Lecture 10
Magnetization Dynamics

Introduction: LLG equation and various torques

Landau-Lifshitz-Gilbert, spin transfer and spin-orbit torques

Adiabatic theory of magnetization dynamics

Berry curvature in magnetization space, Faraday torque

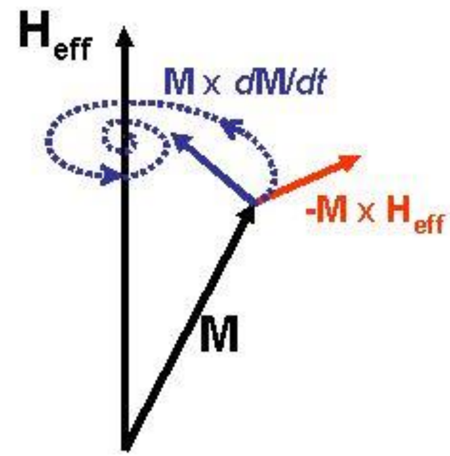
Electronic effects on magnetization dynamics

magnet coupled with Dirac electrons

Traditional form of magnetization dynamics

Laudan-Lifshitz-Gilbert equation

$$\frac{d\boldsymbol{M}}{dt} = -\gamma(\boldsymbol{M} \times \boldsymbol{H}_{\text{eff}} - \eta \boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt})$$



g gyromagnetic ratio

$\boldsymbol{H}_{\text{eff}}$ effective fields

h Gilbert damping

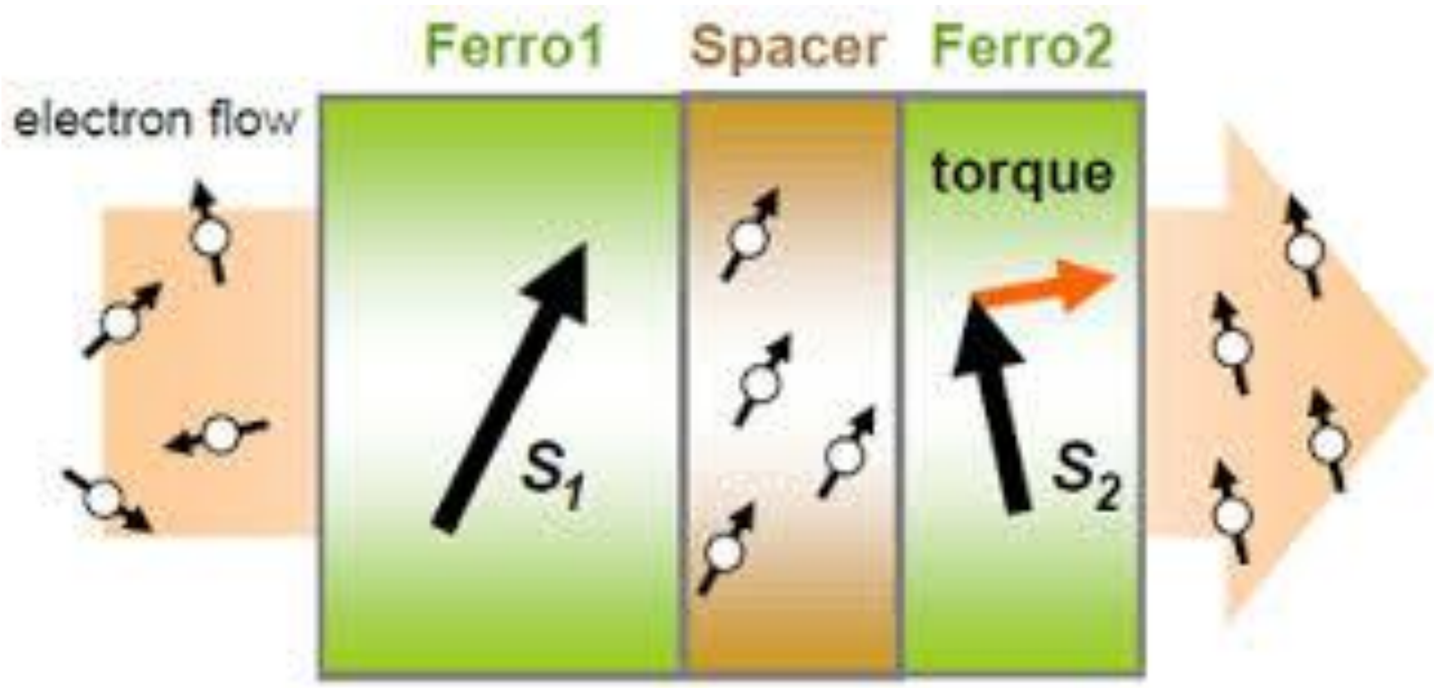
additional torques from charge & spin currents

