneer can allocate spectrum efficiently to buyers who value it the most.

2. Individual Rationality. A double multi-channel spectrum auction is individual rational if no winning seller is paid less than its bid and no winning buyer pays more than its bid:

\[
p^w_m \cdot c^w_m \geq s_m \cdot c^w_m, \quad p^p_n \cdot d^p_n \leq b_n \cdot d^p_n
\]

(1)

Here, we assume the pricing is uniform for each seller \(m\) and buyer \(n\), which is in accordance with our design.
This property guarantees non-negative utilities for bidders who bid truthfully, providing them incentives to participate.

3) **Ex-post Budget Balance.** A double multi-channel spectrum auction is ex-post budget balanced if the auctioneer’s profit $\Phi \geq 0$. The profit is defined as the difference between the revenue collected from buyers and the expense paid to sellers:

$$\Phi = \sum_{n=1}^{N} p_n^b \cdot d_n^w - \sum_{m=1}^{M} p_m^s \cdot c_m^w \geq 0$$  \hspace{1cm} (2)

This property ensures that the auctioneer has incentives to set up the auction.

### III. True-MCSA: Concept And Design

In this section, we first describe the concept of designing True-MCSA, then present our design in detail, finally, analyze the computational complexity.

#### A. Concept

The most challenging problem in designing True-MCSA is how to deal with the multi-channel requests of both buyers and sellers, while guaranteeing that the double spectrum auctions are truthful, and the reuse of spectrum is well exploited. We borrow ideas from McAfee’s design [7] and TRUST [1], and propose a novel auction framework that meets all the above requirements. Specifically, we form buyer groups independently on buyer bids to exploit the reuse of spectrum; design VBG splitting and VBG bidding algorithms to solve the problem of multi-channel bidding; bring forward a novel winner determination mechanism to ensure truthful double spectrum auctions.

1) **Bid-independent Buyer Group Formation:** We group multiple conflict-free buyers together so that each of them can be assigned the same channels. This spectrum allocation process can be dependent on bids, like VERITAS [2]. However, a bid-dependent allocation could allow bid manipulation and make auctions untruthful [1]. Therefore, we take the same policy as TRUST, and form buyer groups based on buyers’ interference conditions but independent of their bids. The buyer group formation initially exploits the special reuse of spectrum among buyers located in different places.

2) **Virtual Buyer Group (VBG) Splitting and Bidding:** After forming buyer groups, we can directly treat each buyer group as a super buyer like TRUST, for in multi-channel scenarios each buyer in the same group may request quite different number of channels. It is hard to determine the group bid and how many channels the buyer group should buy. In order to overcome this hardness, we propose the method of virtual buyer group (VBG) splitting and bidding. Our basic idea is that we further split each buyer group into several VBGs, in which each buyer merely requests one channel, and then regard each VBG as a super buyer to bid for one channel. Note that, in this way, a buyer is grouped into as many VBGs derived from its buyer group as the number of channels he requests, and VBGs derived from the same buyer group contains the buyers from the same buyer set, thus VBG bids could be dependent on each other. To achieve truthfulness, we have to properly deal with this dependence when bidding.

Through the VBG splitting and bidding, we convert the problem of multi-channel auctions similarly to that of single-channel auctions, and thus properly solve the multi-channel request problems. Furthermore, as we will see in Section III-B, the VBG splitting and bidding also answers the question of how many channels a buyer group should buy while maximizing the spectrum reuse and auction efficiency.

3) **Winner Determination and Pricing:** To avoid bid manipulation and ensure economic-robustness of spectrum auctions, winner determination and pricing should satisfy the following conditions: (1) quantity condition: each winning seller or buyer has a non-negative utility value, and the auctioneer has a non-negative profit value; (2) independence condition: the per-channel price charged from each winning buyer or paid to each winning seller is independent on the per-channel bid of each winning bidder; (3) uniform condition: the pricing mechanisms for both buyers and sellers should be uniform globally (among all buyers or sellers) or locally (among buyers in the same group).

The reasons are as follows. First, quantity condition is necessary for achieving the goal of individual rationality and ex-post budget balance. Second, if the independence condition were not satisfied, the winning buyers or sellers would be able to increase their utility by changing their bids and the auctions would not be truthful. Finally, if the uniform condition is not satisfied, such as charging each buyer in the same winning group proportionally to its bid, this could make the auctions become untruthful because selective buyers in the group can manipulate their bids to lower their shares in the group charge while still winning the auction. Then by lowering their bids, they improve their utilities, violating the truthfulness requirement.

In Section III-B, we provide a novel winner determination and pricing that meets the above conditions and properly deals with multi-channel scenarios.

#### B. Design

Now, we present the design of True-MCSA in details. During the presentation, the following example is used to illustrate the auction process.

**Example 1:** In an auction, we assume that the seller set $S$ and the buyer set $B$ with their bids are as follows and the conflict graph of buyers are shown as Fig. 1:

$S = \{S_1(11, 2), S_2(4, 2), S_3(5, 3), S_4(6, 2), S_5(3, 1)\}$

$B = \{B_1(10, 3), B_2(8, 5), B_3(5, 1), B_4(3, 2), B_5(11, 2), B_6(9, 4), B_7(5, 1)\}$

![Fig. 1. The conflict graph of the buyers.](image-url)
We will discuss how the example auction proceeds in each step of True-MCSA. True-MCSA consists of the following four steps.

**Step I: Buyer Group Formation**

We assume that all the sellers’ channels are available to all the buyers and use a conflict graph to describe the interference conditions among buyers. Buyers that do not interfere with each other are grouped into the same group and each of them can be assigned to the same channels. The group formation is performed privately by the auctioneer before the actual auction and kept confidential to buyers. Modeling the interference conditions as a conflict graph, the group formation is equivalent to finding the independent sets of the conflict graph [8][9]. It is noted that the group formation only forms buyer groups, but not assigns specific channels to buyers.

In our auctions, the buyer group formation algorithm forms buyer groups by finding independent sets repeatedly. To find an independent set, it recursively chooses a node in the current conflict graph to be included to the set, eliminates the chosen node and its neighbors, and updates the topology of the remaining nodes.

**Example 1(I):** In this step, by repeatedly choosing the node with minimum degree in the current conflict graph, eliminating the node and its neighbor nodes, we can form two buyer groups like: \(G_1 = \{B_1, B_2, B_3, B_4\}\) and \(G_2 = \{B_5, B_6, B_7\}\).

