# 光学现象中的几何相位

物理中的几何相位（专题6）

高阳

## 力学

$$\vec{F} = m\vec{a}$$

$$F = -\frac{G m_1 m_2}{r^2}$$

$$\delta \int_{t_a}^{t_b} \mathcal{L}(\vec{q}, \dot{\vec{q}}, t)\,\mathrm{dt} = 0$$

$$\dot{\vec{q}} = \frac{\partial H}{\partial \vec{p}}$$

$$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{q}}$$

## 电动

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \epsilon\mu \frac{\partial \vec{B}}{\partial t}$$

$$\vec{F} = q\vec{E} + \mathrm{q}\vec{v} \times \vec{B}$$

## 量子

$$E = \hbar\omega$$

$$\vec{p} = \hbar\vec{k}$$

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\,\psi$$

$$\rho(\vec{r}, t) = |\psi(\vec{r}, t)|^2$$

$$[\hat{x}, \hat{p}] = i\hbar$$

## 热统

$$\mathrm{d}U = \delta Q + \delta W$$

$$\oint \frac{\delta Q}{T} \leq 0$$

$$S = k_B \ln\Omega$$

$$Z = \sum e^{-\beta H}$$

$$f = \frac{1}{e^{\beta(\varepsilon - \mu)} \pm 1}$$

# 光与物质的耦合

单色光

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t}$$

$$\boldsymbol{E} = \boldsymbol{E_0}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t}$$

$$\boldsymbol{B} = \frac{\boldsymbol{E}}{c}$$

线偏振

$$\boldsymbol{E} = E_0\, \hat{x}$$

圆偏振

$$\boldsymbol{E} = E_0\, (\hat{x} \pm i\, \hat{y})$$

速度规范，基于薛定谔方程

$$H = H_0 + H',$$

$$H_0 = \frac{p^2}{2m} + U(r)$$

$$H' = \frac{e\boldsymbol{p}}{m} \cdot \boldsymbol{A} + \frac{e^2}{2m} A^2$$

速度规范，基于紧束缚模型

$$H = H_0 + H',$$

$$H_0 = H(k)$$

$$H' = \frac{\partial H}{\partial k_i} \frac{e}{\hbar} A_i + \frac{1}{2} \frac{\partial^2 H}{\partial k_i \partial k_j} \frac{e^2}{\hbar^2} A_i A_j + \cdots$$

# 电子的响应形式

- 电流

  线性阶 $J_i(\omega, q) = \sigma_{ij}(\omega, q)\, E_j(\omega, q)$

  非线性阶 $J_i(\omega_1, \omega_2; \omega_1 + \omega_2) = \alpha_{ijk}(\omega_1, \omega_2; \omega_1 + \omega_2) E_j(\omega_1) E_k(\omega_2)$

  q经常被忽略: $q = \frac{2\pi}{\lambda}$ $\lambda \sim 300\, nm$

  相比于晶格: $k = \frac{2\pi}{a}$ $a \sim 1 - 2$ Å

- 磁化强度

$$M_i = \beta_{ij} E_j + \mu_{ij} B_j$$

- 自旋流

$$J_i^{S_j} = \gamma_{ijk} E_k + \theta_{ijk\ell}\, E_k E_\ell$$

# 计算方法一：线性响应理论

- 物理量的期望值：
  $\langle O\rangle = Tr\ (\hat{\rho}\ \hat{O})$
- 海森堡绘景，分布函数不变而算符演化

$$\hat{O}(t) \to e^{i\hat{H}t/\hbar}\ \hat{O}\ e^{-i\hat{H}t/\hbar}$$

$$\frac{d\hat{O}}{dt} = -\frac{i}{\hbar}[\hat{O},\hat{H}]$$

- 等价变换

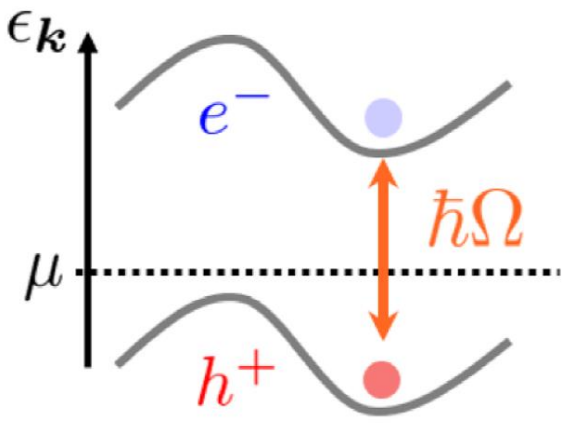
$$\langle O\rangle = Tr\left(\hat{\rho}e^{\frac{i\hat{H}t}{\hbar}}\hat{O}e^{-\frac{i\hat{H}t}{\hbar}}\right) = Tr\left(e^{-\frac{i\hat{H}t}{\hbar}}\hat{\rho}e^{\frac{i\hat{H}t}{\hbar}}\hat{O}\right)$$

- 密度算符的演化

$$\hat{\rho}(t) \to e^{-i\hat{H}t/\hbar}\ \hat{\rho}\ e^{i\hat{H}t/\hbar}$$

$$\frac{d\hat{\rho}}{dt} = +\frac{i}{\hbar}[\hat{\rho},\hat{H}]$$

$$\rho = \rho^0 + \rho^{(1)} + \rho^{(2)} + \cdots$$



(a)