$$\frac{d\boldsymbol{M}}{dt} = -\gamma(\boldsymbol{M} \times \boldsymbol{H}_{\text{eff}} - \eta \boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}) - \gamma(\boldsymbol{\tau}_{stt} + \boldsymbol{\tau}_{sot} + ...)$$

Spin transfer torque, Spin orbital torque

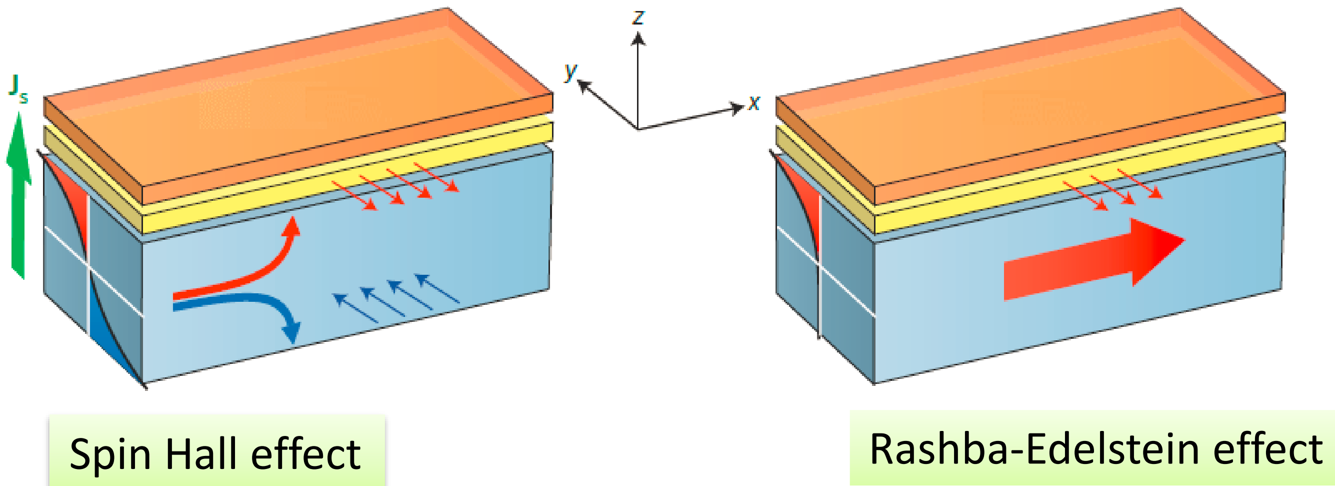
field-like torque, antidamping-like torque

spin transfer torque



spin orbital torque

spin accumulation due of itinerant electrons



A. Manchon, Nature Physics 10, 340 (2014)

SHE & MTJ: spin-polarized current => spin accumulation

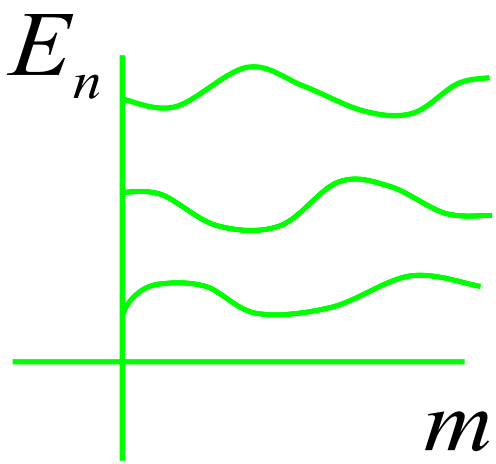
Rashba-Edelstein: inversion symmetric broken + spin orbital coupling

