Lecture 5
Magneto-electronic Applications

- Artificial gauge fields in magnetic textures
  - Spin Faraday effect: emf by changing spins
- Dynamics under a gradient force
  - Density response: polarization
  - Current response: inverse spin Hall effect

Magneto-electronics

- **Electricity: transport of electron charges**
- **Magnetism: alignment of electron spin and orbital moments**
- **Magneto-electronics: effects on charge transport by magnetic fields and structures**

  e.g. Hall effects, magneto resistance,
  spin Faraday, inverse spin Hall.

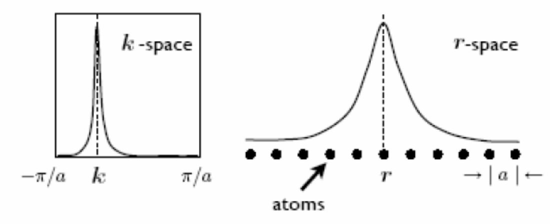
Electron dynamics in phase space

(Suderam and Niu 1999)

- Crystal under slowly varying perturbations

$$H[\boldsymbol{r}, \boldsymbol{p}; \beta_1(\boldsymbol{r}, t), \ldots \beta_g(\boldsymbol{r}, t)]$$

- Local approximation and wave packet in a Bloch band



- Semiclassical dynamics of center of charge

$$\dot{\mathbf{r}} = \frac{\partial \mathcal{E}}{\hbar \partial \mathbf{k}} - (\Omega_{\mathbf{kr}} \cdot \dot{\mathbf{r}} + \Omega_{\mathbf{kk}} \cdot \dot{\mathbf{k}}) - \mathbf{\Omega}_{\mathbf{k}t}$$

$$\dot{\mathbf{k}} = -\frac{\partial \mathcal{E}}{\hbar \partial \mathbf{r}} + (\Omega_{\mathbf{rr}} \cdot \dot{\mathbf{r}} + \Omega_{\mathbf{rk}} \cdot \dot{\mathbf{k}}) + \mathbf{\Omega}_{\mathbf{r}t}$$

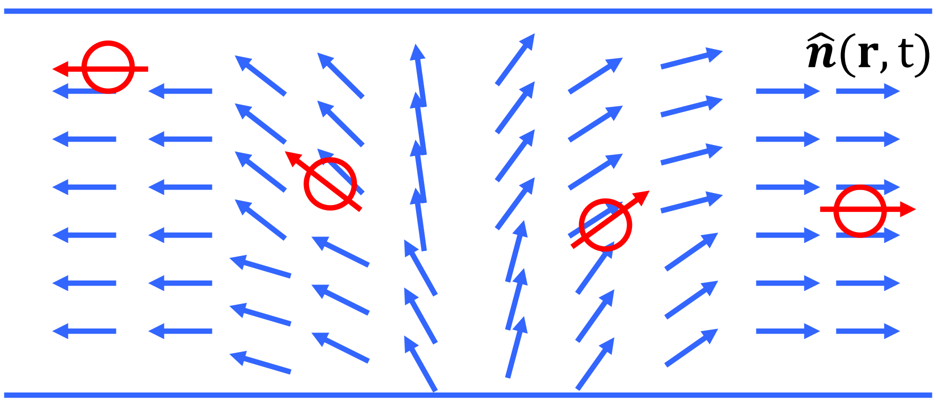
Artificial gauge fields

G. E. Volovik, J. Phys. C 20, L83 (1987)

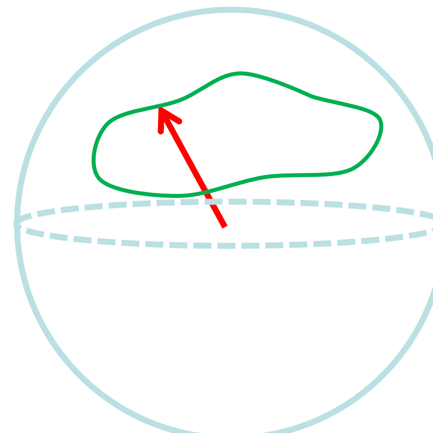
Effective magnetic field in a magnetic texture

s-d coupling

$$H = H_0[\mathbf{q} + (e/\hbar)\mathbf{A}(\mathbf{r},t)] - J\mathbf{n}(\mathbf{r},t)\cdot\boldsymbol{\sigma}$$