# 计算方法二：半经典理论

- 半经典运动方程

$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon}{\hbar \partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}$$
$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} - e\dot{\boldsymbol{r}} \times \boldsymbol{B}$$

- 玻尔兹曼方程

$$\frac{\partial f}{\partial t} + \dot{\boldsymbol{k}} \cdot \frac{\partial f}{\partial \boldsymbol{k}} + \dot{\boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{r}} = -\frac{f - f_0}{\tau}$$

- 傅里叶分解

$$f = f_0 + f_1 + f_2 + \cdots$$

- 物理量计算， 适用于带内过程，无法计算跃迁过程

$$\langle O \rangle = \int dk \, \langle \psi | \hat{O} | \psi \rangle \, f$$

# 线性光响应：光在介质中的传播（1）

电子的响应提供源

$$\nabla \cdot \boldsymbol{E} = -\frac{\rho}{\varepsilon_0}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_{\boldsymbol{0}}\boldsymbol{J}_{\boldsymbol{0}} + \mu_0 \nabla \times \boldsymbol{M} + \mu_0 \frac{\partial \boldsymbol{P}}{\partial t}$$

自由部分，可在真空存在

源，由介质提供，coarse graining方法，

$$\boldsymbol{J} = \boldsymbol{J}_{\boldsymbol{0}} + \nabla \times \boldsymbol{M} + \nabla \times \left(\partial_i Q_{ij}^M \, \hat{e}_j\right) + \cdots$$

$$+ \frac{\partial \boldsymbol{P}}{\partial t} + \frac{\partial}{\partial t}\, \partial_i Q_{ij}^E \, \hat{e}_j + \cdots$$

# 线性光响应：光在介质中的传播（2）

单色光

$$\boldsymbol{E} = \boldsymbol{E_0}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t}$$

折射率

$$k = n\frac{\omega}{c}$$

介质中的传播方程

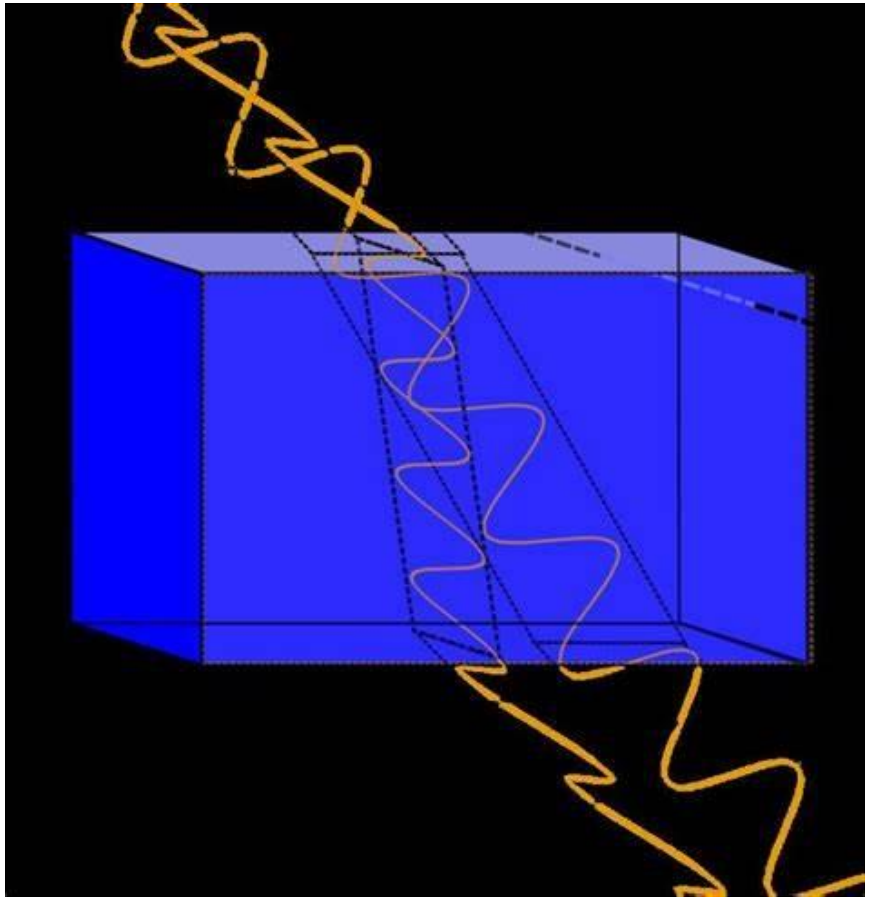
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}$$