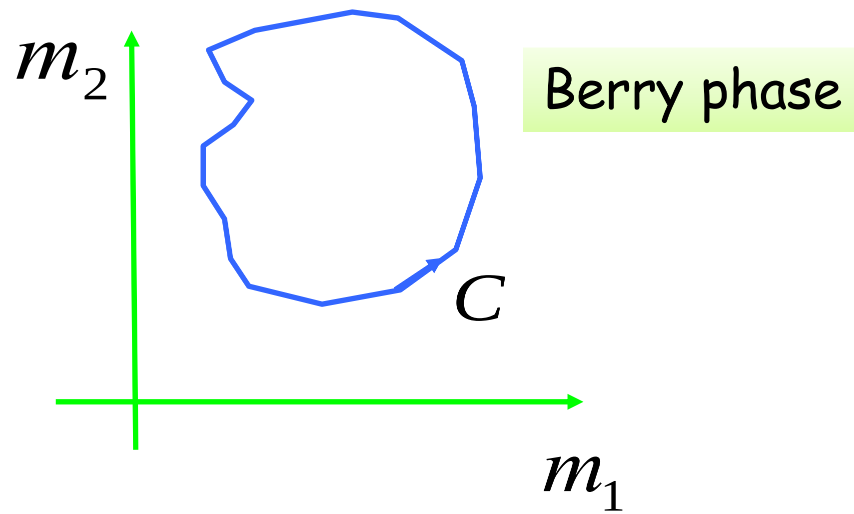
$$\delta \boldsymbol{s}_i = \chi_{ij} E_j$$

Miron et al., Nature 476, 189 (2011)
L. Liu et al. Science 336, 555 (2012).

Berry curvature in magnetization space

magnetization-dependent electron state





Berry curvature:

$$W_{m_1 m_2} = i\frac{\partial}{\partial m_1}\langle y|\frac{\partial}{\partial m_2}|y\rangle - i\frac{\partial}{\partial m_2}\langle y|\frac{\partial}{\partial m_1}|y\rangle$$

antisymmetric tensor or pseudovector in magnetization space

Berry curvature in magnetization dynamics

Magnetization dynamics in the adiabatic approximation:

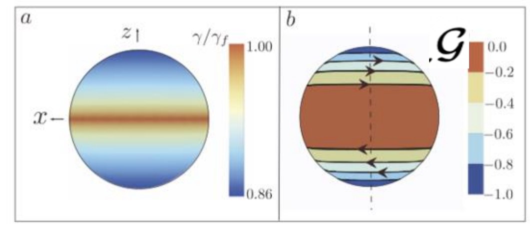
$$-\hbar\Omega_{ij}\dot{m}_j + \frac{\partial E_0}{\partial m_i} = 0$$

Niu & Kleinman, PRL 80,2205 (1998)

Niu et al., PRL 83,207 (1999)

For a uniform magnetization:

$$-\partial_{\boldsymbol{m}}\mathcal{G} + \dot{\boldsymbol{m}} \times \boldsymbol{\Omega}_{\boldsymbol{m}} = 0$$



Comparison with Landau-Lifshitz equation:

$$\dot{m} = -\gamma m \times H_{eff} \longrightarrow \boldsymbol{\Omega}_{\boldsymbol{m}} = \frac{\boldsymbol{m}}{\gamma m^2}$$

Consideration of electron dynamics:

$$\Omega_{mt} - \partial_{\boldsymbol{m}}\mathcal{G} + \dot{\boldsymbol{m}} \times \boldsymbol{\Omega}_{\boldsymbol{m}} - \eta\dot{\boldsymbol{m}} = 0$$

Faraday Conservative Lorentz damping and gain

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PRB 2018, 2021

Magnetization coupling to Bloch electrons

- **Electronic Hamiltonian:**

$$\hat{H} = \hat{H}_e(\boldsymbol{q} + e\boldsymbol{A}; \boldsymbol{m}) - e\phi.$$

-a general magnetization-dependent electronic Hamiltonian

-m assumed to be uniform in space

- **Semiclassical-adiabatic dynamics:**

-Electron dynamics

$$\begin{aligned} \dot{\boldsymbol{k}} &= -e\boldsymbol{E}, \\ \dot{\boldsymbol{x}} &= \frac{\partial \varepsilon}{\partial \boldsymbol{k}} + \dot{\boldsymbol{k}} \cdot \Omega_{\boldsymbol{kk}} + \dot{\boldsymbol{m}} \cdot \Omega_{\boldsymbol{mk}}. \end{aligned}$$

-Magnetization dynamics:

Lorentz — Faraday - $\boldsymbol{\Omega}_{mt}$ — Conservative

$$\int [d\boldsymbol{k}] f \left[\dot{\boldsymbol{m}} \cdot \Omega_{\boldsymbol{mm}} + \dot{\boldsymbol{k}} \cdot \Omega_{\boldsymbol{km}} + \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \right] = 0.$$

- **Non-equilibrium electrons** $\delta f = -\tau \frac{\partial f_0}{\partial \varepsilon} \left(\dot{\boldsymbol{k}} \cdot \frac{\partial \varepsilon}{\partial \boldsymbol{k}} + \dot{\boldsymbol{m}} \cdot \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \right)$

Damping and gain

Equation of motion at zero electric field

magnetization dynamics:

$$\dot{\boldsymbol{m}} \cdot (\bar{\Omega}_{\boldsymbol{mm}} + \eta_{\boldsymbol{mm}}) - \boldsymbol{H} = 0,$$

-Electronic contribution to damping (negative definite)

$$\eta_{\boldsymbol{mm}} = \int [d\boldsymbol{k}] \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \cdot (-\tau) \frac{\partial f_0}{\partial \varepsilon},$$

-Electronic contribution to Berry curvature

$$\bar{\Omega}_{\boldsymbol{mm}} = \int [d\boldsymbol{k}] f_0 \Omega_{\boldsymbol{mm}},$$

-Electronic contribution to free energy and H field

$$\boldsymbol{H} = -\frac{\partial G}{\partial \boldsymbol{m}}, \qquad G = -\beta^{-1} \sum_n \int [dk] ln(1 + e^{-\beta(\varepsilon_n - \mu)})$$

Electric field effect on magnetization dynamics

$$\dot{\boldsymbol{m}} \cdot (\bar{\Omega}_{\boldsymbol{mm}} + \bar{\Omega}^E_{\boldsymbol{mm}} + \eta_{\boldsymbol{mm}} + \eta^E_{\boldsymbol{mm}}) - \boldsymbol{H} - \boldsymbol{H}^E = 0,$$

intrinsic (f_0) and extrinsic (τ) **corrections by the electric field**:

$$\boldsymbol{H}^E = e\boldsymbol{E} \cdot \int [d\boldsymbol{k}] \left(\Omega_{\boldsymbol{km}} f_0 - \tau \frac{\partial \varepsilon}{\partial \boldsymbol{k}} \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \frac{\partial f_0}{\partial \varepsilon} \right),$$

$$\bar{\Omega}^E_{m_i m_j} = e\tau \boldsymbol{E} \cdot \int [d\boldsymbol{k}] \left[\frac{\partial \varepsilon}{\partial \boldsymbol{k}} \Omega_{m_i m_j} - \left(\Omega_{\boldsymbol{k} m_i} \frac{\partial \varepsilon}{\partial m_j} \right)_{\mathrm{A}} \right] \frac{\partial f_0}{\partial \varepsilon},$$

$$\eta^E_{m_i m_j} = e\tau \boldsymbol{E} \cdot \int [d\boldsymbol{k}] \left(\Omega_{\boldsymbol{k} m_i} \frac{\partial \varepsilon}{\partial m_j} \right)_{\mathrm{S}} \frac{\partial f_0}{\partial \varepsilon}$$

damping or gain

-all contribute to torques in the LLG equation

H field from electric field

$$\boldsymbol{H}^E = e\boldsymbol{E}\cdot\left[\int [dk] f_0 \Omega_{\boldsymbol{km}} + \int [dk] \frac{\partial \varepsilon}{\partial \boldsymbol{k}} \frac{\partial \varepsilon}{\partial \boldsymbol{m}} \cdot (-\tau) \frac{\partial f_0}{\partial \varepsilon}\right]$$

Both the km Berry curvature and velocity are odd in k, yielding zero result, if there is spatial inversion symmetry
or time reversal symmetry in the orbital degree of freedom.

Need spin-orbit coupling to break the latter symmetry.