Berry phase on the unit sphere

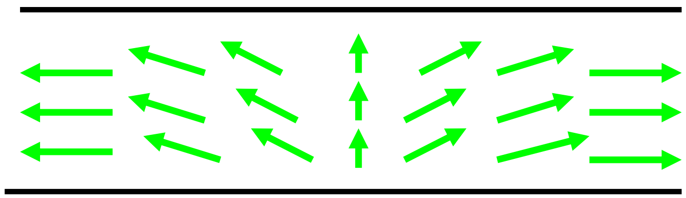


θ and ϕ are the spherical angles of $\hat{\boldsymbol{n}}(\mathbf{r},\mathrm{t})$

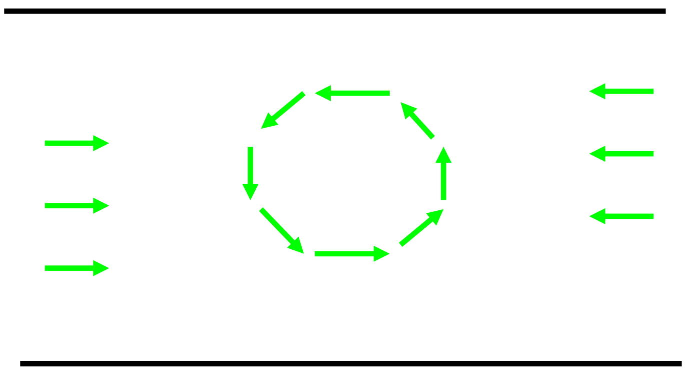
- **Effective magnetic field on conduction electrons**

$$\Omega_{\mathbf{rr}} \longrightarrow \frac{1}{2}\sin\theta\,(\nabla\theta\times\nabla\phi)$$

Domain wall in ferromagnetic wires



Transverse



Vortex

domain wall can be moved by applying a real magnetic field along the wires

Universal Electromotive Force Induced by Domain Wall Motion

Shengyuan A. Yang, Geoffrey S. D. Beach, Carl Knutson, Di Xiao, Qian Niu, Maxim Tsoi, and James L. Erskine
Phys. Rev. Lett. **102**, 067201 – Published 9 February 2009

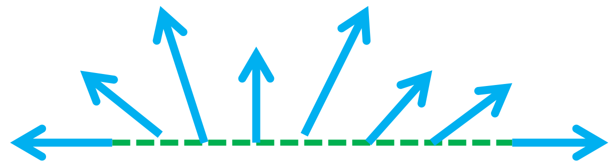
Physics See Viewpoint: A new connection between electricity and magnetism

Effective electric field: spin Faraday effect

- **With time dependence**

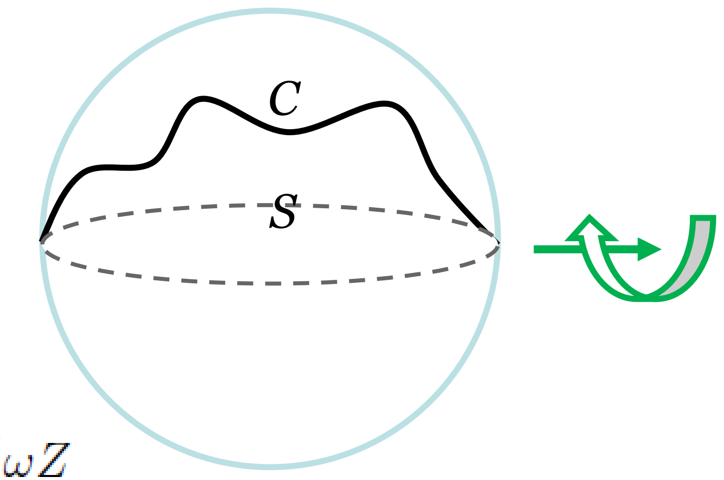
$$\Omega_{xt}(\mathbf{r},t) = \frac{1}{2}\sin\theta\left(\frac{\partial\theta}{\partial t}\frac{\partial\phi}{\partial x} - \frac{\partial\theta}{\partial x}\frac{\partial\phi}{\partial t}\right)$$