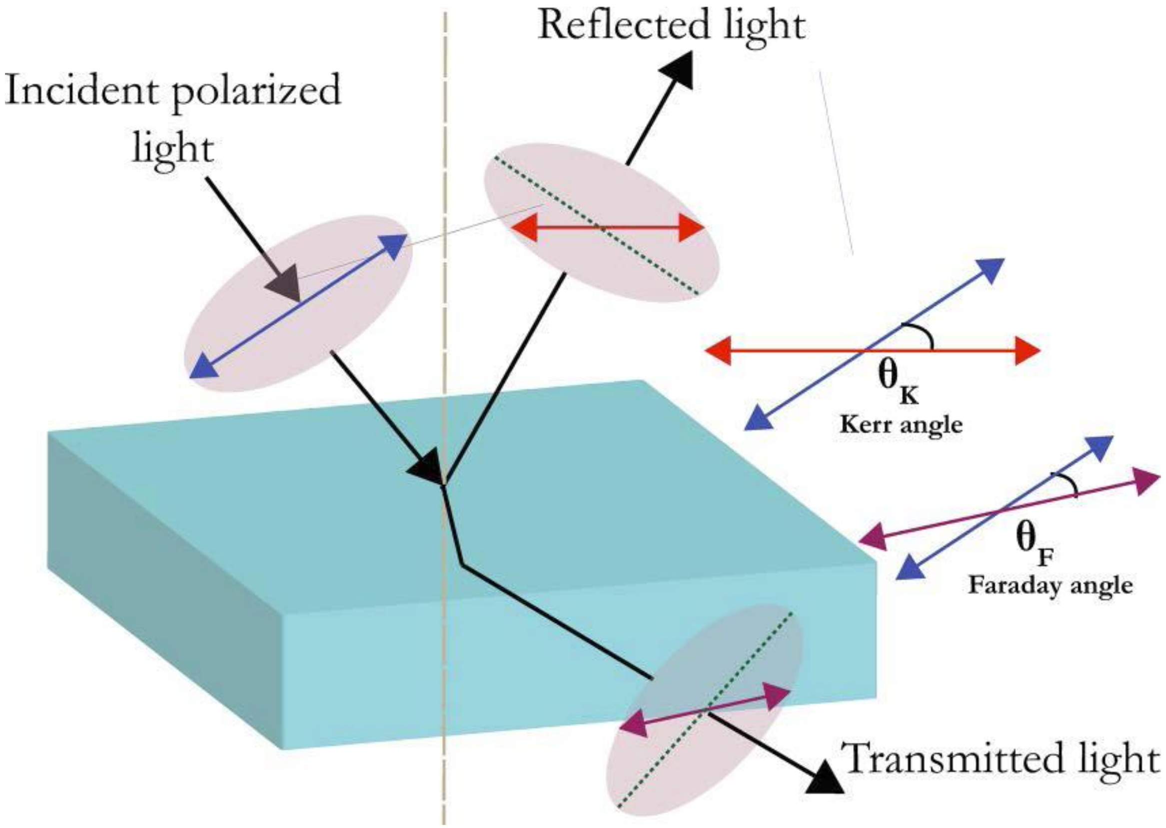
$$J_i(\omega, q) = \sigma_{ij}(\omega, q)\, E_j(\omega, q)$$

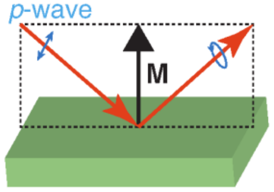
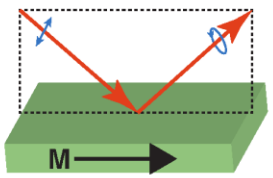
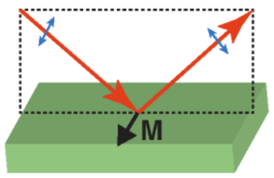
折射率与光轴

$$M_{ij}(n, \omega) E_j(\omega) = 0$$



# 旋光二向色性与磁光克尔效应：表象

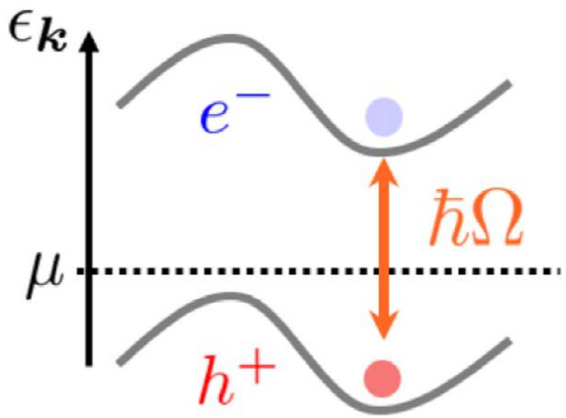



| Name | (a) Polar | (b) Longitudinal | (c) Transverse |
|---|---|---|---|
| Geometry |  |  |  |
| Detection | Out-of-plane | in-plane | in-plane |
| Polarization Variation | Rotation Ellipticity | | None |
| Measurement | Polarization Analysis | | Intensity measurement |

# 旋光二向色性与磁光克尔效应：起源

传播方程的形式

$$\begin{pmatrix} n^2 - n_0^2 & -a & 0 \\ a & n^2 - n_0^2 & 0 \\ 0 & 0 & n^2 - n_1^2 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = 0$$



(a)