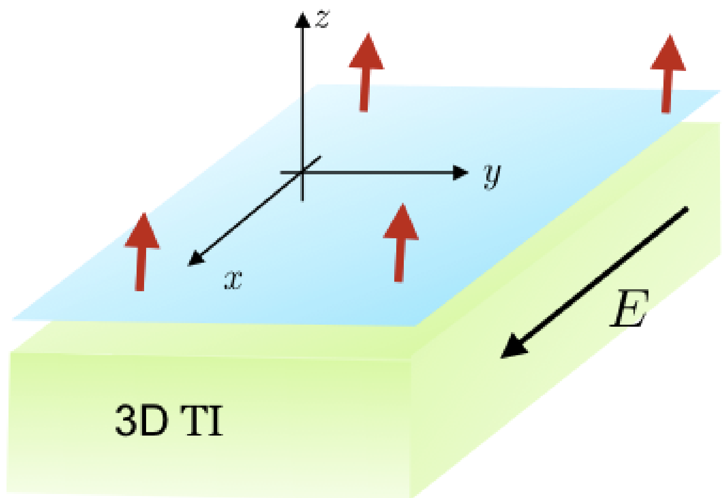
- spin-orbit torque:

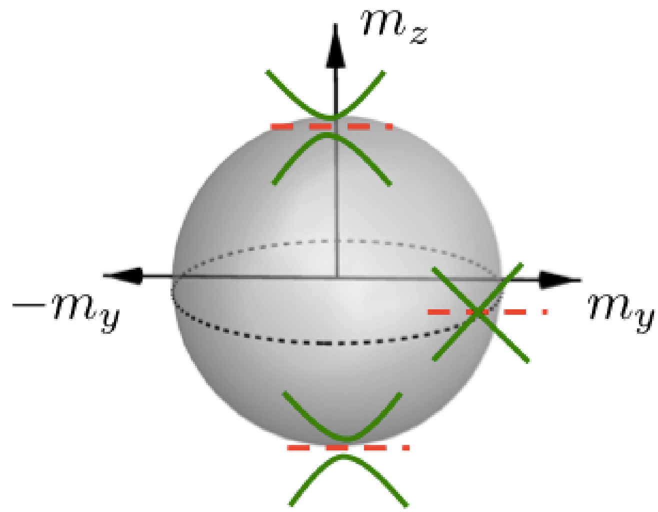
$$\boldsymbol{\tau}_{so}^{H} = \boldsymbol{m} \times \boldsymbol{H}^E$$

FM/TI heterostructure

Ferromagnet coated on the surface of topological insulator:

$$\hat{H}(m) = \hbar v(-k_y\sigma_x + k_x\sigma_y) + Jm \cdot \sigma$$



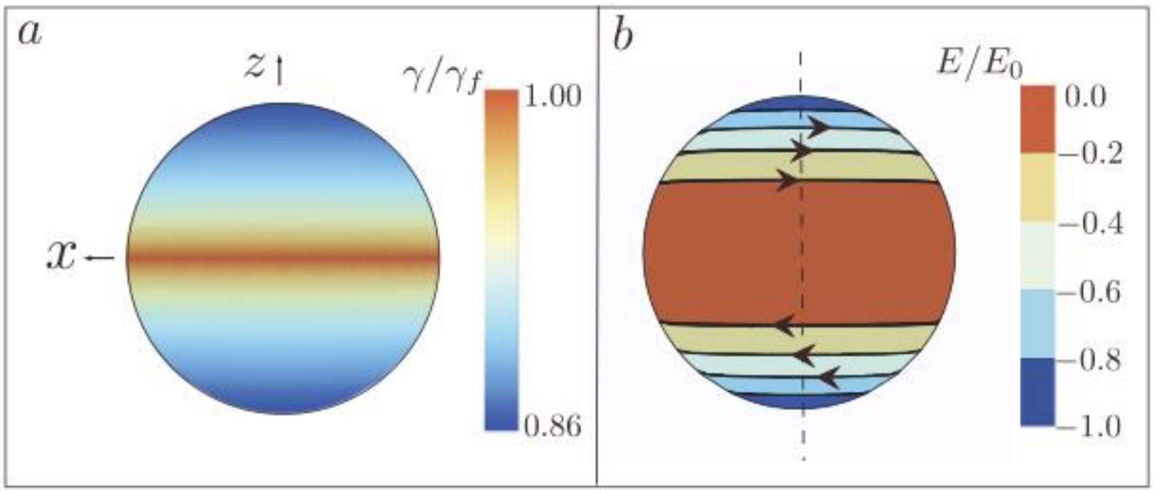


$$\varepsilon = \pm\sqrt{(Jm_x - \hbar v k_y)^2 + (Jm_y + \hbar v k_x)^2 + (Jm_z)^2}$$