effective electric field



- **Electromotive force along the wire**

$$\mathcal{E} = -\frac{\hbar}{e}\int_l d\mathbf{r}\cdot\mathbf{\Omega}_{\mathbf{r}t} = -\frac{\hbar}{e}\int_l dx\,\Omega_{xt}$$

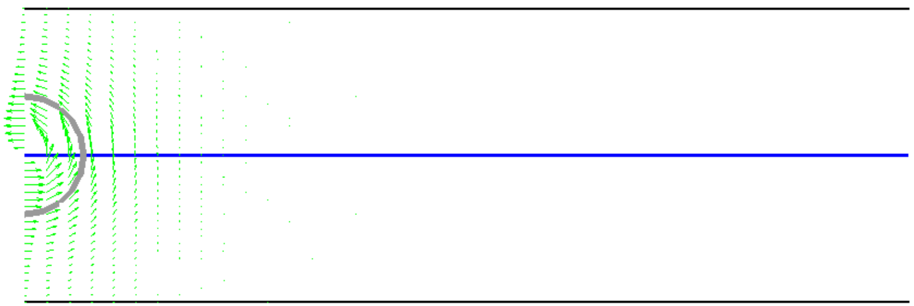


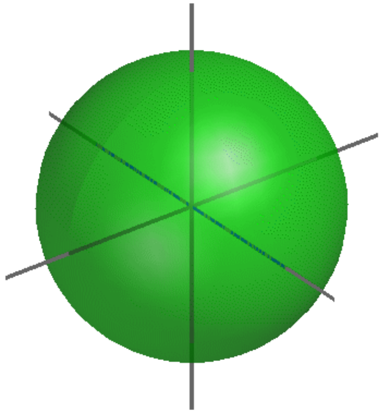
- **Topological invariant:**

$$\int_0^T dt\,\mathcal{E} = \frac{h}{e}Z \longrightarrow \overline{\mathcal{E}} = \frac{\hbar}{e}\omega Z$$