本征值与本征矢量

$$n^2 = n_0^2 \pm ia \qquad E_\pm = E_0(\hat{x} \pm i\,\hat{y})$$

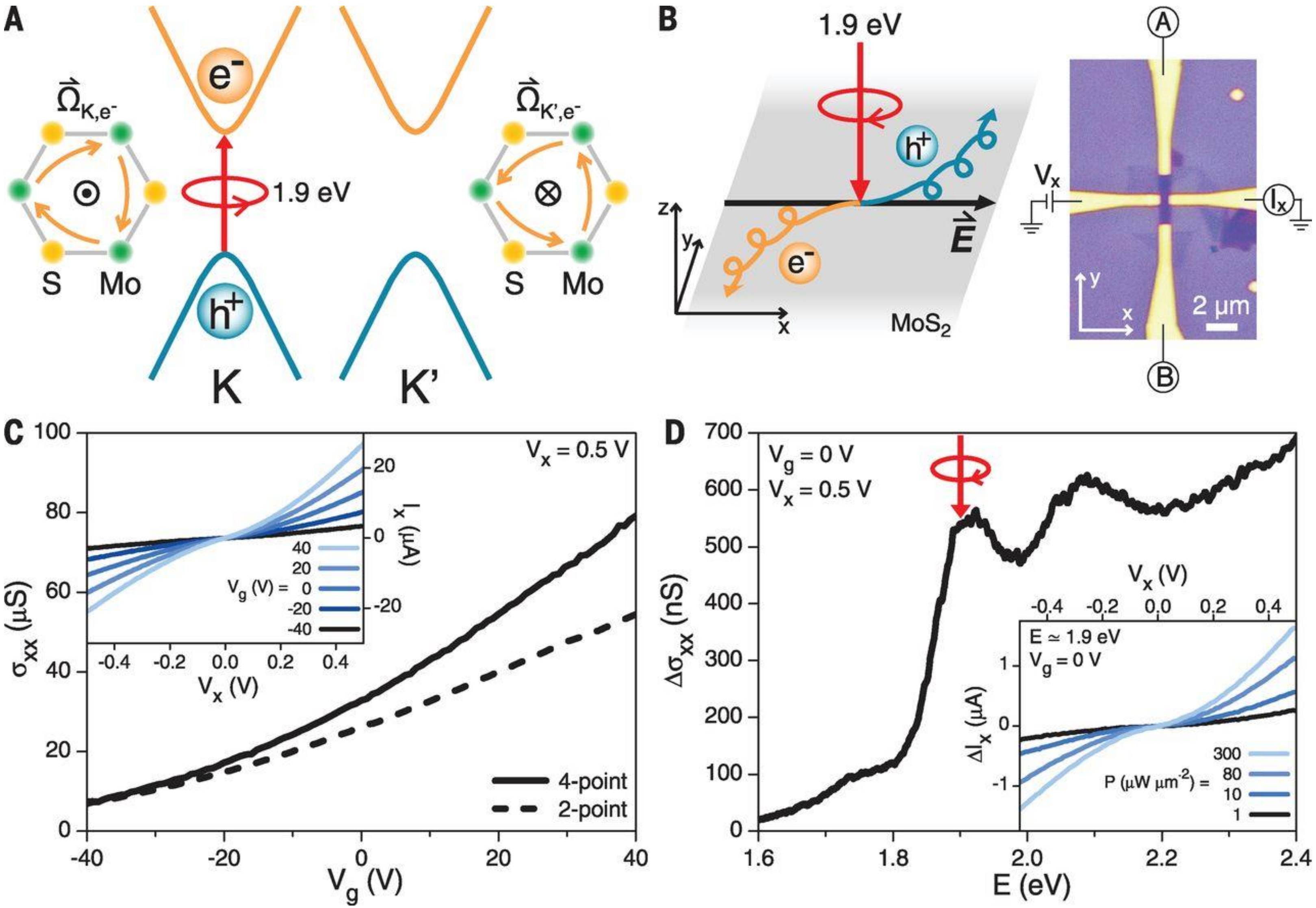
反对称部分的起源

$$\sigma_{xy} = \frac{e^2}{2\hbar^2\omega} \sum_{mn} \int \frac{dk}{8\pi^3} \frac{(f_m - f_n)(\varepsilon_m - \varepsilon_n)^2 (\Omega_z)_{mn}}{\varepsilon_m - \varepsilon_n + \hbar\omega + i\eta}$$