**Step II: VBG Splitting and Bidding**

Buyers in groups request to buy multiple channels. Recall that we denote the bid of buyer \(i\) by \((b_i, d_i)\), where \(b_i\) is the per-channel bid and \(d_i\) is the number of channels requested. Assuming that buyer \(i\) belongs to group \(G_n\), the maximal number of channels requested in the group is denoted by \(K_n = \max_{g \in G_n} d_i\). We split the buyer group \(G_n\) into \(K_n\) VBGs in which each buyer requests only one channel as follows:

- The 1\(^{st}\) VBG consists of the buyers in group \(G_n\) who request their 1\(^{st}\) channel;
- the 2\(^{nd}\) VBG consists of the buyers in group \(G_n\) who request their 2\(^{nd}\) channel;
- ...
- the \(K_n\)\(^{th}\) VBG consists of the buyers in group \(G_n\) who request their \(K_n\)\(^{th}\) channel.

Note that, here the “\(i^{th}\) channel” does not mean the channel from seller \(i\) or the channel labeled \(i\). Instead, it means “the \(i^{th}\) channel requested by some buyers in the buyer group (those request not less than \(i\) channels)”, which can be bought from any winning seller.

Fig. 2 illustrates the VBG splitting procedures. Buyer group \(G = \{1, 2, 3, 4\}\) is split into 5 VBGs according to the number of requested channels of each buyer. Since the VBGs with lower indexes are always the super sets of those with higher indexes and thus have higher bids, they have greater probabilities to win in the auction. In other words, buyers in the buyer group tend to win the first several channels while losing the last ones. The reason is that as the increase of the number of channels the buyer group wins, the spectrum reuse decreases in the group, and the auction should find a proper traded channel number for each buyer group by bid competition among VBGs derived from all the buyer groups, maximizing the total spectrum reuse and the auction efficiency.

From above, we can see that the VBG splitting equivalently transforms a buyer group to a number of VBGs, decomposing the complicated multi-channel cases to simple one-channel cases. Next, we get to design the VBG bidding algorithms.

It is worth to note that we cannot apply the bidding algorithm of TRUST directly. According to TRUST, the bid of a VBG is the value of its size multiplying its minimum buyer bid, with no buyer eliminated. However, the bids of the VBGs derived from a same buyer group could not be independent at all. Some winning buyer in the buyer group can manipulate its bid to increase its utility, as shown in Example 2.

**Example 2** Given 2 buyer groups \(G_1 = \{B_1(5,2), B_2(6,1)\}\) and \(G_2 = \{B_3(3,2), B_4(4,1)\}\), 4 sellers \(S_1(1,1), S_2(2,1), S_3(3,1)\), and \(S_4(4,1)\), we can split the buyer groups into 4 VBGs \(G_{11} = \{B_1(5), B_2(6)\}\), \(G_{12} = \{B_1(5)\}\), \(G_{21} = \{B_3(3), B_4(4)\}\) and \(G_{22} = \{B_3(3)\}\), which bid 10, 5, 6, 3 respectively. According to the winner determination of TRUST, the bid 5 of VBG \(G_{12}\) is the price that the winning VBGs (\(G_{11}, G_{21}\)) should pay. The utility of buyer \(B_1\) is \(5 - 5/2 = 2.5\). But if buyer \(B_1\) lowers its bid to 4 without changing the auction results, he will increase his utility to \(5 - 4/2 = 3\).

Therefore, we design a new bidding mechanism based on critical buyers. The notion of critical buyer is defined as follows.

**Definition:** A Critical Buyer is a buyer whose per-channel bid determines the per-channel price for the buyers in his buyer group if winning. Our main bidding idea is selecting a buyer from each buyer group as the critical buyer, eliminating buyers with smaller bids together with the critical buyer from all the VBGs derived from the buyer group, and letting each VBG bid the value of its size after eliminating multiplying the per-channel bid of the critical buyer.

From the definition of critical buyer and the bidding idea, we can see that the per-channel bid of a critical buyer determines the bid of each VBG derived from his buyer group, which is also each VBG’s payment if winning. The critical buyer provides a critical value (per-channel bid) for the buyers in the same buyer group to bid. If a buyer other than the critical buyer in a winning VBG bids not less than the critical value, he will wins in the VBG; otherwise, he will lose. As
we will see later, this property of the bidding is useful for achieving monotonic winner determination, which is necessary for achieving truthfulness.

According to the way of selecting critical buyers, we design two methods for VBG bidding, namely member-minimized bidding and group-maximized bidding. The two bidding methods meet different design goals.

Algorithm 1 VBG Splitting and MMIN Bidding

**Input:**

The buyer group $G_n$

**Output:**

The VBG set $G^n$ derived from the input buyer group

1. $b_{\min} = \min_{i \in G_n} b_i$
2. $i_{\min} = \arg \min_{i \in G_n} b_i$
3. $G'_n = G_n - \{i_{\min}\}$
4. $K_n = \max_{i \in G'_n} d_i$
5. $G_n = \emptyset$, $\pi = \emptyset$
6. for $i = 1 \rightarrow K_n$ do
7. \hspace{1em} $G^n_i = \emptyset // \text{ VBG}$
8. \hspace{1em} foreach $j \in G'_n$ do
9. \hspace{2em} if $d_j \geq i$ then
10. \hspace{3em} $G^n_i = G^n_i \cup \{j\}$
11. \hspace{1em} end if
12. \hspace{1em} end foreach
13. \hspace{1em} $\pi = \min \cdot |G^n_i|$
14. \hspace{1em} $G^n = G^n \cup \{G^n_i\}$
15. \hspace{1em} $\pi = \pi \cup \{\pi_i\}$
16. end for
17. return $G^n$, $\pi$

1) **Member-Minimized (MMIN) Bidding**

MMIN bidding targets to maximize the number of buyers selected in each buyer group to participate in the winner determination. In MMIN bidding, the critical buyer in buyer group $G_n$ is identified to be the buyer with minimal per-channel bid in the group (if more than one, one is randomly selected) and the per-channel bid is denoted by $b_{\min}$. The critical buyer is then eliminated from each VBG derived from $G_n$ if existing. Then, the bid of VBG $G^n_i$ is calculated by multiplying $b_{\min}$ to the number of its virtual buyers after eliminating.

$$\pi_i = b_{\min} \cdot |G^n_i|$$  \hspace{1em} (3)

Algorithm 1 shows the algorithm of VBG splitting and bidding using MMIN bidding. The algorithm outputs the set $G^n$ of VBGs derived from buyer group $G_n$ and its corresponding bid set $\pi^n$.