Frequency of the vortex motion

$$2\pi f_y = \gamma H$$



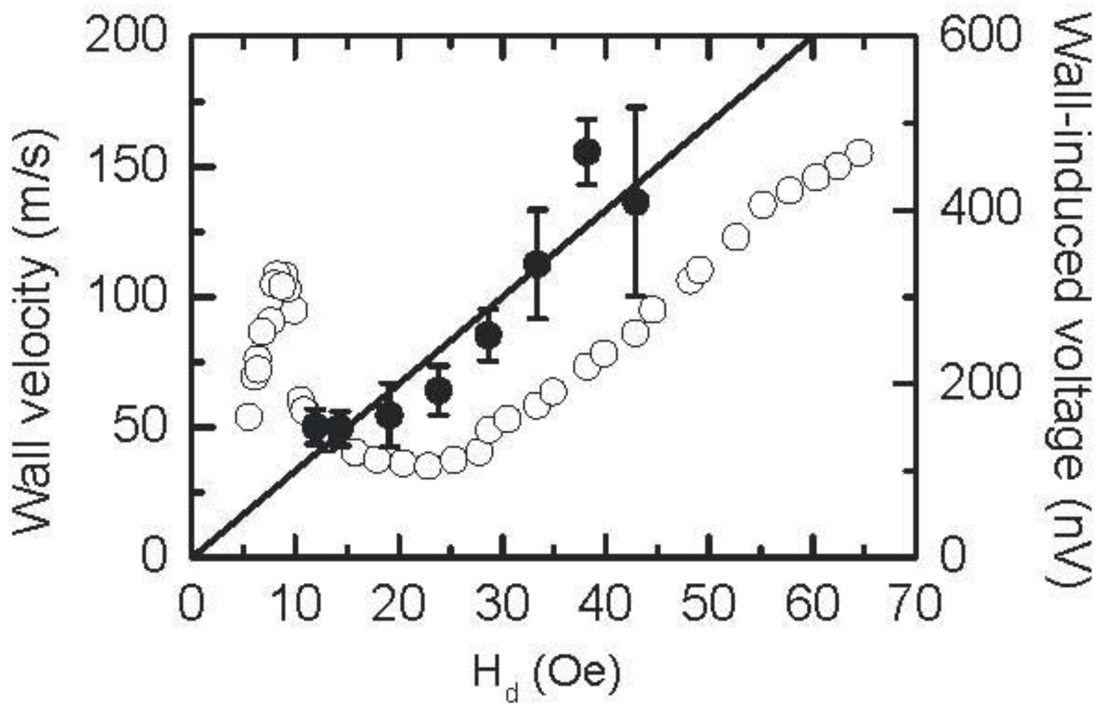


M. Hayashi, et al. Nature Physics (2007)
J.-Y. Lee, et al. cond-mat/07062542 (2007)

Experimental Observation

Shengyuan Yang, G. S. D. Beach, C. Knutson, D. Xiao, Q. Niu, M. Tsoi, and J. L. Erskine, PRL **102**, 067201 (2009);
Shengyuan Yang et al., PRB **82**, 054410 (2010).

$$\bar{V}_x = \frac{\hbar}{e}\gamma H$$



Gradient Forces

- Contrast with adiabatic pump:
  - time vs. position dependence in parameters

$$H[\boldsymbol{r}, \boldsymbol{p}; \beta_1(\boldsymbol{r}, t), \ldots \beta_g(\boldsymbol{r}, t)]$$

- Examples of parameters and gradient forces:
  - Scalar potential – Electrical force
  - Vector potential – Lorentz force
  - Zeeman field – Spin force

$$\dot{\mathbf{r}} = \frac{\partial \mathcal{E}}{\hbar \partial \mathbf{k}} - (\Omega_{\mathrm{kr}} \cdot \dot{\mathbf{r}} + \Omega_{\mathrm{kk}} \cdot \dot{\mathbf{k}}) - \boldsymbol{\Omega}_{\mathrm{k}t}$$

$$\dot{\mathbf{k}} = -\frac{\partial \mathcal{E}}{\hbar \partial \mathbf{r}} + (\Omega_{\mathrm{rr}} \cdot \dot{\mathbf{r}} + \Omega_{\mathrm{rk}} \cdot \dot{\mathbf{k}}) + \boldsymbol{\Omega}_{\mathrm{r}t}$$

Gradient forces

- Applications:
  - Density response: polarization
  - Current response: inverse spin Hall effect

Dynamics under a gradient force

For simplicity, assume 1D & time-independent

$$\beta(x,t) = \beta(x)$$

Equations of motion

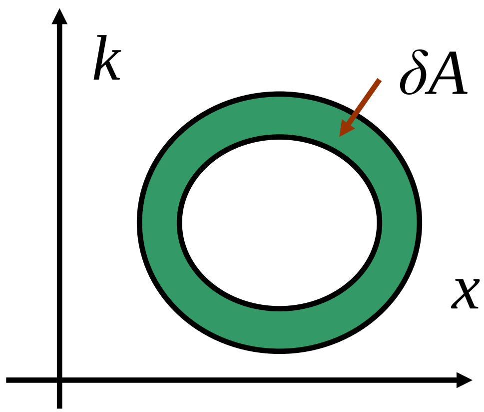
$$\dot{x} = \frac{\partial \varepsilon}{\partial k} - \Omega_{kx}\dot{x} \ , \qquad \dot{k} = -\frac{\partial \varepsilon}{\partial x} + \Omega_{xk}\dot{k}$$

Gradient correction in energy

$$\varepsilon = \varepsilon_0 + \Delta\varepsilon \ , \qquad \Delta\varepsilon = -\Im\Big[\Big\langle\frac{\partial u}{\partial x}\Big|(\varepsilon - H)\Big|\frac{\partial u}{\partial k}\Big\rangle\Big]$$