半经典动力学，反常速度

$$J_x = -\frac{e^2}{\hbar} \int \frac{dk}{8\pi^3} (\Omega_z)_m f_m \, E_y$$

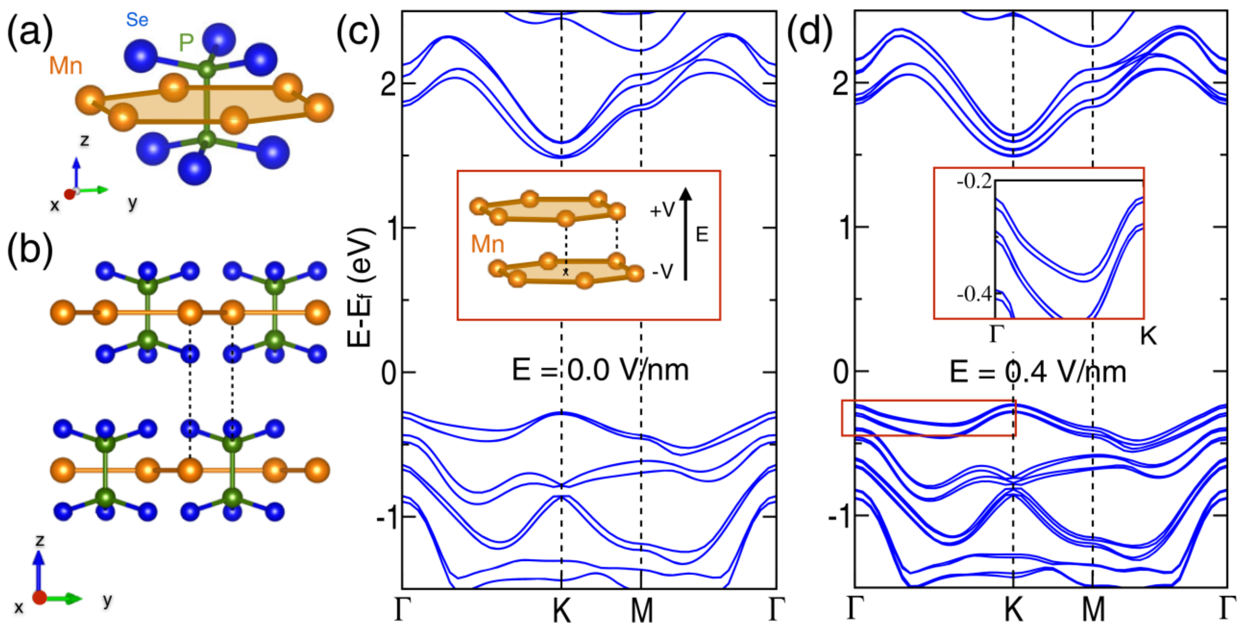
# 谷霍尔效应的实验验证

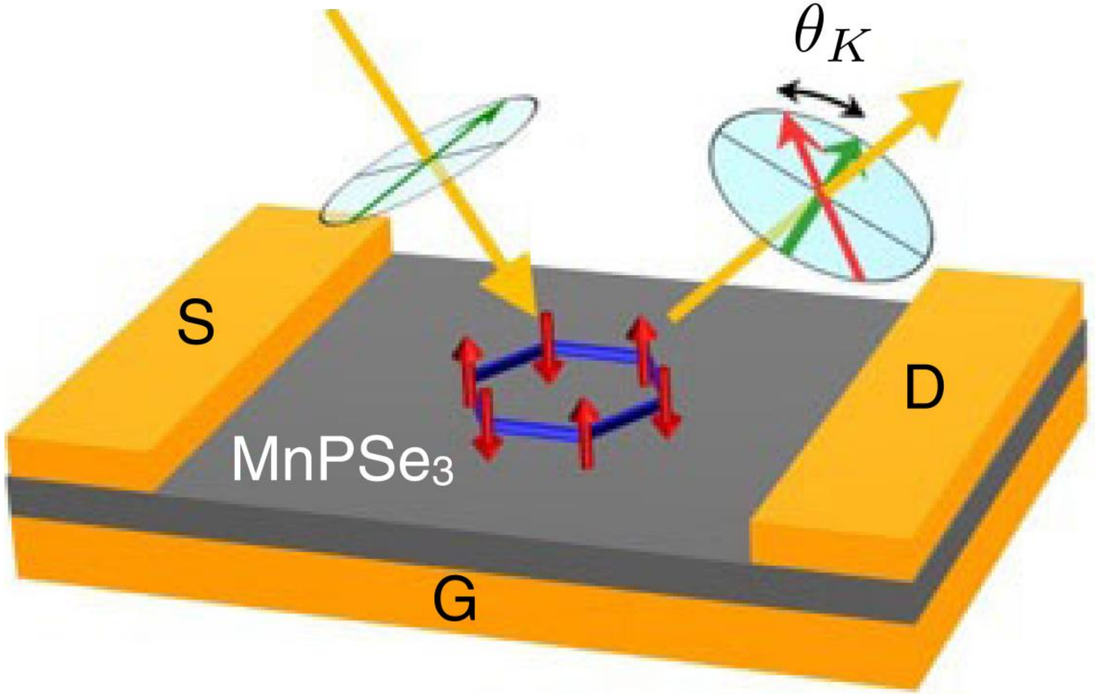



K.F. Mak, et al., Science 344, p1489-1492 (2014)