2) **Group-Maximized (GMAX) Bidding**

GMAX bidding aims to maximize the first VBG bid by selecting a proper critical buyer. The bid of the first VBG of buyer group $G_n$ is defined as

$$\pi_1 = \max_{i \in G^n_{i_2}, i \geq 2} b_i \cdot (i-1)$$  \hspace{1em} (4)

with $I_n = \arg \max_{i \in G^n_{i_2}, i \geq 2} b_i \cdot (i-1)$.

Where $G_n^{\pi}$ is obtained from the first VBG $G^n_1$ by sorting its buyers in non-increasing order in term of per-channel bid and $i$ is the buyer rank starting from 1. Then buyer $l_n$ in $G^n_1$ is the critical buyer. The buyers in buyer group $G_n$ with per-channel bids smaller than that of the critical buyer, together with the critical buyer itself are eliminated for their low bids, from each VBG derived from $G_n$. Doing this guarantees that the first VBG bids a maximized bid and the entire VBGs bid independently on the per-channel bid of each buyer left after eliminating. The bids of other VBGs of buyer group $G_n$ can be calculated by multiplying the number of their buyers to the per-channel bid of the critical buyer.

The MMIN and GMAX bidding methods seem similar to those proposed in TRUST [1] and TDSA [3], respectively. Indeed, we have got some inspirations from the two designs. However, our bidding methods are essentially different from the previous methods. In our methods, some buyers are eliminated in the bidding process to make the VBG bids independent on the remaining buyers. In the previous methods, no buyers are eliminated in the bidding process. Note that in our auctions the VBGs derived from each buyer group could bid dependently on each other if directly applying the previous methods. Example 2 shows that the direct application of TRUST leads to bid manipulation. An example can also be constructed to illustrate the similar result for TDSA.

Additionally, according to the bidding methods described above, it is obvious that all the buyer groups containing only one buyer will be eliminated from participation in the winner determination.

**Example 1(II):** In this step, we take MMIN bidding as an example. Firstly, the critical buyers in both $G_1$ and $G_2$ are identified to be $B_4$ and $B_7$, respectively. Then the two groups are split into VBGs according to the buyers’ requested numbers of channels and critical buyers $B_4$ and $B_7$ are eliminated from these VBGs. Finally, the bid of each VBG is calculated by multiplexing the size of selected set to the per-channel bid of its buyer group’s critical buyer, as illustrated in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Group</th>
<th>VBG</th>
<th>Member Set</th>
<th>Selected Set</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1(B_4)$</td>
<td>$G_{11}$</td>
<td>${B_{1}(10), B_{2}(8), B_{3}(5), B_{4}(3)}$</td>
<td>${B_{1}, B_{2}}$</td>
<td>9</td>
</tr>
<tr>
<td>$G_1(B_4)$</td>
<td>$G_{12}$</td>
<td>${B_{1}(10), B_{2}(8)}$</td>
<td>${B_{1}, B_{2}}$</td>
<td>6</td>
</tr>
<tr>
<td>$G_1(B_4)$</td>
<td>$G_{13}$</td>
<td>${B_{1}(10), B_{2}(8)}$</td>
<td>${B_{1}, B_{2}}$</td>
<td>6</td>
</tr>
<tr>
<td>$G_2(B_7)$</td>
<td>$G_{21}$</td>
<td>${B_{4}(11), B_{6}(9), B_{7}(5)}$</td>
<td>${B_{6}, B_{7}}$</td>
<td>10</td>
</tr>
<tr>
<td>$G_2(B_7)$</td>
<td>$G_{22}$</td>
<td>${B_{4}(11), B_{6}(9)}$</td>
<td>${B_{6}, B_{7}}$</td>
<td>10</td>
</tr>
<tr>
<td>$G_2(B_7)$</td>
<td>$G_{23}$</td>
<td>${B_{6}(9)}$</td>
<td>${B_{6}}$</td>
<td>5</td>
</tr>
<tr>
<td>$G_2(B_7)$</td>
<td>$G_{24}$</td>
<td>${B_{6}(9)}$</td>
<td>${B_{6}}$</td>
<td>5</td>
</tr>
</tbody>
</table>

In Step III: **Winner Determination**

In this step, we determine the winning VBGs, and thus the winning buyers together with the number of channels they buy. Recall that we assume the transmission channels are homogenous and a seller provides multiple channels bidding $(s_m, c_m)$. The buyers are grouped in VBGs to bid. Similar to McAfee’s design, first of all, the sellers’ per-channel bids $s_m$ are sorted in non-decreasing order and the buyer (namely VBG in this step) bids are sorted in non-increasing order in the winner determination:
Algorithm 2 Winner Determination

Input:

\[ S = \{s_1, s_2, \ldots, s_M\}, \text{ s.t. } s_1 \leq s_2 \leq \ldots \leq s_M \]
\[ C = \{c_1, c_2, \ldots, c_M\} / \text{ The set of requested number for } S \]
\[ \pi = \{\pi_1, \pi_2, \ldots, \pi_K\}, \text{ s.t. } \pi_1 \geq \pi_2 \geq \ldots \geq \pi_K \]

Output:

The last seller winner \( j \) and the last buyer winner \( k \)

1. \( L = \sum_{i=1}^{M} c_i \)
2. for \( i = 1 \rightarrow \min[L, K] \) do
3. \( j = 1 + \arg \max_{0 \leq i \leq M} \{ \sum_{t=1}^{i} c_t < i \} \)
4. \( \text{sum}_1 = \sum_{t=1}^{i} \pi_t \)
5. \( \text{sum}_2 = i \cdot s_j \)
6. if \( \text{sum}_1 < \text{sum}_2 \) then
7. break
8. end if
9. end for
10. \( i = i - 1 \) // last profitable trade
11. \( j = \arg \max_{0 \leq l \leq M-1} \{ \sum_{t=1}^{l} c_t < l \} \)
12. \( k = \sum_{t=1}^{j} c_t \)
13. return \( j, k \)

\( S : s_1 \leq s_2 \leq \ldots \leq s_M \)
\( \pi : \pi_1 \geq \pi_2 \geq \ldots \geq \pi_K \)

In the case of ties, the ordering is random, with each tied seller or buyer bidder having an equal probability of being ordered prior to the other ones. Note that, for simplicity of presentation, we “relabel” the bids in the sorted order (i.e. non-decreasing order for sellers and non-increasing order for VBGs), while keeping in mind that we arrange the orders of sellers and VBGs according to their bid values, but not their labels.