Degenerate lines on the equator

Magnetization Dynamics

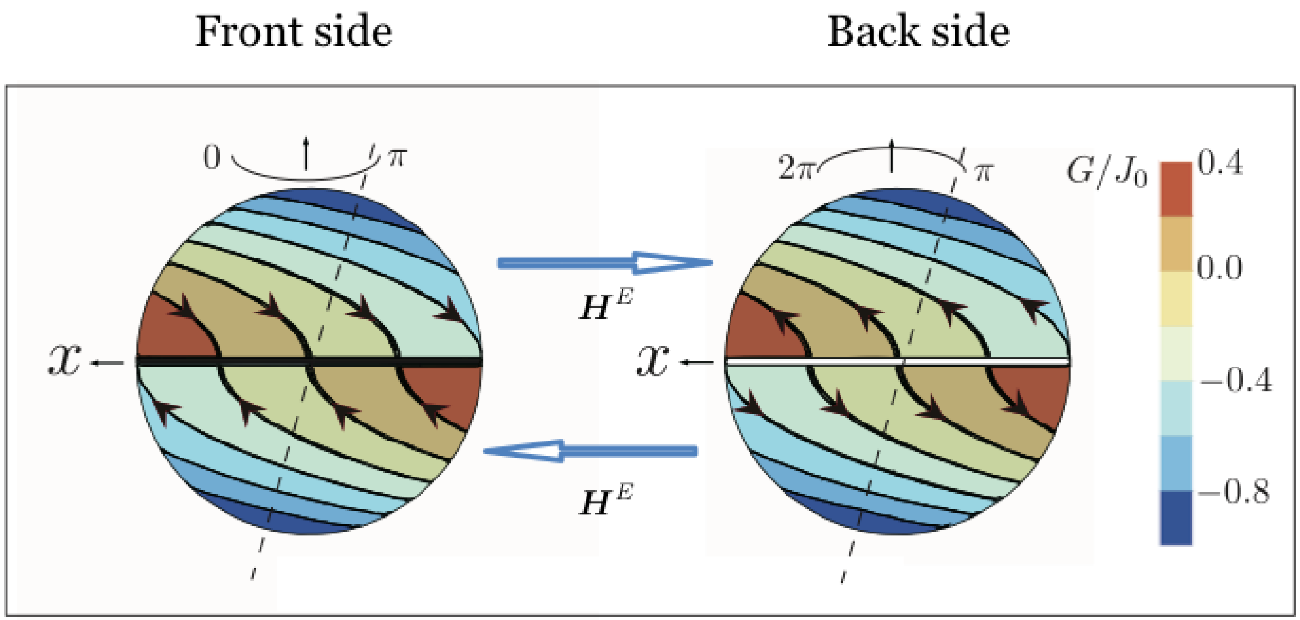
The influence of TI electrons in the absence of electric field,



(a) Anisotropic gyromagnetic ratio renormalization; (b) uniaxial anisotropic energy and magnetization trajectories.

Magnetization Dynamics

The influence of TI electrons in the presence of electric field,



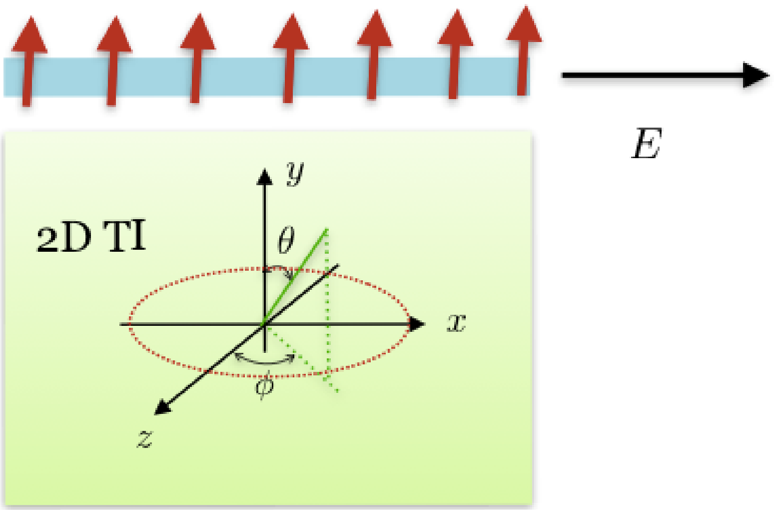
HE is in opposite directions on the north and south hemispheres, resulting in