# 门电压调控的磁光克尔效应

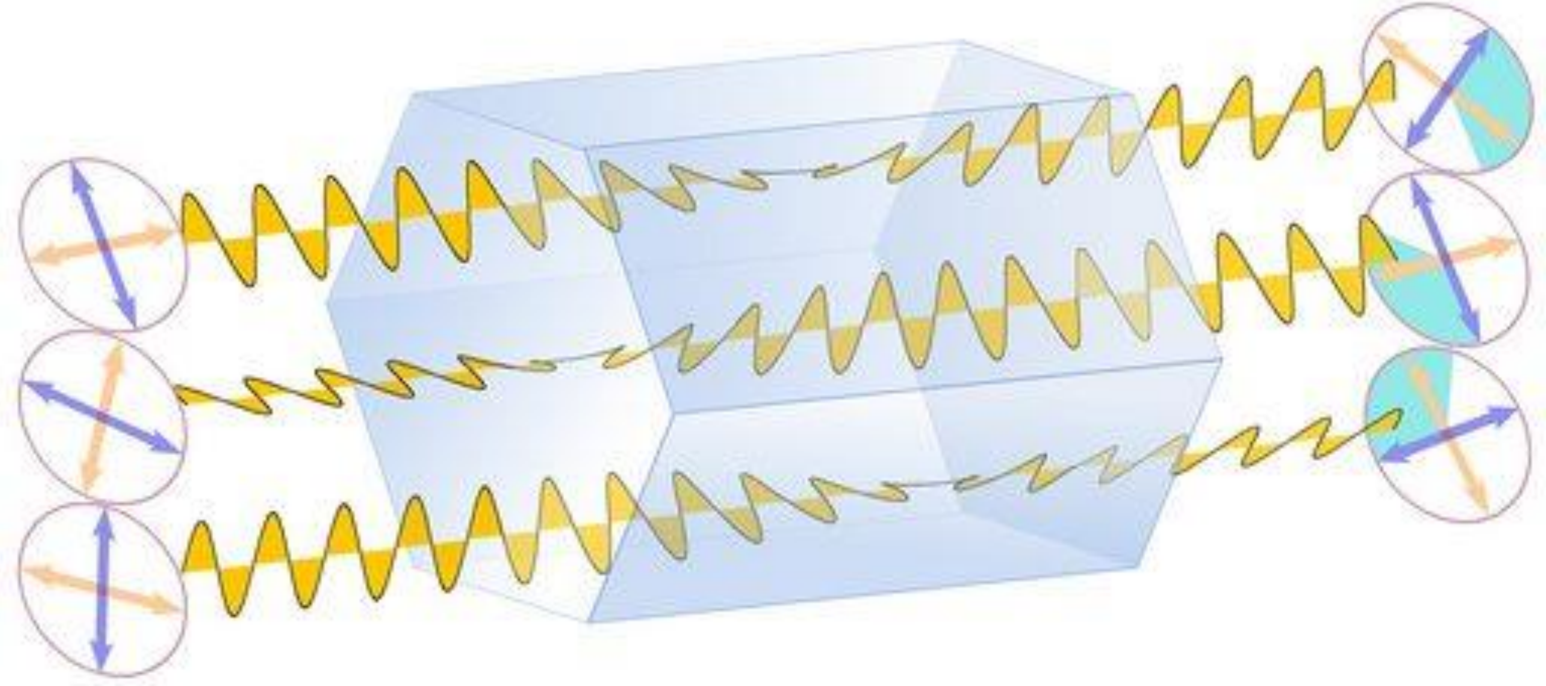
$$\theta_K + i\eta_K = \frac{2(Z_0 d\sigma_{xy})}{1-(n_s + Z_0 d\sigma_{xx})^2},$$




Sivadas, Okamoto, and Xiao, PRL 117, 267203(2016)

# 空间色散现象1：自然光活性

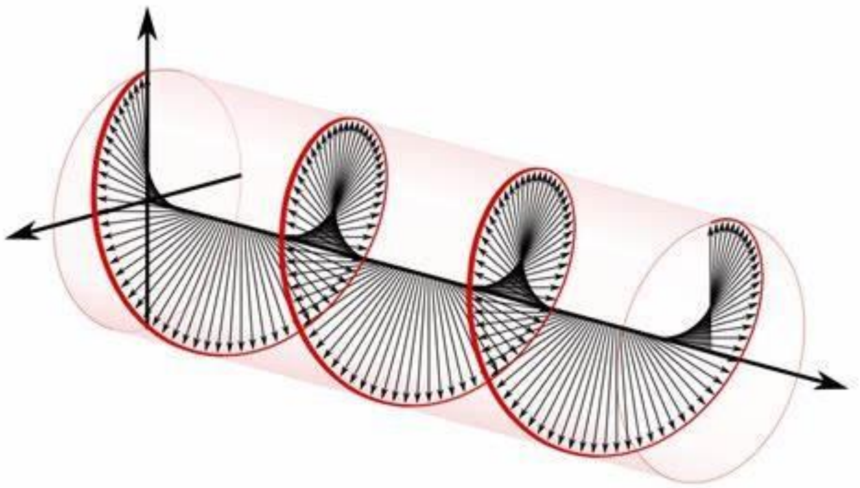


破坏空间反演而不是时间反演
材料需具有手性
如alpha石英

# 自然光活性：起源

传播方程的形式

$$\begin{pmatrix} n^2-n_0^2 & -a & 0 \\ a & n^2-n_0^2 & 0 \\ 0 & 0 & n^2-n_1^2 \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = 0$$



反对称部分的起源

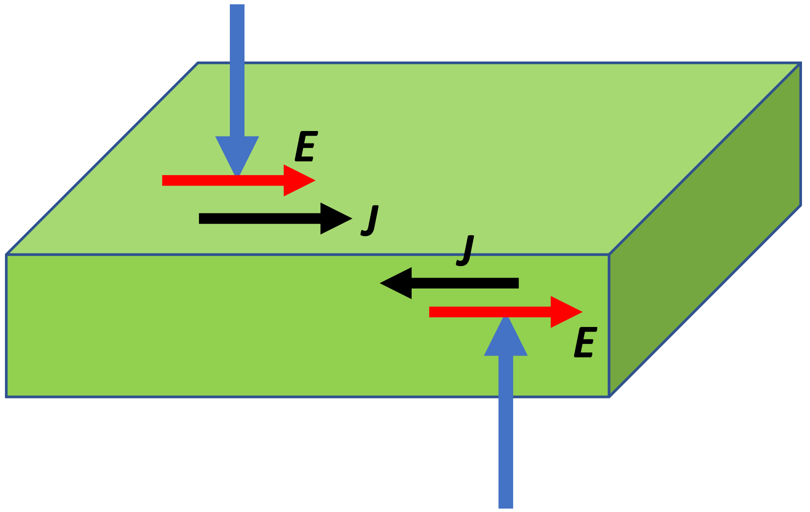
$$\sigma_{xy}(\omega,q)=\sigma_{xy}(\omega,0)+q_z\sigma_{xyz}(\omega,0)+\cdots$$