Then, we rewrite each seller’s per-channel bid \( s_m \) as many times as the number \( c_m \) of channels he bid, resulting the bid mapping between sellers and buyers (VBGs) as follows:

\( S : s_1 \leq s_2 \leq \ldots \leq s_M \)
\( \pi : \pi_1 \geq \pi_2 \geq \ldots \geq \pi_K \)

Let \( L \) denote the total number of channels provided by 

\[ L = \sum_{j=1}^{M} c_j \]  

(5)

\[ j(i) = 1 + \arg \max_{0 \leq i \leq M-1} \{ \sum_{t=1}^{i} c_t < i \} \]  

(6)

We define the last profitable trade \( k_l \) as:

\[ k_l = \arg \max_{i \leq \min(L, K)} \{ \sum_{t=1}^{i} \pi_t \geq i \cdot s_{j(k)} \} \]  

(7)

As a result, the last profitable seller is \( j(k_l) \). In order to achieve truthfulness, we sacrifice the last profitable seller to price the winning sellers, so all trades involving the last profitable seller have to be abandoned. Then the auction winners are the first \( (j(k_l) - 1) \) sellers in \( S \) and the first \( k = \sum_{t=1}^{j(k_l)-1} c_t \leq k_l - 1 \) VBGs in \( \pi \). The winning sellers lease channels to the winning VBGs on the basis of the bid mapping, one for each. The winner determination process is described in Algorithm 2.

It is worth to note that, according to the winner determination, the number of channels that each winning buyer buys is the number of winning VBGs to which he belongs to; the number of channels that each winning seller sells is always the number of channels he bids. It is impossible that a seller sells part of his channels. Because, according to the bid mapping, we sell all channels from one winning seller to winning buyers before we move on to those from the next seller, and sacrifice trades involving the last profitable seller for truthfulness purpose.

\textbf{Example 1(III)}: Table II shows the procedure of winner determination. According to the bid accumulation (BA), at the place of \((S_d, G_{14})\), Equation (7) is satisfied. Thus, we get the last profitable trade \( k_l=8 \), the last profitable seller \( j(k_l) = 4 \), the last winning seller \( (j(k_l) - 1) = 3 \), and the last winning VBG \( k = 1 + 2 + 3 = 6 \). So the number of winning VBGs is 6, and the number of winning sellers is 3. The shaded cells in the table indicate that the last two profitable trades are sacrificed for truthfulness. The results (WS: Winning Sellers, WVBG: Winning Virtual Buyer Groups, WB: Winning Buyers) of winner determination are summarized in the last three rows of Table II, where the winning channel number and the requesting channel number are separated by “,” e.g. \( B_2(8,3/5) \) means that \( B_2 \) bids a per-channel bid 8 and win 3 channels out of 5 requesting channels.

\textbf{Step IV: Pricing}

Each buyer in the same winning VBG is charged an even share of the VBG bid and each channel is paid by the price \( s_{j(k)} \). Then, each buyer is charged the sum of what it is charged in all the winning VBGs it belongs to and each seller is paid by the product of multiplying the number of channels he bid to the price \( s_{j(k)} \).

\textbf{Example 1(IV)}: Table III lists the calculation of utility for

\textbf{TABLE II

The Procedure of Winner Determination}

\begin{tabular}{|l|c|c|c|c|c|c|c|c|c|c|}
\hline
NO. & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
Seller & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \\
\hline
Bid & 3 & 4 & 4 & 5 & 5 & 6 & 6 & 11 & 11 & - \\
\hline
BA & 3 & 8 & 12 & 20 & 25 & 30 & 42 & 48 & 99 & 110 \\
\hline
VBG & G_{21} & G_{22} & G_{11} & G_{12} & G_{13} & G_{21} & G_{24} & G_{14} & G_{15} & - \\
\hline
Bid & 10 & 10 & 9 & 6 & 5 & 5 & 3 & 3 & - & - \\
\hline
BA & 10 & 20 & 29 & 35 & 41 & 46 & 51 & 54 & 57 & - \\
\hline
\end{tabular}

\textbf{TABLE III

The Utility Calculation of Each Seller and Buyer}

\begin{tabular}{|l|c|c|c|c|c|c|}
\hline
Seller & S_1 & S_2 & S_3 & - & - & - \\
\hline
Pay & 6 \times 1 = 6 & 6 \times 2 = 12 & 6 \times 3 = 18 & - & - & - \\
\hline
Util. & 6 - 3 = 3 & (6 - 4) \cdot 2 = 4 & (6 - 5) \cdot 3 = 3 & - & - & - \\
\hline
Buyer & B_1 & B_2 & B_3 & B_4 & B_5 & - \\
\hline
Charge & 3 \times 3 = 9 & 3 \times 3 = 9 & 3 \times 1 = 3 & 5 \times 2 = 10 & 5 \times 3 = 15 & - \\
\hline
Util. & (10 - 3) \cdot 3 = 21 & (8 - 3) \cdot 3 = 15 & 5 - 3 = 2 & (11 - 5) \cdot 2 = 12 & (9 - 5) \cdot 3 = 15 & - \\
\hline
\end{tabular}
each seller and buyer. It is obvious that the utility of each seller and buyer is positive, so the individual rationality is satisfied. From Table II, we can see that ex-post budget balance is also satisfied.

In the auction framework, there are some buyers and one seller sacrificed, which is indispensable for truthfulness reason. In step II, we have shown in Example 2 that direct application of TRUST after VBG splitting leads to bid manipulation. To prevent this, we have to let each VBG bid and be priced independently on its buyers. So we eliminate critical buyer and buyers with smaller bids from each VBG, let it bid and be priced based on the critical buyer. In step III, we have to abandon all trades involving the last profitable seller. Otherwise, we can not achieve truthfulness. For example, in Table II, if trade 7 is reserved, seller \( S_4 \) can lower his bid to 5.1 without changing the auction results, and raise his utility from 0 to 0.9, violating the truthfulness requirement.

C. Analysis of Computational Complexity

The computational complexity of True-MCSA mainly includes three parts as follows.

