Lecture 4. 半经典理论框架

1. 波包.

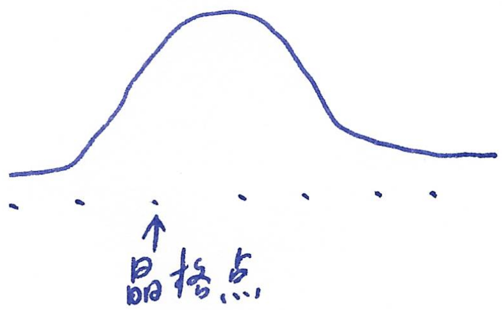
$$|W_n\rangle = \int d^3\vec{k}\; C(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}\, |u_n(\vec{k})\rangle$$

$|C|^2 \sim \delta(\vec{k}-\vec{k}_c)$

局域在布里渊区的某个 $\vec{k}_c$ 点附近.

可理解为展宽很小的高斯波包. $\rightarrow \vec{k}_c$

实空间较宽



2. 运动方程.

$\vec{r}_c = \langle W|\vec{r}|W\rangle \longrightarrow \vec{r}_c$ $\quad$ $(\vec{r}_c, \vec{k}_c)$ 构成相空间.

$$\dot{r}_c = \frac{\partial \varepsilon}{\hbar \partial k_c} - \dot{k}_c \times \Omega$$

$$\hbar \dot{k}_c = -\frac{\partial \varepsilon}{\partial r_c} - e\,\dot{r}_c \times \vec{B}$$

$$\varepsilon = \varepsilon_n(\vec{k}_c) + \delta\varepsilon(k_c; \vec{r}_c; \vec{B})$$

线性阶: $\delta\varepsilon_c = -\vec{B}\cdot\vec{m}$ 磁矩.

非线性阶: $\delta\varepsilon_c = \alpha_{ij} E_i E_j + \beta_{ij} E_i B_j + \gamma_{ij} B_i B_j + \cdots$

$\vec{\Omega} = \vec{\Omega}_n(k_c) + \delta\vec{\Omega}$, $\quad \delta\vec{\Omega} = \overleftrightarrow{\alpha}\cdot\vec{E} + \overleftrightarrow{\beta}\cdot\vec{B}$

3. 平衡态性质.

自由能

$$F=\int\frac{d^3k}{(2\pi)^3}D\,g(\varepsilon)$$

态密度. 准确的能量. , $g(\varepsilon)=-k_BT\ln(1+e^{\frac{\mu-\varepsilon}{kT}})$ 自由能密度.

$M=-\frac{\partial F}{\partial B}$. 磁化强度

$\chi=-\frac{\partial^2 F}{\partial B\partial B}$ 磁化率.

4. 输运性质.

$$\vec{J}=-e\int\frac{d^3k}{(2\pi)^3}D\dot{r}_c\,f$$

分布函数.

5. 态密度如何求?

相空间体积元. $\Delta V=\Delta r\,\Delta k$

一维. $\Delta V=\Delta x\,\Delta k_x$

$$\frac{d\Delta V}{dt}=\Delta\dot{x}\,\Delta k_x+\Delta x\,\Delta\dot{k}_x$$

$$=(\partial_x\dot{x})\Delta x\Delta k_x+\Delta x\Delta k_x(\partial_{k_x}\dot{k}_x)$$

$$=(\partial_x\dot{x}+\partial_{k_x}\dot{k}_x)\Delta x\Delta k_x$$

三维: $\frac{d\Delta V}{dt}=(\nabla_r\cdot\dot{r}+\nabla_k\cdot\dot{k})\Delta r\Delta k$

$$\Rightarrow\frac{1}{\Delta V}\frac{d\Delta V}{dt}=\nabla_r\cdot\dot{r}+\nabla_k\cdot\dot{k}$$

代入运动方程.

$\nabla_r \cdot \dot{r} + \nabla_k \cdot \dot{k}$

$= \nabla_r \cdot (\frac{\partial \varepsilon}{\hbar \partial k} - \dot{k} \times \Omega) + \nabla_k \cdot (-\frac{\partial \varepsilon}{\hbar \partial r} - \frac{e}{\hbar} \dot{r} \times B)$

$= - \nabla_r \cdot (\dot{k} \times \Omega) - \frac{e}{\hbar} \nabla_k \cdot (\dot{r} \times B)$

$= - \nabla_r \cdot (-\frac{\partial \varepsilon}{\hbar \partial r} \times \Omega - \frac{e}{\hbar} (\dot{r} \times B) \times \Omega) - \frac{e}{\hbar} \nabla_k \cdot (\frac{\partial \varepsilon}{\hbar \partial k} \times B - (\dot{k} \times \Omega) \times B)$

Ω不与r相关, B不与k相关.