空间色散：

$$J_i=\beta_{ij}B_j$$

$$\beta_{ij}\propto\int dk\, f' v_i m_j$$

轨道磁矩




Zhong, Moore, and Souza, PRL 116, 077201(2016)

# 自然光活性：经典对应

运动方程

$$\dot{\boldsymbol{r}} = \frac{\partial \varepsilon}{\hbar \partial \boldsymbol{k}} - \dot{\boldsymbol{k}} \times \boldsymbol{\Omega}$$
$$\hbar \dot{\boldsymbol{k}} = -e\boldsymbol{E} - e\dot{\boldsymbol{r}} \times \boldsymbol{B}$$

加上态密度之后

$$D\dot{\boldsymbol{r}} = \frac{\partial \varepsilon}{\hbar \partial \boldsymbol{k}} + e\boldsymbol{E} \times \boldsymbol{\Omega} + \frac{e}{\hbar}(\boldsymbol{v} \cdot \boldsymbol{\Omega})\boldsymbol{B}$$

手性磁电流：

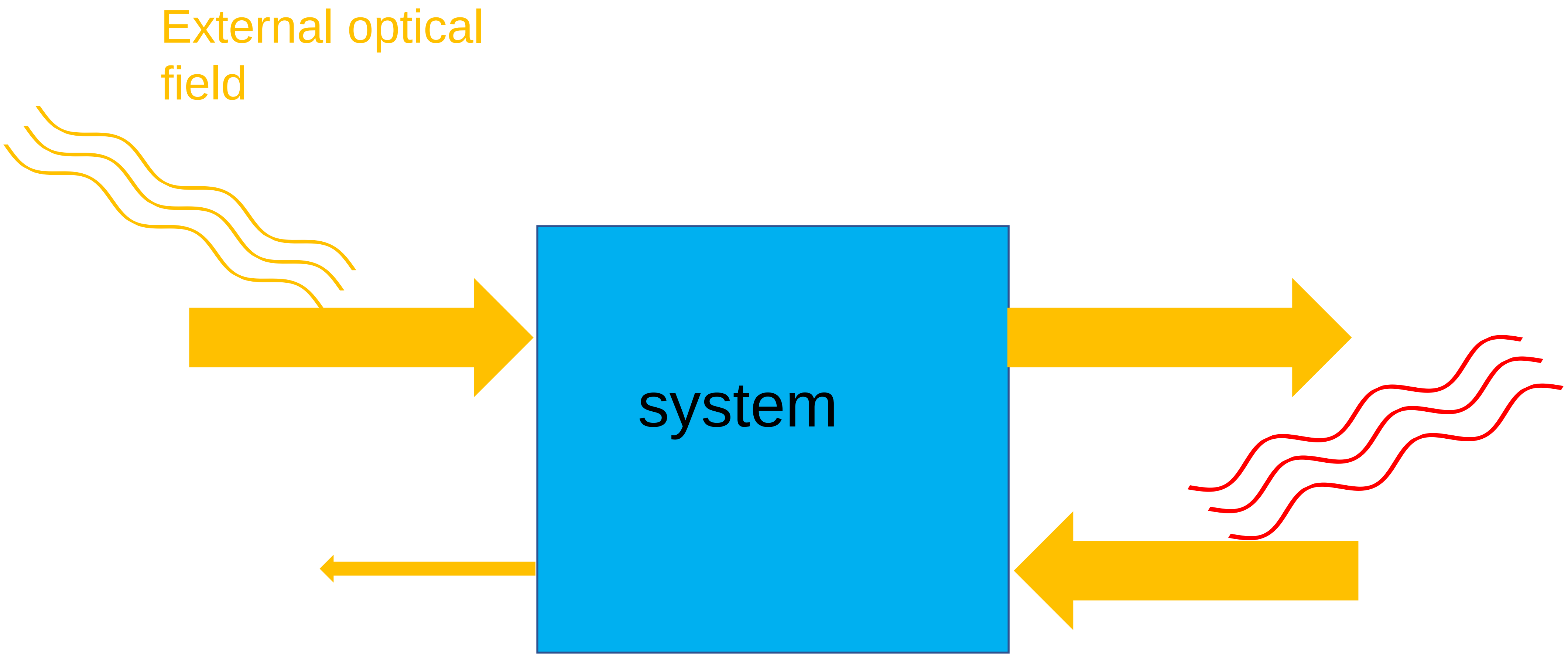
$$\boldsymbol{J} = -\frac{e^2}{\hbar} \int dk \, (\boldsymbol{v} \cdot \boldsymbol{\Omega}) \, f_0 \, \boldsymbol{B}$$