Buyer Group Formation: it examines each buyer for his conflict relation with other buyers and forms independent sets. So the worst computational complexity is \( O(N^3) \).

VBG Splitting and Bidding: it splits each buyer group into VBGs according to how many channels its buyers bid, and calculates the bid of each VBG. The worst computational complexity is \( O(ND) \), where \( D = \max_i d_i \).

Winner Determination: it sorts the sellers’ per-channel bids in non-decreasing order with complexity \( O(N \log M) \), sorts the VBG bids in non-increasing order with complexity \( O(ND \log (ND)) \).

As a result, the computational complexity of True-MCSA is \( O(N^2 + M \log M + ND \log (ND)) \), which is determined by \( N, M, \) and \( D \). In practice, if \( D \) becomes very large, the sort of VBG bids can be optimized by sorting only the largest \( L \) ones, so the complexity becomes \( O(N^2 + M \log M + L \log L) \), where \( L \) is the total number of channels auctioned and \( L = \sum_{i=1}^{M} c_i \).

IV. PROOF OF AUCTION PROPERTIES

In this section, we prove that True-MCSA satisfies the properties of ex-post budget balance, individual rationality and truthfulness for both sellers and buyers. We only prove the case when using MMIN VBG bidding, while the proof of the case when using GMAX VBG bidding is similar and we omit it for limitation of space.

1) Proof of Ex-post Budget Balance

**Theorem 1**: True-MCSA is ex-post budget balanced i.e. \( \Phi \geq 0 \).

**Proof**: Because \( k \) is the number of winning VBGs and the number of the channels traded in the auctions, and \( k \) satisfies \( \sum_{i=1}^{k} \pi_i \geq k \cdot s_{\text{sum}} \), thus \( \Phi = \sum_{i=1}^{k} \pi_i - k \cdot s_{\text{sum}} \geq 0 \).

2) Proof of Individual Rationality

**Theorem 2**: True-MCSA is individual rational.

**Proof**: By the definition of individual rationality, we need to show that no winning seller will be paid less than its bid, and no winning buyer will be charged more than its bid.

First, because True-MCSA sorts seller’s per-channel bids in a non-decreasing order and pays each winning seller \( m \) with last profitable seller \( j(k) \)'s per-channel bid, the payment to \( m \) is \( p_m^b \cdot c_m^w s_{j(k)}^w \cdot c_{j(k)}^w \geq c_{m'}^w \cdot c_{m'}^w \), where \( c_{m'}^w \) is the number of channels that seller \( m \) manages to sell and it always satisfies \( c_{m'}^w \geq c_m^w \) in True-MCSA. Second, for each winning buyer \( n \), the per-channel price charged to \( n \) is \( p_n^b \cdot d_n^w \leq b_n \cdot d_n^w \), where \( d_n^w \) is the number of channels that buyer \( n \) wins.

3) Proof of Truthfulness

To prove True-MCSA’s truthfulness, we need to show that for any buyer \( n \) or seller \( m \), it can not improve its utility by bidding other than its true valuation. For this, we first show that its winner determination is monotonic for both sellers and buyers and the pricing is bid-independent. Using these two claims, we then prove the truthfulness.

1) Monotonic winner determination

The following two lemmas summarize the monotonicity of True-MCSA’s winner determination.

**Lemma 1**: Given \( \{(s_m, c_m)\}_{m=1}^{M} \) and \( \{(b_1, d_1), ..., (b_{n-1}, d_{n-1}), (b_n, d_n)\} \), if buyer \( n \) wins \( d_n^w(0 < d_n^w < d_n) \) channels by bidding \((b_n, d_n)\), then, by bidding \((b', d_n')\) with \( b' \geq b_n \) and \( d_n' \geq d_n \), buyer \( n \) also wins the same number of channels.

**Proof**: Since buyer \( n \) wins \( d_n^w \) channels by bidding \((b_n, d_n)\), it is not eliminated from its buyer group and \( d_n^w \) of its VBGs win the auction. When bidding \((b', d_n')\) with \( b' \geq b_n \) and \( d_n' \geq d_n \), buyer \( n \) is still not eliminated from its buyer group and the bids of its first \( d_n^w \) VBGs remain the same as before, while the bids of its last \( (d_n' - d_n) \) VBGs must be no greater than that of its \( d_n^w \) VBG. So, it must be that the same \( d_n^w(0 < d_n^w < d_n) \) of its VBGs win the auction. Lemma 1 holds.

**Lemma 2**: Given \( \{(bs_m, c_m)\}_{m=1}^{M} \) and \( \{(s_1, c_1), ..., (s_{m-1}, c_{m-1}), (s_m, c_m)\} \), if seller \( m \) wins the auction by bidding \((s_m, c_m)\), then, by bidding \((s_m', c_m)\) with \( s_m' < s_m \), seller \( m \) also wins the auction.

**Proof**: Since sellers are ranked in non-decreasing order in term of per-channel bid, seller \( m \) wins the auction by bidding \((s_m, c_m)\), it must also win by bidding \((s_m, c_m)\) with \( s_m' < s_m \), seller \( m \) also wins the auction.

2) Bid-independent pricing

We show that the pricing is bid-independent for both winning buyers and sellers.

**Lemma 3**: Given \( \{(s_m, c_m)\}_{m=1}^{M} \) and \( \{(b_1, d_1), ..., (b_{n-1}, d_{n-1}), (b_n, d_n)\} \), if buyer \( n \) wins the same number \( d_n^w \) of channels by bidding \((b_n, d_n)\) and \((b_n', d_n')\), then the utility \( u_n^b \) for \( n \) is the same for both.

**Proof**: It is easy to show that the bids of all the winning VBGs of buyer \( n \) remain the same in both cases, then the utility \( u_n^b \) for \( n \) is the same for both.

**Lemma 4**: Given \( \{(bs_m, c_m)\}_{m=1}^{M} \) and \( \{(s_1, c_1), ..., (s_{m-1}, c_{m-1}), (s_m, c_m)\} \), if seller \( m \) wins the auction by bidding \((s_m, c_m)\) and \((s_m', c_m)\), then the payment \( p_m^b \) to \( m \) is the same for both.

**Proof**: The proof is similar to that of Lemma 3.

3) True-MCSA’s truthfulness

Using the above claims, we now prove the main results on True-MCSA’s truthfulness.