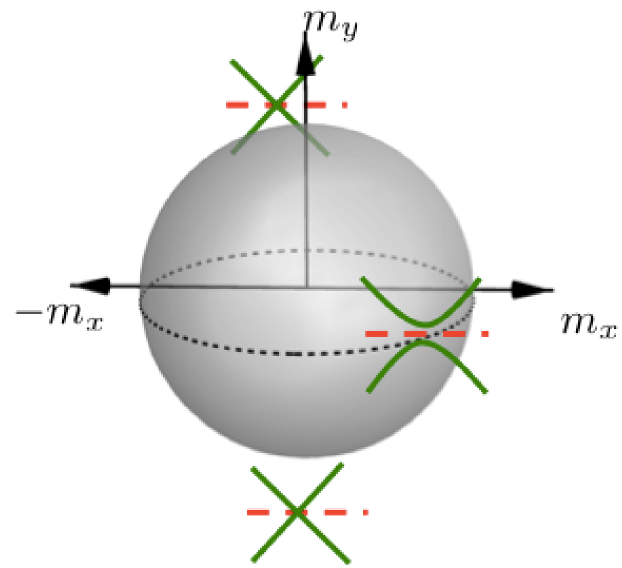
1) Stable fixed points tilted away from north and south poles
2) Front side of the equator becomes attractive and back side repulsive.
3) H field cannot be defined as the gradient of a globally defined free energy

FM/TI heterostructure II

Ferromagnet attached with 2D topological insulator:

$$\hat{H}(\boldsymbol{m}) = \hbar v k \sigma_y + J\boldsymbol{m}\cdot\boldsymbol{\sigma}.$$





$$\varepsilon = \pm\sqrt{(Jm_x)^2 + (\hbar v k + Jm_y)^2 + (Jm_z)^2}$$

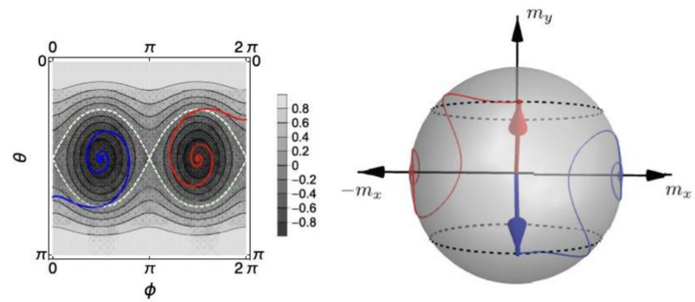
Two degenerate points at: $m_y = \pm m$

Hard axis along y: $\varepsilon_{ti} = -K_{ti}\hat{m}_y^2 = -K_{ti}\cos^2\theta$ $\quad K_{ti} < 0$

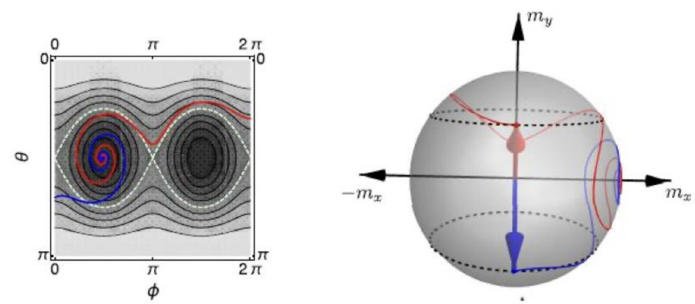
Magnetization trajectories

Free energy contours and typical trajectories with Gilbert damping

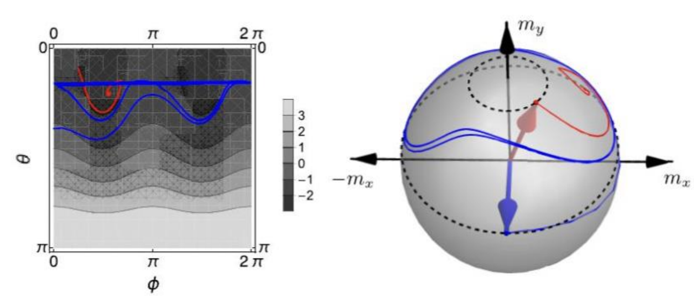
Zero E:



Finite E:



Finite E and Zeeman field in the y direction:



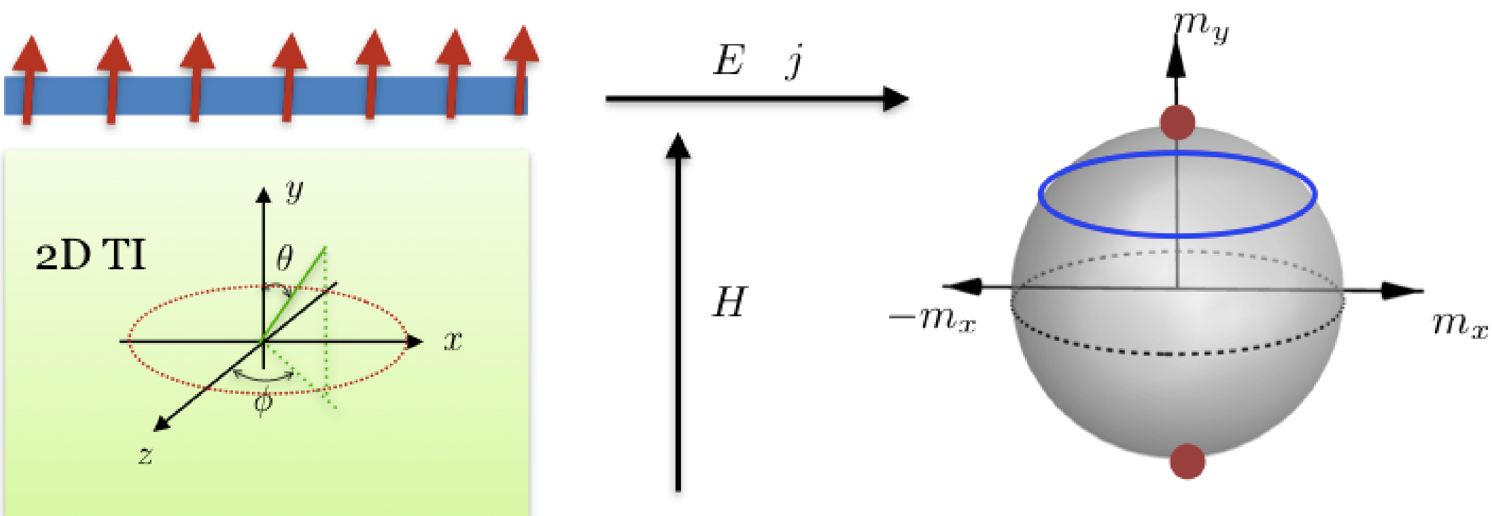
Electric Work

The H^E field is related to the pumped current

$$\boldsymbol{j}_p = e \int [d\boldsymbol{k}]\, \Omega_{\boldsymbol{km}} \cdot \dot{\boldsymbol{m}}$$

Electric work is quantized for a close loop, $W = \oint \boldsymbol{j}_p \cdot \boldsymbol{E} dt = \oint \boldsymbol{H}^E \cdot d\boldsymbol{m}$ = *eE C*

(per unit length) *C: Chern number*



static H field.

The electric work over one cycle is nonzero, and cancels the energy dissipation!

Conclusion

- **Adiabatic theory of magnetization dynamics**

$$\boldsymbol{\Omega}_{mt} - \partial_{\boldsymbol{m}}\mathcal{G} + \dot{\boldsymbol{m}} \times \boldsymbol{\Omega}_{\boldsymbol{m}} - \eta\dot{\boldsymbol{m}} = 0$$

Faraday Conservative Lorentz damping and gain

- **Semi-classical formulation of electronic contributions**

$$\dot{\boldsymbol{m}} \cdot (\bar{\Omega}_{\boldsymbol{mm}} + \bar{\Omega}^{E}_{\boldsymbol{mm}} + \eta_{\boldsymbol{mm}} + \eta^{E}_{\boldsymbol{mm}}) - \boldsymbol{H} - \boldsymbol{H}^{E} = 0,$$

1. Zero electric field:
   Berry curvature, damping, and conservative H field
2. With electric field:
   corrections on Berry curvature, damping and Faraday magnetomotive force