Wilkinson-Rammal term

Modified Density of States



Semi. Quantization $\oint k\, dx = 2\pi(n + \frac{1}{2} - \frac{\gamma_B}{2\pi})$

Berry phase

$$\gamma_B = \int dx dk\, \Omega_{kx}$$

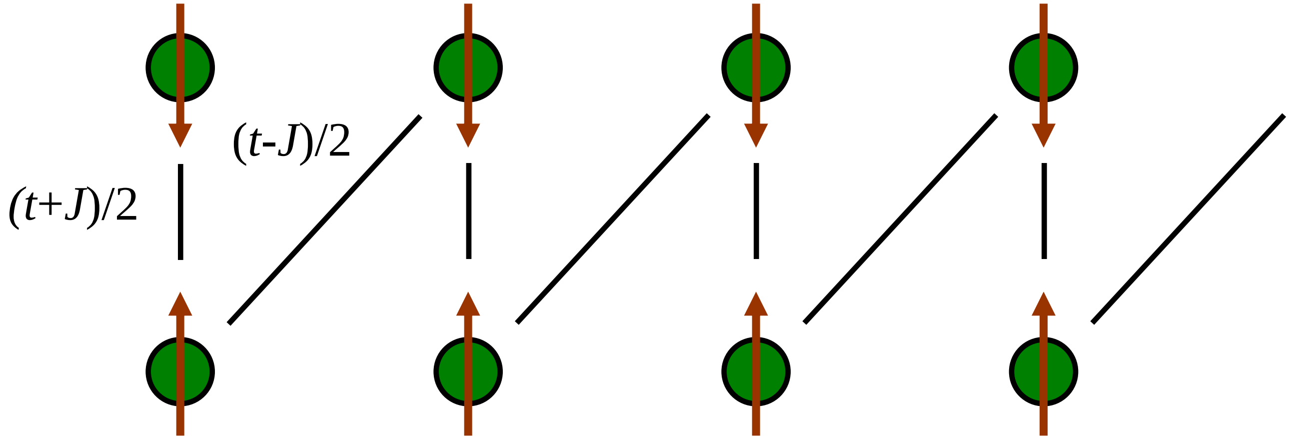
$$\delta A = \frac{2\pi}{1 + \bar{\Omega}_{kx}}$$

Phase-space volume of a quantum state

Average Berry curvature over δA

Antiferromagnetic Spin Chain: multiferroic application

(Quasi 1D Ferroelectric)



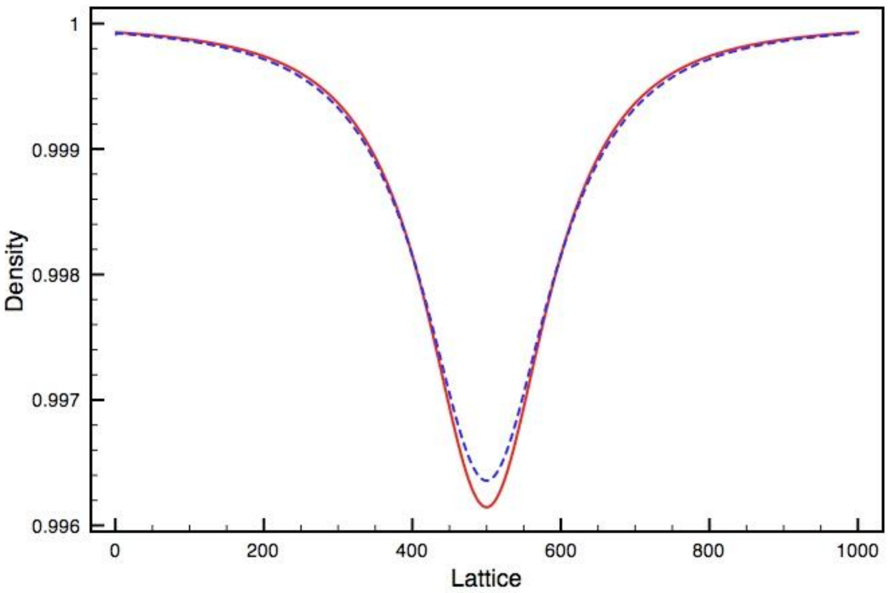
Hamiltonian $$H(k,x) = t\cos\frac{k}{2}\sigma_x + J\sin\frac{k}{2}\sigma_y + h(x)\sigma_z$$