$\downarrow$

$= (\nabla_r \times \frac{\partial \varepsilon}{\hbar \partial r}) \cdot \Omega - \frac{e}{\hbar} (\nabla_k \times \frac{\partial \varepsilon}{\hbar \partial k}) \cdot B$

$+ \frac{e}{\hbar} \nabla_r \cdot [(\dot{r} \cdot \Omega) B - (B \cdot \Omega) \dot{r}] + \frac{e}{\hbar} \nabla_k \cdot [(\dot{k} \cdot B) \Omega - (B \cdot \Omega) \dot{k}]$

$= \frac{e}{\hbar} \nabla_r \cdot [(\frac{\partial \varepsilon}{\hbar \partial k} \cdot \Omega) B - (B \cdot \Omega) \dot{r}] + \frac{e}{\hbar} \nabla_k [-(\frac{\partial \varepsilon}{\hbar \partial r} \cdot B) \Omega - (B \cdot \Omega) \dot{k}]$

$= \frac{e}{\hbar^2} (\frac{\partial \varepsilon}{\partial k} \cdot \Omega) \overset{=0}{\nabla_r \cdot B} - \frac{e}{\hbar^2} (\frac{\partial \varepsilon}{\partial r} \cdot B) \overset{=0}{\nabla_k \cdot \Omega}$

$- \frac{e}{\hbar} (B \cdot \Omega) (\nabla_r \cdot \dot{r} + \nabla_k \cdot \dot{k}) - \frac{e}{\hbar} \dot{r} \cdot \nabla_r (B \cdot \Omega) - \frac{e}{\hbar} \dot{k} \cdot \nabla_k (B \cdot \Omega)$

$\Rightarrow D (\nabla_r \cdot \dot{r} + \nabla_k \cdot \dot{k}) = - \frac{dD}{dt}.$

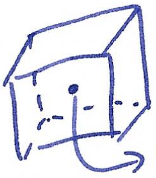
$D = 1 + \frac{e}{\hbar} B \cdot \Omega$

$\Rightarrow \frac{1}{\Delta V} \frac{d \Delta V}{dt} = - \frac{1}{D} \frac{dD}{dt} \Rightarrow \frac{d}{dt} (D \Delta V) = 0$

$\Rightarrow D \Delta V$ 守恒.

态数目正比于 $D \Delta V$

6. 分布函数. ?



→ 相空间元中的粒子数.

$(\vec{r},\vec{k})$

$\Delta N = f(\vec{r},\vec{k}) \cdot D\Delta r\Delta k$. , f 而引含时.

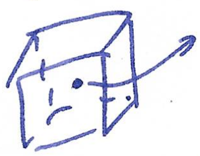
$= f(\vec{r},\vec{k},t)\ D\Delta r\Delta k$.

$$\frac{d\Delta N}{dt} = \frac{df}{dt} D\Delta r\Delta k + f \underbrace{\frac{dD\Delta V}{dt}}_{\to =0}$$

$$\Rightarrow \frac{d\Delta N}{dt} = \frac{df}{dt} D\Delta V$$ → 暂时忽略 $D\Delta V$.

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{r}\cdot\frac{\partial f}{\partial r} + \dot{k}\cdot\frac{\partial f}{\partial k}.$$ → 增多的量 或进入的量.

散射 $(r,k) \to (r',k')$.



一般认为与 r 无关.

$$\left.\frac{df}{dt}\right|_{sc} = \sum_{k'} \Big[\underbrace{W_{kk'} \overset{k\text{有}}{f_k} (1-\overset{k'\text{空}}{f_{k'}})}_{\text{出}} - \underbrace{W_{k'k} \overset{k'\text{有}}{f_{k'}} (1-\overset{k\text{空}}{f_k})}_{\text{进}} \Big]$$

$$\Rightarrow \frac{\partial f}{\partial t} + \dot{r}\cdot\frac{\partial f}{\partial r} + \dot{k}\cdot\frac{\partial f}{\partial k} = -\sum_{k'} \left[W_{kk'} f_k (1-f_{k'}) - W_{k'k} f_{k'} (1-f_k) \right]$$

迟豫时间近似

$$\frac{\partial f}{\partial t} + \dot{r}\cdot\frac{\partial f}{\partial r} + \dot{k}\cdot\frac{\partial f}{\partial k} = -\frac{f-f_0}{\tau}.$$

f_0 平衡分布 (费米分布).