**Theorem 3**: True-MCSA is truthful for buyers.
TABLE IV
FOUR POSSIBLE AUCTION RESULTS WHEN BIDDING TRUTHFULLY AND UNTRUTHFULLY,
WHERE X MEANS THE BIDDER LOSES AND √ MEANS IT WINS

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The bidder lies</td>
<td>X</td>
<td>X</td>
<td></td>
<td>√</td>
</tr>
<tr>
<td>The bidder bids truthfully</td>
<td>X</td>
<td>√</td>
<td>X</td>
<td>√</td>
</tr>
</tbody>
</table>

Proof: We need to show that any buyer $n$ cannot obtain higher utility by bidding $(b_n', d_n')$, where $b_n' \neq v_n'$ or $d_n' \neq d_n'$ or both. Table IV lists the four possible auction results for one buyer when it bids truthfully and untruthfully. We now examine these cases one by one.

CASE 1: For both bids, buyer $n$ is denied and charged with zero, leading to the same utility of zero.

CASE 2: This happens only if $b_n' < v_n'$. Theorem 2 ensures a non-negative utility when $n$ bids truthfully and wins the auction. Thus, its utility is no less than that when $n$ bids untruthfully and loses (zero utility).

CASE 3: This happens only if $b_n' > v_n'$ while the number of channels bid can be $d_n' = d_n'_{o}$ or $d_n' = d_n'_{u}$. Let $\pi_1, \ldots, \pi_{q}$ and $\pi_{q}'$, $\ldots, \pi_{q}'$ represent the bids of the $q (0 < q \leq d_n')$ VBGs, whose auction results are changed from losing (i.e., $n$ wins none of the channels) to winning (i.e., $n$ wins $q$ channels), when $n$ bids truthfully and untruthfully. Then this case can be divided into the following two further cases.

(A) Case $d_n' = d_n'_{o}$

In this case, we have $0 < q \leq d_n'$. Because $n$ changes the auction results of these VBGs from losing to winning by bidding higher than $v_n'$, $n$ must be eliminated from its VBGs (no matter its VBGs lose or win the auctions) when he bids truthfully, i.e., $\pi_i = n \cdot v_i'$, $n_i$ is the size of each VBG, for $i = 1, 2, \ldots, q$. Conversely, when $n$ bids untruthfully, it is easy to show that the untruthful bids for its VBGs, $\pi_i'$, must satisfy the following condition: $\pi_i \cdot b_n' \geq \pi_i' \geq \pi_i \cdot v_n'$. Therefore, the utility when $n$ bids $(b_n', d_n')$ is $\sum_{i=1}^{q} (v_n' - \pi_i'/n_i) \leq 0$, which is not more than that when $n$ bids truthfully (0). Theorem 3 holds.

(B) Case $d_n' \neq d_n'_{o}$

This case can only affect the value of $q$. When $d_n' < d_n'_{o}$, it is the same as the case of (A) except that $q$ may become smaller, so the utility when $n$ bids $(b_n', d_n')$ is not more than that when he bids truthfully. When $d_n' > d_n'_{o}$, $q$ may become greater and if $0 < q \leq d_n'$, the result is the same as case (A), and if $q > d_n'$, i.e., $q = d_n' + q'$ ($q' > 0$), the utility when $n$ bids $(b_n', d_n')$ is $\sum_{i=1}^{d_n'_{o}} (v_n' - \pi_i'/n_i) + u_{n'_{u}, q'}(q') \leq 0$, where $u_{n'_{u}, q'}(q')$ is the utility of the extra $q'$ channels which is not greater than 0 according to our problem assumption. So Theorem 3 holds.

CASE 4: The following two further cases must be examined.

(A) Case $d_n' = d_n'_{u}$

According to Lemmas 1 and 3, it is easy to show that buyer $n$ wins the same number of channels and is charged by the same price in both cases, leading to the same utility.

(B) Case $d_n' \neq d_n'_{u}$

This case can only affect the winning number of channels. When $d_n' < d_n'_{u}$, winning number may become smaller, so the utility when $n$ bids $(b_n', d_n')$ is not more than that when he bids truthfully. When $d_n' > d_n'_{u}$, if $0 < d_n' < d_n'_{o}(d_n'_{u})$ is the winning number when $n$ bids truthfully), the winning number of channels must remain the same when $n$ lies according to Theorem 3, so the result is the same as case (A); and if $d_n' = d_n'_{o}$, the winning number of channels when $n$ lies according to Lemmas 1, so the result is the same as case (A); and if $d_n' > d_n'_{o}$, the utility of channels when $n$ lies according to Lemmas 1, so the result is the same as case (A); and if $d_n' = d_n'_{u}$, the winning number of channels when $n$ lies according to Lemmas 1, so the result is the same as case (A); and if $d_n' > d_n'_{u}$, $\Delta u_{n'_{u}, q'}(\Delta d)$ is not greater than that when $n$ bids truthfully, where $u_{n'_{u}, q'}(d)$ is the utility when $n$ wins $d$ channels and $u_{n'_{u}, q'}(\Delta d)$ is the utility of the $\Delta d$ extra winning channels. Theorem 3 holds.

From the above, we show that no buyer can improve its utility by bidding untruthfully. Our proof is completed. □

Theorem 4: True-MCSA is truthful for sellers.

Proof: Similarly, we need to show that any seller $m$ cannot obtain higher utility by bidding $(s_m', c_m')$, where $s_m' \neq v_m'$. Again, the four cases listed in Table IV are examined.

CASE 1: The same as the buyer case. Theorem 4 holds.

CASE 2: This happens only when $s_m' > v_m'$. The utility of the seller $m$ is non-negative when bids truthfully and wins (Theorem 2), while it is zero when $m$ wins and loses. Theorem 4 holds.

CASE 3: This happens only when $s_m' < v_m'$. First, let $p$ be the per-channel payment to the auction winners when $m$ bids truthfully. Because $m$ loses in this case, $p \leq v_m'$. Second, let $p'$ be the per-channel payment to the winners (including $m$) when $m$ bids $(s_m', c_m')$. It is easy to show that because $m$ lowers its bid and wins, $p' \leq p$. Combine the two, we have $p' \leq v_m'$ and hence $m$'s per-channel utility is $p' - v_m' \leq 0$ when bidding untruthfully. Thus, no matter the value of $c_m'$ is, the utility when $m$ lies is not greater than that when it bids truthfully. Theorem 4 holds.