与动量空间的磁单极子相关

Weyl半金属

$$\boldsymbol{J} = \frac{e^2}{h^2} \, (\mu_R - \mu_L)\boldsymbol{B}$$

# 空间色散现象2：方向二向色性



# 方向二向色性：起源

线偏振的响应部分的高阶依赖

$$\sigma_{xx}(\omega, q) = \sigma_{xx}(\omega, 0) + q_z \sigma_{xxz}(\omega, 0) + \cdots$$

折射率

$$n_z = ic\mu_0\sigma_{xxz}/2 + \sqrt{(n_0 + i\kappa_0)^2 - c^2\mu_0^2\sigma_{xxz}^2/4}$$

↑

传播方向翻转时这一项将反号

经典对应：四极矩电场产生的电流

$$J_x^{\mathrm{tr}} = -2e \int \frac{d\boldsymbol{k}}{8\pi^3} f_m \partial_{k_x} \delta\varepsilon = \gamma_{xxz} \partial_z E_x$$

$$\gamma_{jik} = \frac{e^2}{\hbar} \sum_m \int \frac{d\boldsymbol{k}}{8\pi^3} G_{ijk,m} f'_m$$

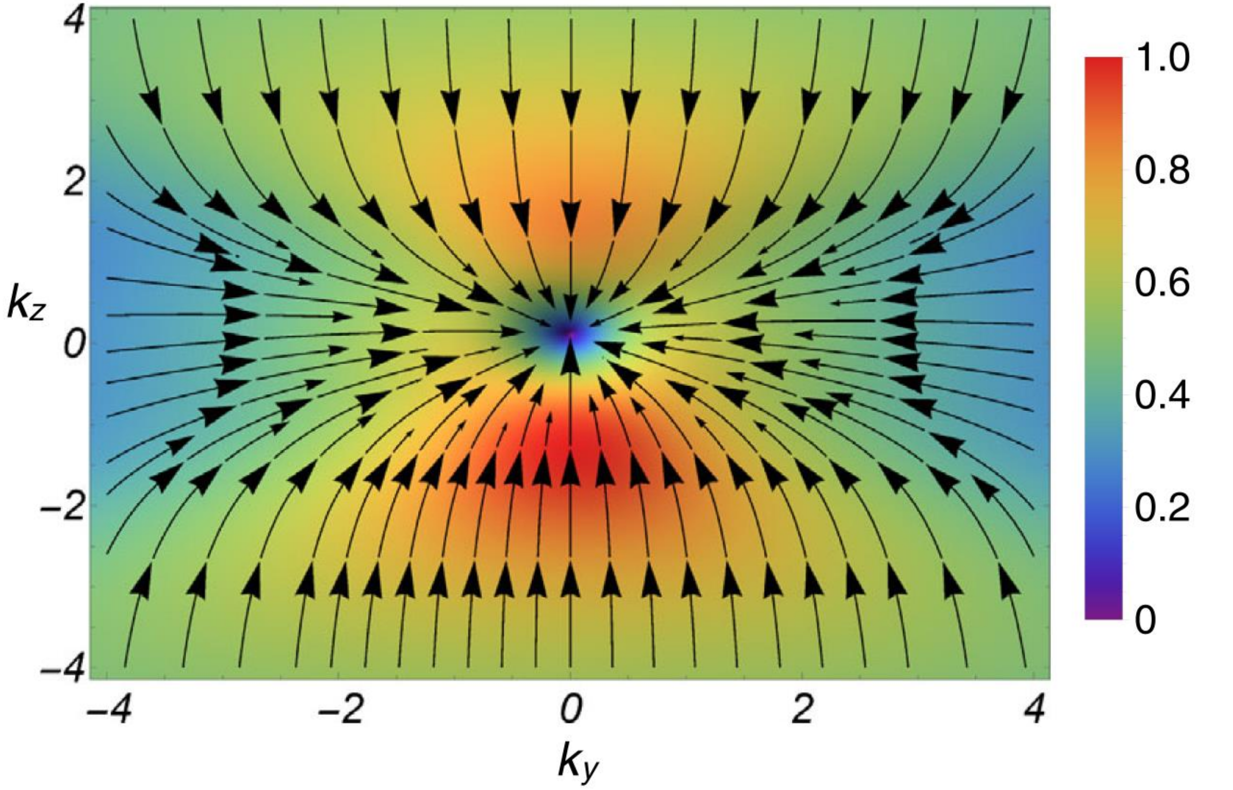
$$G_{ijk,m} = v_{i,m} g_{jk,m}$$

Gao and Xiao, PRL 122, 227402 (2019)

# 量子度规的偶极矩

量子度规：布洛赫态间的距离

$$d(|\psi_k\rangle, |\psi_{k+dk}\rangle) = \left||\psi_{k+dk}\rangle - |\psi_k\rangle\right|^2$$
$$= g_{ij} dk_i dk_j$$




量子度规与贝利曲率的关联

$$\langle \partial_i u_k | \partial_j u_k \rangle - \langle \partial_i u_k | u_k \rangle \langle u_k | \partial_j u_k \rangle$$
$$= g_{ij} + \frac{i}{2} \epsilon_{ijk} \Omega_k$$

Gao and Xiao, PRL 122, 227402 (2019)