$$\Omega_{kx} = \frac{tJ\nabla h}{2(h^2 + t^2\cos^2\frac{k}{2} + J^2\sin^2\frac{k}{2})^{3/2}}$$

position-dependent Zeeman field

Electron Density Response

For a filled band (insulator), $t = 2$, $J = 1$, h: -5 →5; 1000 sites



Theory (solid line)

$$n = \int_{-\pi/a}^{\pi/a} \frac{dk}{2\pi}\Big(1 + \frac{tJ\nabla h}{2(h^2 + t^2\cos^2\frac{k}{2} + J^2\sin^2\frac{k}{2})^{3/2}}\Big)$$

$$\Omega_{kx}$$

Polarization from Charge Accumulation

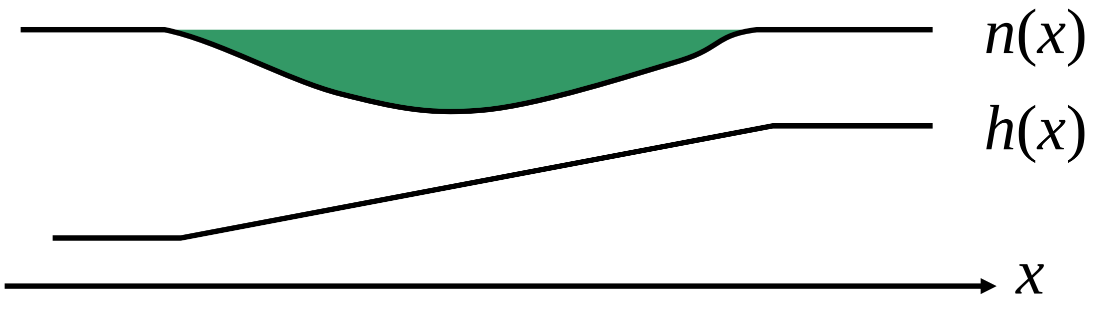
$$\nabla \cdot \mathcal{P}(\boldsymbol{r}) = -\rho(\boldsymbol{r}) = \delta n(\boldsymbol{r})\, e$$

A new derivation of polarization formula

$$\Delta\mathcal{P} = \int_{x_1}^{x_2} e\, dx\, \delta n = e \int_{x_1}^{x_2} dx \int_0^{2\pi/a} \frac{dk}{2\pi} \Omega_{kx}$$

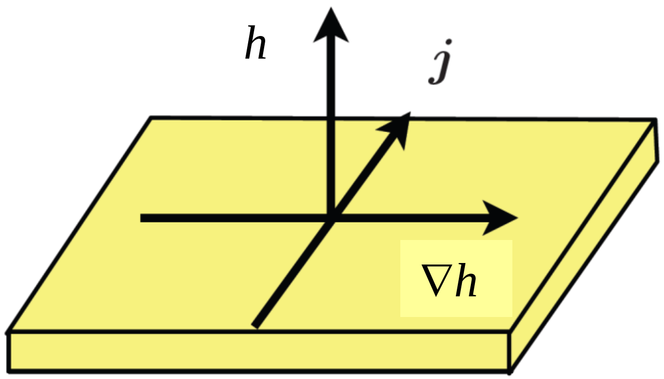
$$= e \int_{h_1}^{h_2} dh \int_0^{2\pi/a} \frac{dk}{2\pi} i \left[\left\langle \frac{\partial u}{\partial k} \middle| \frac{\partial u}{\partial h} \right\rangle - \left\langle \frac{\partial u}{\partial h} \middle| \frac{\partial u}{\partial k} \right\rangle \right]$$