CASE 4: According to Lemmas 2 and 4, seller $m$ wins the same number of channels and the payment for it does not change, leading to the same utility in both cases. Theorem 4 holds.

Having shown that no seller can improve its utility by bidding other than its true value, our proof completes. □

V. EXPERIMENTAL RESULTS

In this section, we use network simulations to evaluate the performance of True-MCSA. We study the auction efficiency of our framework and the impacts of allocation algorithms, bidding algorithms, bidding patterns and buyer distributions on the efficiency.

A. Simulation Setup

We study the performance of True-MCSA under different settings. The key factors that affect True-MCSA’s performance are the underlining spectrum allocation algorithms, the bidding algorithms, bidding patterns, and buyer distributions. We assume that the buyer interference conditions are modeled by a conflict graph and all buyers are distributed in an area of $100 \times 100$. All the results are averaged over 500 rounds.

1) Allocation algorithms

In our auction design, the allocation algorithms form buyer groups by finding independent sets repeatedly. We study two ways of selecting a node during finding an independent set.

Greedy Allocation: To form a group, it recursively chooses a buyer with the minimal degree in the current conflict graph,
eliminates the chosen node and its neighbors, and updates the degree values of the remaining buyers.

**Random Allocation:** To form a group, it recursively chooses a buyer randomly in the current conflict graph, eliminates the chosen node and its neighbors, and updates graph of the remaining buyers.

2) Bidding algorithms

In Section III-B, we have proposed two VBG bidding algorithms: MMIN bidding and GMAX bidding. In our experiments, we compare the impacts of the two algorithms on auction efficiency in different settings.

3) Bidding Patterns

We assume that buyers’ bids are randomly distributed over (0, 1] and those of sellers over (0, 2]. Also, we assume that auctions can have a base bid value \( b_0 \) and each bid is then defined by \( b = b_0 + \alpha \cdot (b_{\text{max}} - b_0) \), where \( \alpha \) is a random number uniformly distributed over (0, 1], and \( b_{\text{max}} = 1 \) for buyers and \( b_{\text{max}} = 2 \) for sellers. Then, we assume that the channel numbers requested by buyers are randomly distributed over \([1..b_{\text{max}}]\) and those of sellers over \([1..c_{\text{max}}]\), where \([1..X]\) represents the integer number set from 1 to \( X \). We use a triple \((c_{\text{max}}, d_{\text{max}}, b_{\text{0}})\) to represent a bidding pattern for the auctions and examine the impacts of different bidding patterns on the auction efficiency.

4) Buyer Distributions

The auction efficiency depends on the interference conditions among buyers. We model the interference conditions using a conflict graph, and apply a distance-based criterion to determine whether two buyers conflict with each other. In this case, the interference conditions depend mainly on the buyer distribution. We consider two types of distributions:

**Random Distribution:** We randomly distribute a set of buyers in a given area, with a variety of conflict degrees.

**Clustered Distribution:** We randomly place some buyers in a given area and gradually add buyers in some small center areas, creating some hotspots.

The performance metrics are auction efficiency (or social welfare, \( \alpha \)), the number of channels traded \((N_i)\), the per-channel spectrum efficiency \((\beta)\), and the efficiency degradation \((\eta)\) over pure allocation (PA). Here, auction efficiency is defined as the bid-weighted sum of all the channels won by buyers minus that of all the channels won by sellers as Equation (8). Auction efficiency is in fact the value created by the auction and shared by sellers, buyers and the auctioneer, and thus has another name called social welfare. Since it reflects not only the spectrum reuse but also the financial efficiency of the auctions, we use it as the main performance metric instead of spectrum utilization. What is more, according to [1], spectrum utilization and auction efficiency reflect coarsely the same conclusions, thus we omit the spectrum utilization results. The other metrics are defined as Equations from (9) to (11).

\[
\alpha = \sum_{n=1}^{N} b_n \cdot d_n^w - \sum_{m=1}^{M} s_m \cdot c_m^w \tag{8}
\]

\[
N_i = \sum_{m=1}^{M} c_m^w \tag{9}
\]

\[
\beta = \alpha / N_i \tag{10}
\]

\[
\eta = 1 - \alpha / \alpha_{\text{PA}} \tag{11}
\]

Where \( d_n^w (0 \leq d_n^w \leq d_0) \) and \( c_m^w (0 \leq c_m^w \leq c_m) \) are the numbers of winning channels of buyer \( n \) and seller \( m \), respectively, and \( d_n^w = 0 \) or \( c_m^w = 0 \) means buyer \( n \) or seller \( m \) loses; \( \alpha_{\text{PA}} \) is the auction efficiency of PA.

By default, the experimental settings are as follows: True-MCSA uses Random allocation algorithm, MMIN bidding algorithm, random buyer distribution, bidding pattern(3, 5, 0), and the numbers of sellers and buyers are 10 and 100, the protection distance of buyers is 10.

B. Auction Efficiency

We start from studying the auction efficiency of True-MCSA in different settings. We use PA as a benchmark and change the number of sellers, the number of buyers, and the protection distance of buyers (i.e. the maximum radius of buyers’ interference range) to evaluate the four performance metrics mentioned above. PA repeatedly selects the unassigned VBG of the biggest size and assigns it a channel regardless of the buyers’ bids. For a fair comparison, we implement PA using the same allocation algorithm assuming the number of channels available is equal to the number of channels traded in True-MCSA.

Fig. 3 shows that as the increasing of the number of sellers (other factors are fixed as default), both the auction efficiency and the number of channels traded increase while the per-channel efficiency and efficiency degradation decrease. Since increasing the number of sellers means raising the supplies, we can conclude that raising the supplies increases the auction efficiency of True-MCSA though decreases the per-channel efficiency, and diminishes the efficiency degradation caused by achieving economic-robustness. Furthermore, the efficiency degradation values are within 30% when the number of sellers varies from 10 to 100.

Fig. 4 illustrates that as the increasing of the number of buyers, the entire performance metrics trend to increase while the degradation over PA has a small-scale fluctuation in its curve. This indicates that raising the demands increases both the per-channel efficiency and number of channels traded, and thus the auction efficiency, but the efficiency degradation caused by achieving economic-robustness is also increase. Still, the degradation values are within 30% when the number of buyers varies from 20 to 300.

In Fig. 5, we can see that as the protection distance of buyers increase, the per-channel efficiency and number of channels traded decrease, and thus the auction efficiency decreases, but the efficiency degradation over PA fluctuates within 30%. Therefore, heavy interferences among buyers severely affect the auction efficiency of both True-MCSA and PA.

C. Impact of Allocation Algorithms

We compare the performances of two allocation algorithms, Greedy and Random. Table V shows that the performance differences between the two algorithms are small. Overall, the Greedy algorithm performances slightly better than the Random algorithm, but it suffers slightly greater efficiency degradation over the PA.
D. Impact of Bidding Algorithms

In this part, we compare the performances of the two VBG bidding algorithms: MMIN bidding and GMAX bidding.

In Fig. 6, as the increasing of the number of sellers, the per-channel efficiency of MMIN bidding is still greater than that of GMAX bidding, and the traded channel number of MMIN bidding exceeds that of GMAX bidding when the number of sellers is more than 80. As a total effect, the auction efficiency of MMIN bidding exceeds that of GMAX bidding when the number of sellers is more than 30. Furthermore, the efficiency degradation of MMIN bidding decreases more rapidly than that of GMAX bidding and the degradation values of the former are still much smaller than that of the latter. Therefore, we can conclude that MMIN bidding algorithm is more suitable for auctions with more supplies than GMAX bidding algorithm.

In Fig. 7, it is shown that as the increasing of the number of buyers, the per-channel efficiencies of both MMIN bidding and GMAX bidding increase at the same time, while the former is still greater than the latter; the traded channel numbers of both MMIN bidding and GMAX bidding also increase simultaneously, but the latter is still far greater than the former. As a total effect, the auction efficiency of GMAX bidding is still greater than that of MMIN bidding. Additionally, the efficiency degradation of GMAX bidding becomes smaller than that of MMIN bidding when the number of buyers is greater than 150. Thus, the conclusion is that GMAX bidding algorithm is more suitable for auctions with more demands than MMIN bidding algorithm.

Fig. 8 shows that as the protection distance increases, the auction efficiency, the per-channel efficiency, and the number of channels traded of both MMIN bidding and GMAX bidding decrease similarly, while the efficiency degradations of both fluctuate similarly, too. Furthermore, all the performance metrics converge to one point as the protection distance is 50. So it can be concluded that the impacts of protection distance for both two bidding algorithms are analogy.

E. Impact of Bidding Patterns

We also study the impact of different bidding patterns \( (c_{\text{max}}, d_{\text{max}}, b_0) \). Table VI lists the performance metrics when applying different bidding patterns. From the table, we can see that: (1) a positive base bid value like \( b_0 = 0.1 \) can greatly improve all the performance metrics; (2) increasing the value of \( c_{\text{max}} \) leads to decreasing of per-channel efficiency but increasing of traded channel number, and thus improving
the auction efficiency while slightly decreasing the efficiency degradation; (3) increasing the value of $d_{\text{max}}$ leads to increasing of both the per-channel efficiency and the traded channel number, and thus improving the auction efficiency while slightly affecting the efficiency degradation.

These results can be explained as follows. For result (1), the value of $b_0$ determines the lowest bid of buyers, so raising $b_0$ reduces the effect of the lowest bid on the group bid, and makes the group size gradually become the dominate factor. As a result, the performance of True-MCSA improves and converges to that of PA. But normally, when raising $b_0$, the number of buyers satisfying the bidding pattern will falls. So, the auctioneer should trade off the above two sides, and properly design the base bid (also referred to as preserved price). For results (2) and (3), raising either $c_{\text{max}}$ or $d_{\text{max}}$ means raising either market supply or demand, so the change of performance metrics is similar to that of Figs. 3 and 4.

Auctions have been widely used to allocate spectrum [10], including transmit power auctions [11], spectrum band auctions [12][13][14][15], and spectrum pricing [16][17][18][19]. However, these schemes did not consider truthfulness. Paper [2] proposed the first truthful spectrum auction design VERTAS, but only addressed single-sided multi-channel auctions and a direct extension of VERTAS to double auctions is untruthful [1]. For double spectrum auctions, paper [20] proposed a hierarchical design based on McAfee’s design without spectrum reuse. Paper [1] proposed TRUST, the first truthful double auction design with spectrum reuse for multi-party spectrum trading, but TRUST only dealt with one-channel requirements for both sellers and buyers. Paper [3] improved TRUST by redesigning the group bidding and winner determining mechanisms and still only dealt with one-channel requirements as TRUST. Although paper [4] brought forward a simple illustration for solving buyers’ two-channel

F. Impact of Buyer Distribution

We randomly place 60 buyers in a given area of size 100×100 and gradually add 20 buyers in two randomly selected small center areas of size 20×20, creating 2 hotspots. Table VII shows that the performance metrics suffer declines in different degrees when using cluster distribution.
problems worth exploring in the future.

7. Concluding Remarks and Future Work

We have proposed True-MCSA, a framework for truthful double multi-channel spectrum auctions to support multi-party spectrum trading. True-MCSA achieves truthfulness, individual rationality and ex-post budget balance, the three key economic properties required for economic-robust auctions. Furthermore, True-MCSA successfully deals with the multi-channel requests from both sellers and buyers, while enabling spectrum reuse to significantly improve auction efficiency. From the design and evaluation of True-MCSA, we can see that the auction efficiency is impacted by economic factors leading to certain efficiency degradations. However, the experimental results suggest that we can improve the auction efficiency by choosing a proper bidding algorithm and using a base bid. True-MCSA makes an important contribution on enabling spectrum reuse to improve auction efficiency in multi-channel cases.

In this paper, we have assumed that all the spectrum channels are homogeneous and designed True-MCSA to achieve multi-channel spectrum auctions. Through the auctions, sellers sell all the channels they provide when winning but sell nothing when losing, buyers buy at most all the channels they require when winning while buy nothing when losing. However, there are other request formats [2], e.g. requesting continuous channels, requesting restrict number of channels, which should be addressed in future work. Finally, how to resist collusion and what is the tradeoff between auction efficiency and collusion-resisting economic robustness in double truthful multi-channel spectrum auctions are also interesting problems worth exploring in the future.

### Table VII

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<tbody>
<tr>
<td>Uniform</td>
<td>40.2154</td>
<td>4.2328</td>
<td>9.3010</td>
<td>0.2743</td>
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<tr>
<td>Cluster</td>
<td>27.6391</td>
<td>5.0223</td>
<td>9.1450</td>
<td>0.3761</td>
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</table>

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