Charge pumping in space



Current response: inverse spin Hall effect

- Charge Hall current driven by the gradient of a Zeeman field
- Reciprocal of the spin Hall effect
- Avoids the problem of measuring spin current directly



2D Model and Berry Curvatures

2D electrons in an asymmetric quantum well

$$H = \frac{k^2}{2m} + \alpha(\boldsymbol{k} \times \boldsymbol{\sigma}) \cdot \hat{z} + h(\boldsymbol{r})\sigma_z$$

Rashba spin-orbit coupling

Zeeman field

Berry curvatures

$$\Omega_{k_x,k_y} = \pm\frac{\alpha^2 h}{2\Delta^3}\,, \qquad \Omega_{k_x,h} = \mp\frac{\alpha^2 k_y}{2\Delta^3}\,, \qquad \Omega_{k_y,h} = \pm\frac{\alpha^2 k_x}{2\Delta^3}$$

$$\Delta \equiv \sqrt{\alpha^2 k^2 + h^2}.$$

Dynamics and Transport

Motion under a spin force $F = \nabla_x h$

$$\hbar \dot{x} = (1 - F\Omega_{k_x,h})\partial_{k_x}\varepsilon + F\partial_{k_x}\delta\varepsilon \; ,$$
$$\hbar \dot{y} = \partial_{k_y}\varepsilon + F\partial_{k_y}\delta\varepsilon - F\Omega_{k_x,k_y}\partial_h\varepsilon - F\Omega_{k_y,h}\partial_{k_x}\varepsilon;$$

Transport charge current

$$j = -e\int d\boldsymbol{k}\, D(\boldsymbol{r},\boldsymbol{k})g(\boldsymbol{r},\boldsymbol{k})\dot{\boldsymbol{r}} - e\boldsymbol{\nabla}\times\int d\boldsymbol{k}\, \boldsymbol{\Omega}_{\boldsymbol{k}}(\mu - \varepsilon)$$

Current from equation of motion

Current from orbital magnetization

Xiao, Yao, Fang & Niu, PRL (2006)

Inverse Spin-Hall Current in Metals

Charge current due to spin force *F*

$$j_y = -\frac{e}{\hbar} F \int^{\mu} [d\boldsymbol{k}] (\Omega_{k_x,h}\partial_{k_y}\varepsilon - \Omega_{k_y,h}\partial_{k_x}\varepsilon + \partial_h \Omega_{k_x,k_y}\varepsilon - \partial_h \Omega_{k_x,k_y}\mu - \Omega_{k_x,k_y}\partial_h \mu + \partial_{k_y}\delta\varepsilon) \; ,$$

for Rashba model $$j_y = -\frac{e}{8\pi} F$$

Spin current due to electric force E_y

$$j_x^s = \frac{e}{8\pi} E_y$$

Onsager relation is satisfied: Shi, Zhang, Xiao & Niu, PRL (2006)

Inverse Spin-Hall Current in Insulators

For a filled band,

$$j_y = \frac{e}{\hbar} F \partial_h (\mu \int_{BZ} [d\boldsymbol{k}] \Omega_{k_x,k_y}) = \frac{e}{h} C F \partial_h \mu$$

- Chern number C *and* chemical potential gradient

- These conclusions remain true for multiple bands, where C should be regarded as the total Chern number.

- If there are species of electrons with different chemical potentials, as in the Kane-Mele graphene model,

$$j_y = \frac{e}{h} F \sum_\alpha C_\alpha \partial_h \mu_\alpha$$

Summary: magneto-electronic applications

- Artificial gauge fields in magnetic textures
  - Spin Faraday effect
- Dynamics under a gradient force
  - Density response: polarization
  - Current response: inverse spin Hall effect

$$\dot{\mathbf{r}} = \frac{\partial \mathcal{E}}{\hbar \partial \mathbf{k}} - (\Omega_{\mathbf{kr}} \cdot \dot{\mathbf{r}} + \Omega_{\mathbf{kk}} \cdot \dot{\mathbf{k}}) - \mathbf{\Omega}_{\mathbf{k}t}$$

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